Emittance exchange in ring coolers

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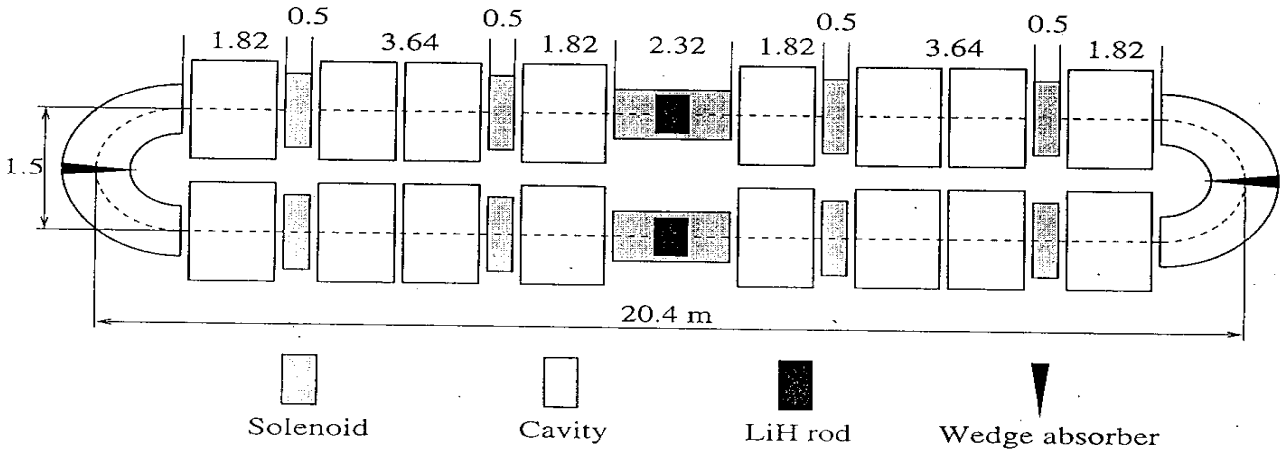
Emittance exchange workshop, BNL, September 2000

Part A: Low frequency ring cooler (RF = 13.5 MHz, single bunch cooling)

A1. Schematic of the cooler (all sizes in meters)



Motivation:

- Cost
- Easy emittance exchange

A2. List of parameters

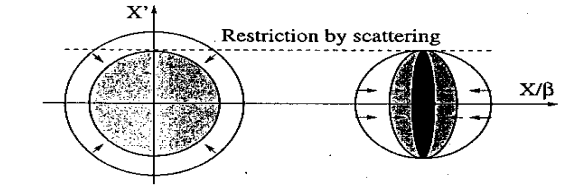
1.	Circumference:	42.47 m
2.	Kinetic energy of muons:	210 – 270 MeV
3.	Revolution frequency:	6.76 MHz
4.	Bending radius:	75 cm
5.	Bending magnets:	field index 0.5
		B = 1.439 T before wedge absorber
		B = 1.489 T after wedge absorber
6.	Bent solenoids:	B = 2.105 T before wedge absorber
		B = –2.035 T after wedge absorber
7.	Central solenoids:	B = –3.315 T, L = 114 cm before absorber
		B = 2.646 T, L = 118 cm after absorber
8.	Short solenoids:	B = 1.95 – 2.36 T, L = 50 cm
9.	Accelerating gradient	12 MeV/m
10.	RF harmonic number:	2
11.	LiH absorber:	L = 36.7 cm
12.	Injected beam:	$\sigma_X = \sigma_Y = 5.5$ cm
		$\sigma_{Px} = \sigma_P = 22$ MeV/c
		$\sigma_Z =$ 150 cm $\sigma_E = 10$ MeV

A3. Matrix of the period

$$M = 0.94\times\ I\times\ (X\text{-}Y \text{ rotation } 22.5^{o})$$

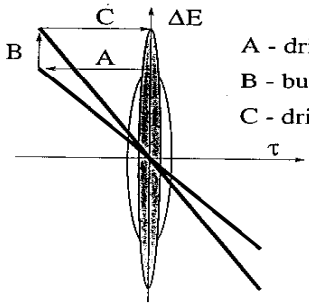
There is no β - function; transverse coordinates and momenta cooled independently

