MuCool Notes

Ionization Cooling on Spiral Orbit

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Note 9

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Sweeping Method to Cool a Large Energy Spread of Initial Muon Beam

Introduction

In this note is treated a possibility to cool a large initial energy spread of muons immediately after the decay channel, i.e. before the phase rotation and beam capture by RF field, — applying spiral-correlated wedge absorbers. The beam will be substantially decelerated, but particles' energies will concentrate in a relatively narrow area. The trick was described earlier in Ref. [1,2]; the related particle dynamics and arrangement for emittance exchange have been treated in Ref. [3,4]. The formal dynamic foundations were considered rigorously in Ref. [5].

Here, I consider an arrangement as follows.

- no acceleration is involved;
- all the particles to drive have the cyclotron wave length shorter than the helical field period $2\pi/k$:

$$\frac{eB}{c\,p_{max}} > k \quad , \quad (p \leq p_{max}) ;$$

(e = 1, c = 1 in further)

- the dispersion $\partial a/\partial p$ and spiral-correlated absorber gradient $\partial n/\partial \rho \equiv \nabla n$ both are positive;

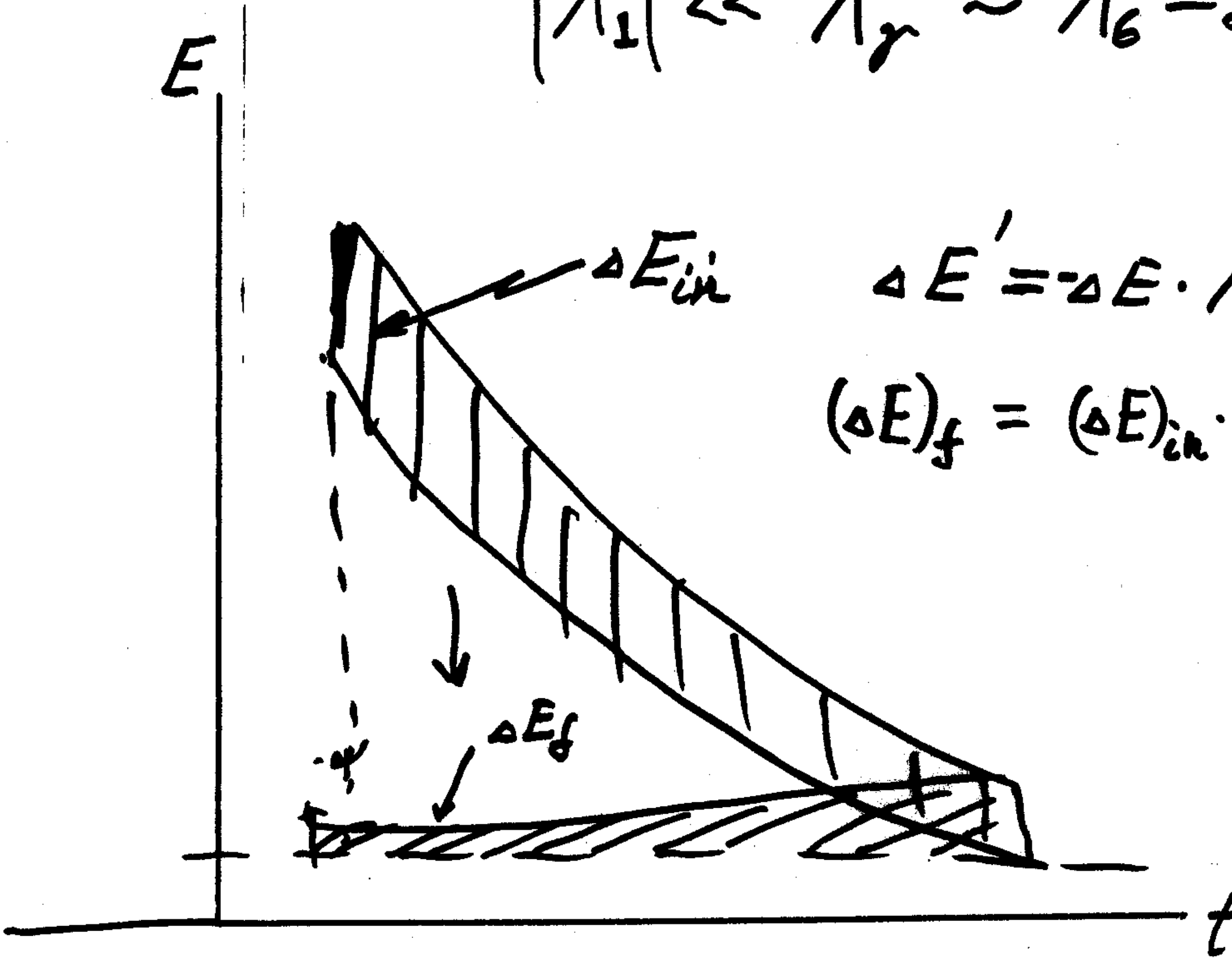
- the optimum behaviour $n(\rho)$ is found under condition of equality between the two transverse decrements (they appear negative but of a modest value);
- solenoid field B ramps (decreases) downstream in tact with the p_{max} deceleration in absorbers:

