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# Helical Ionization Cooling

**Field:** solenoid + helix — dipole / quadrupole / sextupole (same period $2\pi/\kappa$)

$\kappa_c^{-1} = \frac{p_z c}{eB}$

$\kappa \sim \kappa_c$

**Wedge absorbers**

**Features:**

- helical orbits $\vec{a}(p, \kappa z)$
- dispersion introduced: $D = \frac{p}{a}\frac{da}{dp}$
- continuous emittance exchange (ideally...)
- conservative focusing (ideally)
- 3-dimensional cooling

**Issues:**

- helix design
- how to combain with RF ?
- wedges design

<u>Possible substitution</u>: quasi-resonance dipoles, separated (at $\frac{\Delta p}{p} \ll 1$)

$|\kappa_c \quad \kappa| \ll \kappa$

<u>Possible arrangement of a real helix</u>:

- sweeping a large energy spread <u>at no RF</u> -

<u>Helical orbit:</u>


$$\kappa a = \frac{\alpha_0}{1 - \frac{\kappa}{\kappa_0} + \frac{\kappa}{\kappa_0} g + \frac{(\kappa a)^2}{2}}; \qquad \alpha_0 = \frac{b_\varphi}{B}$$

$$\kappa_0 = \frac{eB}{pc}$$

$$\frac{p_\perp}{p_\parallel} \equiv \theta = \kappa a \qquad g = \frac{1}{p\kappa^2} \frac{\partial b_\varphi}{\partial \rho}$$

<u>Dispersion:</u>

$$D_{ds} \equiv \frac{p}{a} \frac{da}{dp} = \frac{\kappa}{\kappa_0} \frac{\theta}{\alpha_0 + \theta^3}$$

<u>Helical tunes:</u> (at $\theta \ll 1$) $Q_+$, $Q_-$; (helical frame)

$$Q_\pm^2 = R \pm \sqrt{R^2 - G};$$

$$R \approx \frac{1+q^2}{2}; \qquad G = (q - g) D^{-1}$$

$$q = \frac{\kappa_0}{\kappa} - 1$$

<u>Stability area:</u>

$0 < G < R^2$

$Q_+$ — drift mode

$Q_-$ — cyclotron mode (genetic)

**Emittance exchange:**

$$\Lambda_\gamma \equiv \frac{\partial}{\partial \gamma}\langle \gamma' \rangle \approx \left[-\frac{2}{\gamma^2} + \left(\frac{a\nabla n}{n} + \frac{\theta^2}{1+\theta^2}\right)D\right]\Lambda_0$$

$$\Lambda_0 = \frac{E'}{E}\sqrt{1+\theta^2}$$

**Wedge models:**

a) axial: $n(\rho)$;

$$\nabla n = \partial n / \partial \rho$$

$$\rho \approx a + \frac{\vec{a}}{a}\vec{\xi}\,; \quad \vec{\xi} = \vec{\xi}_+ + \vec{\xi}_-$$

b) helical wedges:

$$n = n_0 + \nabla n \cdot \frac{\vec{a}}{a}\vec{\rho}$$

**Transverse cooling:**

$$\frac{\Lambda_+ + \Lambda_-}{\Lambda_0} \approx 2 - \left(\frac{a\nabla n}{n} + \frac{\theta^2}{1+\theta^2}\right)D$$

$$\frac{\Lambda_+ - \Lambda_-}{\Lambda_0} = \frac{1}{\sqrt{R^2 - G}}\left[\frac{q^2-1}{1+\theta^2} + (RD-1)\left(\frac{a\nabla n}{n} + \frac{\theta^2}{1+\theta^2}\right)\right]$$

(cooling at $\Delta p \ll p$)

Options: 1) no quadrupole (drift effect)

2) with quadrupole

## Sweeping a large energy spread (initial beam)

- make $\Lambda_+ = \Lambda_- = 0$ (or, even negative)

  then $\Lambda_\gamma = \Lambda_6$ (or, even larger)