Transverse cooling



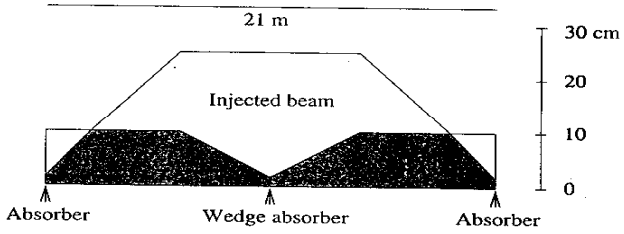
Cooling in an usual periodic system Cooling in the ring

Longitudinal cooling

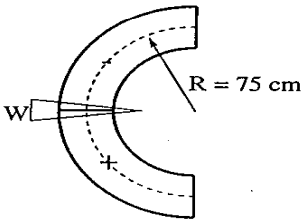


A - drift in straight section
B - bunching
C - drift in bending section

Beam envelope



A4. Bendind part and dispersion

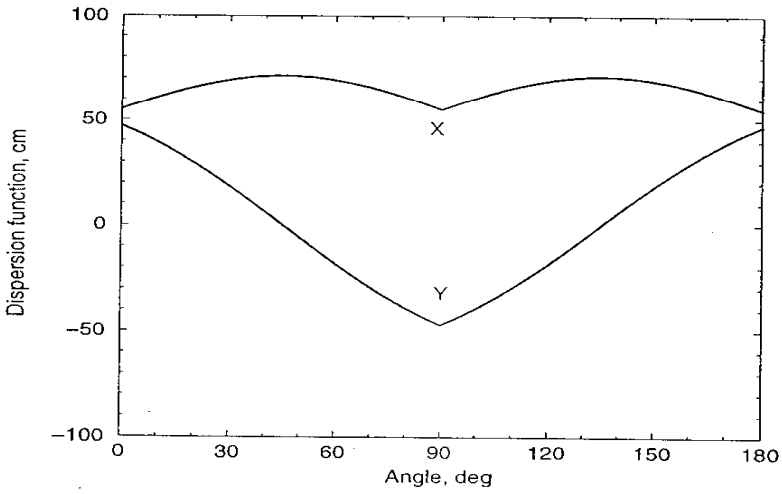


Solenoidal field: B = 2.070 +/- 0.035 T

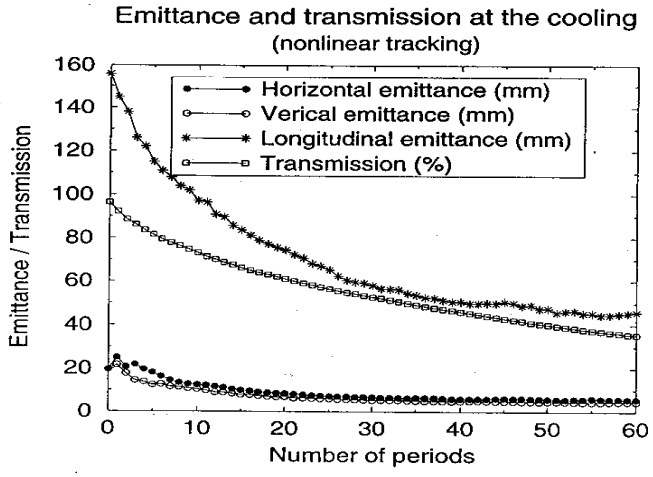
Transverse field: combined functions n = 0.5
B = 1.464 +/- 0.025 T