$$\frac{B(z)}{p_{max}(z)} = \text{Const} ,$$

to maintain the spiral radii $a(p, B)$ and dispersion about near the maximum values.

"Phase rotation" via emittance exchange (+ cooling!)

$$|\Lambda_1| \ll \Lambda_\gamma \approx \Lambda_6 = 2\frac{\gamma'}{\gamma}$$

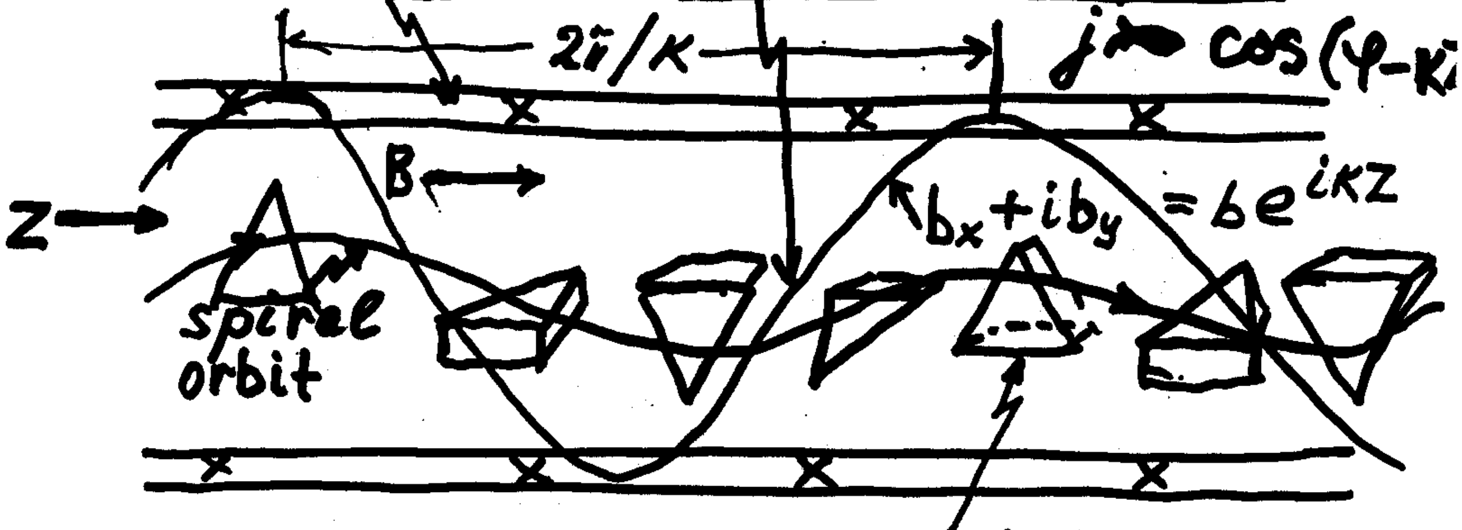


$$\Delta E' = -\Delta E \cdot \Lambda_\gamma$$

$$(\Delta E)_f = (\Delta E)_{in} \cdot \left(\frac{\gamma_{in}}{\gamma_f}\right)^2$$