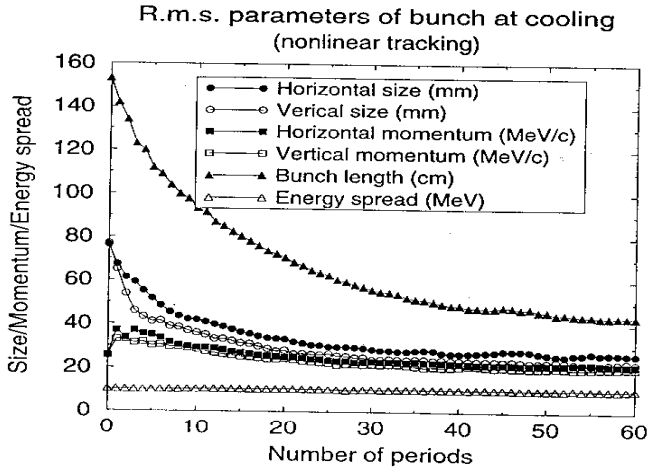
Transverse matrix: M = - I (w/o wedge)

Dispersion function

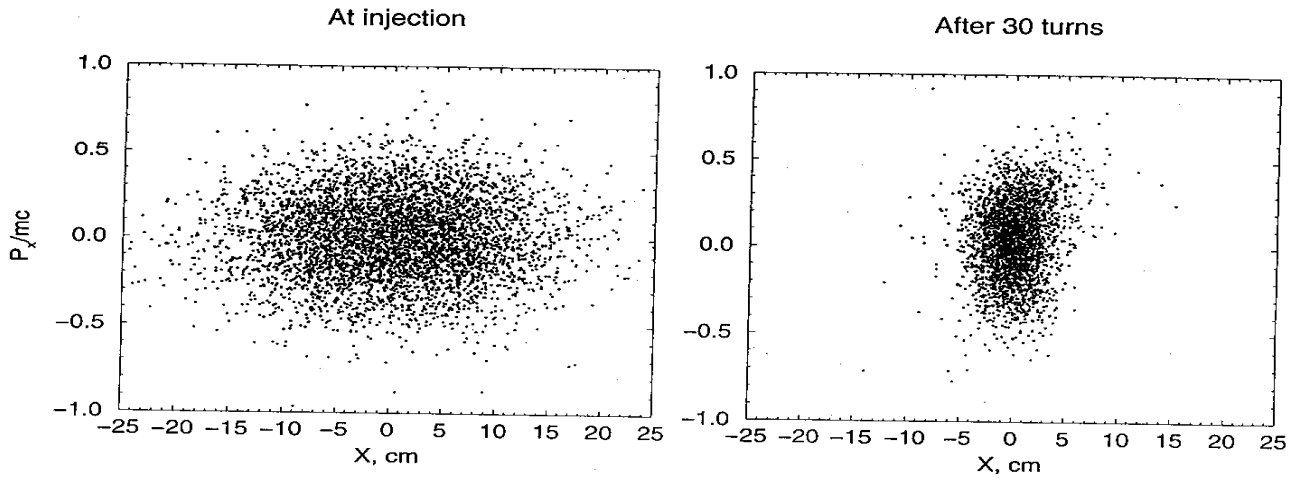


A5. Cooling simulation (nonlinear tracking)