General arrangement

· Solenoid + helical dipole



$b = b_0(1 + \frac{1}{4}K^2\rho^2 + \ldots)$; $\quad K \sim K_c \equiv \frac{eB}{pc}$

· quad and sext can be added (same period)

· typical values: $B = (6 \to 1 \to 20)T$

$b \geq 2\,kG$

$\alpha \equiv \frac{b}{B} \approx (3 \to 20 \to 3)\%$

Why Spiral Transport?

- dispersion introduced
- conservative focusing
- 3-dim Cooling appears
- field can vary along beam pass

I suggest to simulate such process; some analitical support based on earlier considerations is given below. The limitations will be shown, and numerical estimations will be presented.

1. Spiral orbit in solenoid

Helical dipole field of $\underline{\underline{b}}$ magnitude and $2\pi/\kappa$ period excites a spiral orbit of radius $a(p)$:

$$\kappa a(p) = \frac{b}{B - \kappa p}, \qquad (b \ll B) \qquad (1)$$

where B is solenoid field (particle charge and light velocity assumed unit).
Let

$$B > \kappa p \qquad (2)$$

for a particle with <u>maximum</u> p value:

$$B > \kappa p_{max}, \qquad (3)$$

then the condition in (2) is satisfied for all the particles to be captured. Thus, we avoid resonance between helical field and particles cyclotron oscillation:

$$\frac{B}{p} \equiv \kappa_c > \kappa \,. \qquad (4)$$

Note, that product $\kappa a(p)$ is spiral angle, θ:

$$\theta = \kappa a \,; \qquad (5)$$

here we assume

$$\theta \ll 1 \,. \quad (\text{at the beginning}) \qquad (6)$$

More correctly, angle θ is defined by equation

$$\theta = \frac{b}{B - \kappa p\left(1 - \frac{\theta^2}{2}\right)} ; \qquad (7)$$

thus, non-resonance condition means that

$$\frac{\kappa_0}{\kappa} - 1 \equiv \frac{B}{\kappa p} - 1 \equiv q \gg \left(\frac{b}{B}\right)^{2/3} ; \qquad (8)$$

then

$$\theta \ll \left(\frac{b}{B}\right)^{1/3} \quad \left(\text{and, } \theta_\varepsilon \ll \left(\frac{b}{B}\right)^{1/3} \; (9)\right)$$

Under condition in (8), the solution in (7) can be found as

$$\theta = \frac{\alpha}{1 - \frac{\kappa p}{B}} = \alpha + \frac{\alpha}{q} ; \; \alpha \equiv \frac{b}{B} . \qquad (10)$$

2. Dispersion

Dispersion $\hat{D}$ is defined as

$$\hat{D} = \frac{p}{a} \cdot \frac{da}{dp} ;$$

thus, here is

$$\hat{D}^{-1} \approx \frac{B}{\kappa p} - 1 = q \qquad (11)$$

3. Transverse stability

Both the components of particle state of motion in solenoid, drift and cyclotron, are stable under non-resonance condition in (8) (see Ref. [5]).

4. Transverse and longitudinal decrements in spiral-correlated absorber

Formula in (51) of the previous note (Ref. [5]) defines the condition of equality between transverse decrements, Λ_+ and Λ_- (or Λ_d and Λ_c):

$$\frac{a \nabla n}{n} = 2\,\frac{1+q}{1-q}\,; \qquad (12)$$

taking into account the connection $a(p)$ and q definition in (11), we can integrate the equation in (12), to find the $n(a)$ behavior:

$$n(a) = n_0\left(\frac{\kappa a - 2\alpha}{\kappa a - \alpha}\right)^2, \qquad (13)$$

or considering n as function of p (at a given longitudinal position at spiral orbit), we find

$$n(\rho) = \text{Const}\cdot(1-q)^2 = \text{Const}\cdot\left(2 - \frac{K_c}{K}\right)^2 =$$
$$= \text{Const}\cdot\left(2 - \frac{B}{\rho K}\right)^2 \qquad (14)$$

A largest spiral radius a_{max}, and a largest dispersion D can be achieved at a minimum q value (limited by closeness to resonance):

$$\theta^2 < q_{min} = \frac{B}{K\rho_{max}} - 1 \ll 1 , \qquad (15)$$

while $n(q)$ turns to zero at $q = 1$, i.e. at

$$\rho = \frac{B}{2K} \approx \frac{1}{2}\rho_{max} \qquad (16)$$

Thus, the absorber concentration, $n(\rho)$ (averaged along z), can be defined for the interval

$$\frac{B}{2K} \le \rho(a) \le \approx \frac{B}{K} , \qquad (\rho \approx a) \qquad (17)$$

or, in terms of a,

$$2\alpha \le Ka \le Ka_{max} = \frac{\alpha}{1 - \frac{\rho_{max}}{KB}} \approx \frac{\alpha}{q_{min}} , \qquad (18)$$

with $$1 \le \hat{D} \le \frac{1}{q_{min}} = \frac{\theta_{max}}{\alpha} \qquad (19)$$

Using formula (52) in Ref.[5] (or formulaes for Λ_d and $\Lambda_L \equiv \Lambda_c$ of Ref. [3,4]) for transverse decrements ($\Lambda_+ \Longleftrightarrow \Lambda_d$; $\Lambda_- \rightleftarrows \Lambda_c$), we find:

$$\Lambda_d = \Lambda_c \equiv \Lambda_\perp = -\frac{2A}{\gamma\beta^4} q(1-q) < 0\,, \tag{20}$$

while

$$\Lambda_\gamma = \frac{2A}{\gamma\beta^2}(1-q)^2 + \frac{4A}{\gamma\beta^4} q(1-q) > 2\frac{|\gamma'|}{\gamma} \tag{21}$$

in all the absorber-occupied area $0 < q < 1$;

constant A comes from expression for particle deceleration rate:

$$\gamma' = -\frac{A}{\beta^2}(1-q)^2 = \text{const.}\,\frac{n(\rho)}{\beta^2} \tag{22}$$

5. Deceleration / energy compression process

The spiral orbits get down with particle deceleration in absorbers; in order to prevent particles escape the absorbers area, one has to ramp the solenoid strength (maintaining angle

α constant, or even increasing it keeping helical dipole field, b, constant). The ramping rate B' can be found on the condition

$$\frac{B(z)}{p_{max}(z)} = \text{Const}, \qquad (23)$$

or

$$q_{min} = \frac{B(z)}{\kappa\, p_{max}(z)} - 1 = \text{Const}. \qquad (24)$$

The rate $p_{max}(z)$ can be found then using the deceleration equation (22)

$$(\beta^2\gamma)'_{max} = -A(1-q_{min})^2 \approx -A; \qquad (25)$$
$$(q_{min} \ll 1)$$

solution at $A' = 0$ is

$$\left(\gamma + \frac{1}{\gamma}\right)_{max} = \left(\gamma + \frac{1}{\gamma}\right)_{\substack{max\\ z=0}} - A(1-q_{min})^2 z \qquad (26)$$

The q_{min} to be selected avoiding a too low value. Apparantely, the deceleration/ramping process is stable at

$$-B' < -\kappa p'_{max}. \qquad (27)$$

Note, that A parameter (i.e. n_0 in (13)) also can ramp with z ($n_0 \propto \beta^3_{max}(z)$, for instance).

According to result in (21), the energy compression rate exceeds even the double the deceleration rate; thus, the relative energy spread decreases. Finally, the process can be stopped after that p_{max} reached a reasonable minimum value (say, 100 MeV/c, standing at 0.6 – 0.7 GeV/c; then, the lower absorber edge, n=0, $p(z)_{min} \approx p_{max}(z)/2$ reaches ~ 50 MeV/c).
Particle distribution dN/dp after a deep deceleration is expected to have a rather small effective width near a maximum not far off $(p_{max})_{final}$. The distribution can be plotted solving the equations in (22) and (25).

The local compression factor k can be defined as

$$k = \frac{dp_{in}}{dp_{fin}} = \exp^{-1}\left(\cdot\int_0^z \Lambda_\gamma\, dz\right) =$$

$$= \exp^{-1}\left[-2\int_0^z\left(\frac{\gamma'}{\gamma} + \frac{4A}{\gamma\beta^4}q(1-q)\right)dz\right] =$$

$$= \left(\frac{\gamma_{in}}{\gamma_{fin}}\right)^2 \cdot \exp\left[\int_0^z \frac{4A}{\gamma\beta^4}\, q(1-q)\right]dz \equiv k_6 \cdot k_{extra}, \quad (28$$

where

$$k_6 = \left(\frac{\gamma_{in}}{\gamma_{fin}}\right)^2$$

is a compression factor for a 6dimensional phase space cell, which is not effeted by dispersion ~~and energy~~ loss gradient. A maximum k (at $1-\beta^2 \ll 1$) will be achieved for particles near maximum energy (for which q is minimum, see formula (21)).

6. Transverse antidamping

The transverse decrements (being equal) are negative at positive ∇n (since the drift decrement is negative at $\nabla n > 0$; see Ref. [3], [4]) However, since most of particles concentrate to area $q \ll 1$ ($p \to p_{max}(z)$), the transverse increments effectively appear rather low comparatively to the longitudinal decrement. Thus, one can expect that the transverse heating will not impact dramatically the longitudinal sweeping trick (remind that one does not need RF field in this section). Note also, that the transverse emittance-related beam size grows only in power 1/4 comparatively the energy "extra" compression [1,2].

7. <u>Transverse heating due to angle scatter and struggling.</u>

Here we find for emittance growing up:

$$\Delta \mathcal{E}_c = \frac{m_e}{m} \lambda \left[Z \ln\left(\frac{\gamma_{in}}{\gamma_{fin}}\right) + (\gamma K a / q_{min})^2_{in} / 4 \log \right] \quad (2$$

8. Conditions and limitations (short summary)

8.1. $\lambda \equiv \frac{2\pi}{k} > \lambda_c \equiv 2\pi \frac{pc}{eB}$

assume $\boxed{p_{max} = 0.6\ GeV/c,\ B_{max} = 6\ T}$

then

$$(\lambda_c)_{max} \approx 2.5\,m\ ;$$

thus,

$$\boxed{\lambda \approx 2.5\,m\ ;} \quad \lambdabar = 0.4\,m$$

($\lambda = const$ might be a basic option)

8.2. Field B ramp:

$$\frac{B(z)}{p_{max}(z)} = const$$

assume $(p_{max})_{fin} = 100\ MeV/c$
$(E = 40\ MeV)$
$(\gamma_{max})_{fin} = 1.4$

then

$$B_{fin} \approx 1\,T\ ;$$

thus, $B(z) = (6T \rightarrow 1T)$

8.3. Emittance-related beam size:

$$\sigma_\varepsilon = \sqrt{\frac{\varepsilon \cdot mc^2}{eB}}\ ; \quad (\varepsilon - \text{normalized})$$

$$\boxed{\varepsilon = 1.5\ cm}$$

then

$$\sigma_\varepsilon = (3.5 \longrightarrow 8.5)\ \text{cm}$$

$$\tfrac{1}{2}\theta_\varepsilon^2 \approx \sigma_\varepsilon^2 \kappa^2 = (0.35 \rightarrow 2) \times 0.01$$

8.4. Spiral radius max value

$$\kappa a = \frac{\alpha}{q}\ ; \qquad \alpha \equiv b/B, \quad q \equiv \frac{\lambda_c}{\lambda} - 1$$

$$\kappa a_{max} = \frac{b/B}{\kappa \frac{c p_{max}(z)}{e B(z)} - 1} \equiv \frac{b/B}{q_{min}} \qquad (3($$

- a_{max} should exceed σ_ε substantially; assume

$$a_{max} = 3\sigma_\varepsilon,$$

then $\boxed{a_{max}(z) = (10 \rightarrow 25)\text{cm}.}$

- assume $\boxed{b = \text{const} = 2\ \text{k Gauss}}$;

if $a_{max} \propto 1/\sqrt{B}$, we find:
$q_{min} \propto 1/\sqrt{B}$ (both as ramping functions)

hence, at the:

beginning, $q_{min} \approx 0.13$;

end, $q_{min} \approx 0.33$.

8.5. Adiabatic ramp condition:

since q_{min} changes with z, it should be as low as

$$Kq'_{min} \ll (Kq_{min})^2;$$

since we assumed $q_{min} > 1/\sqrt{p}$, then the condition is

$$\frac{1}{2}\frac{p'}{p} \ll Kq_{min}; \qquad (31$$

assume $\frac{p'}{p} = 2\xi Kq_{min}$, $\xi = Const < 1$

then $p' \sim \sqrt{p}$,

$$(p') = 2\xi K(q_{min} \cdot p_{max})_{in}$$

Let $E'_{in} = 30$ MeV/m (l. hydrogen)

then we find

$$\xi = \frac{30 \cdot 0.4}{2 \cdot 0.13 \cdot 600} \approx 0.08$$

(a good adiabaticity!),

while the total deceleration pass is

$$L_{dec} = \int dz = \int \frac{dE}{E'} = \int \frac{dE}{Const \cdot \sqrt{E}} = \frac{2\sqrt{E_{in}}}{Const} =$$

$$= 2\sqrt{E_{in}} \cdot \frac{\sqrt{E_{in}}}{E'_{in}} = \frac{2E_{in}}{E'_{in}} = 2 \cdot \frac{600}{30} =$$

$$= \boxed{40\ m}.$$

To reduce L_{dec}, one can keep $E' = Const$, then

$$\boxed{L_{dec} \approx 20\,m,}$$

while the $\xi(z)$ value to the end will be equal to

$$\xi_{max} = \left(\frac{p'}{2p\,q_{min}}\right)_{fin} \cdot \frac{1}{K} = \frac{30 \cdot 0.4}{2 \cdot 100 \cdot 0.2} = 0.3;$$

(still a good adiabaticity!).

8.6. Transverse heating (formula (29)):

~~at~~

at $Z = 1$, there is

$$\Delta\mathcal{E}_c \approx 0.8\ cm;$$

at $Z = 4$ (Be): $\Delta\mathcal{E}_c \approx 1.6\ cm$

9. Discussion

Benefits of sweeping emittance exchange:

1) a deep compression of the initial large energy spread seems attainable using the sweeping method of ionization cooling in spiral transport at no RF field. The succeeding phase rotation, if still necessary, would be dramatically simplified; but it might be conceivable to avoid beam longitudinal expansion at all; instead, the bunching process could start immediately after sweeping.

2) The induction linac might not be needed

3) A strong drift path reduction seems real.

4) Bunching process might occure efficient, (since $\frac{\Delta p}{p}$ reduction to 15–10 % seems possible), capturing ~ 80% of particles after decay channel