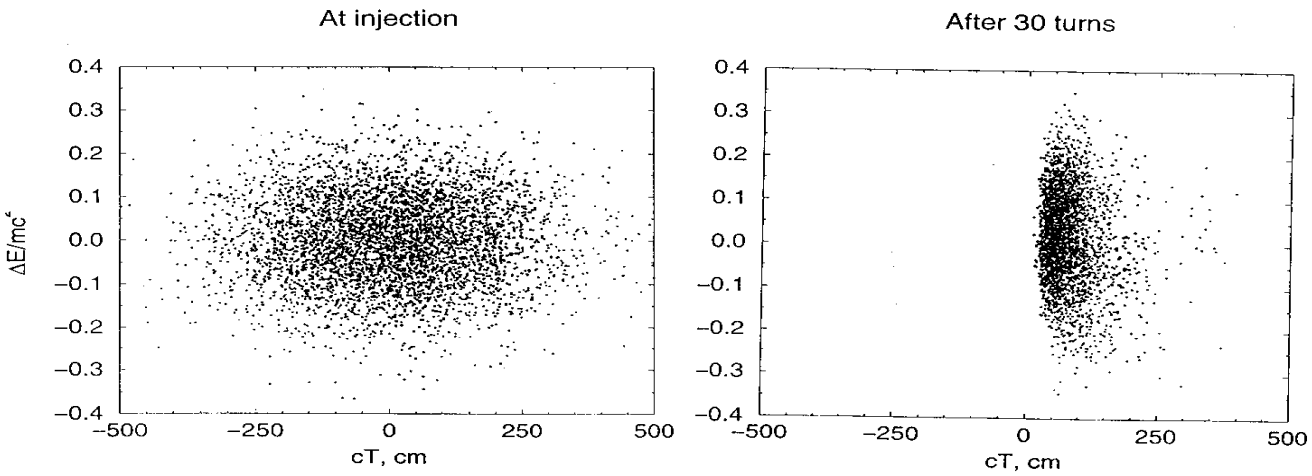




Transverse phase space



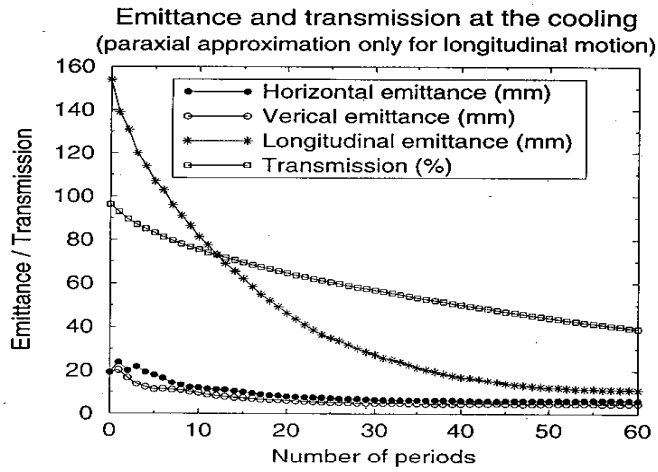
Longitudinal phase space

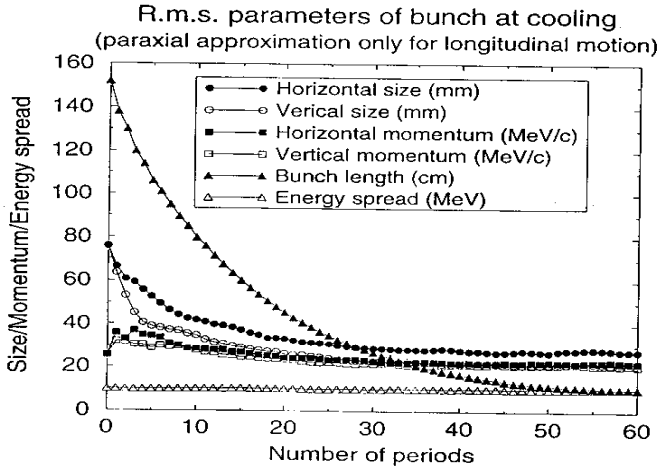


A6. Cooling simulation
(longitudinal motion in paraxial approximation)

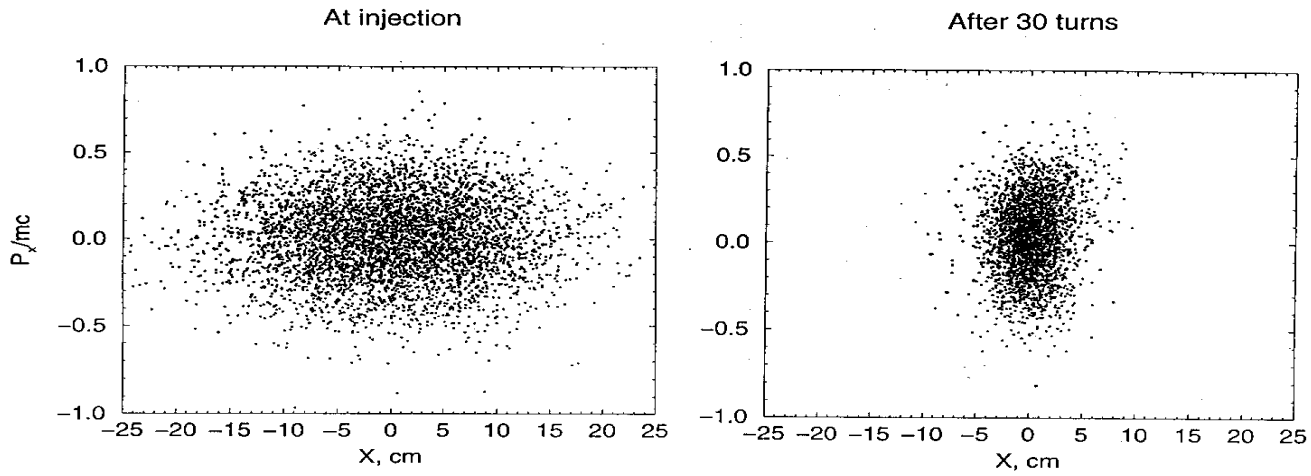
The most dangerous effect is dependence of a trajectory length on angle:

$$\langle L \rangle \approx L_0 \, (1+\langle \theta^2 \rangle/2)$$

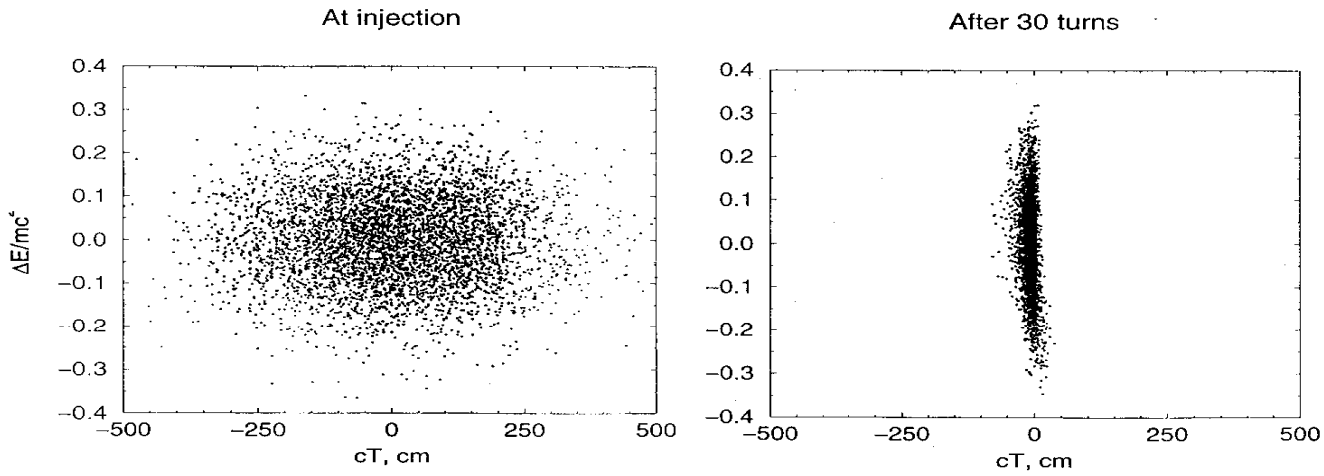




Transverse phase space

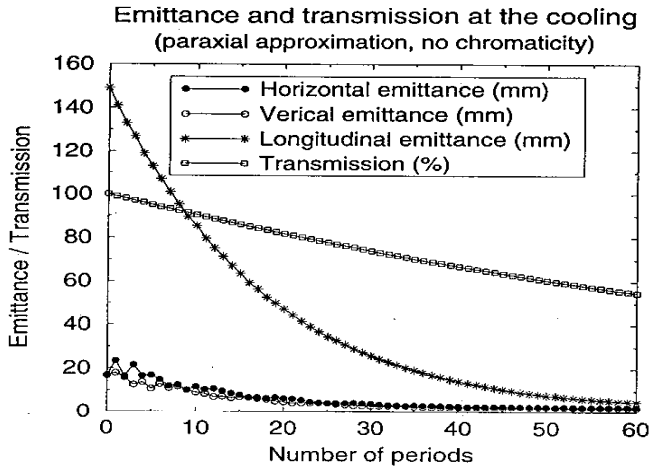


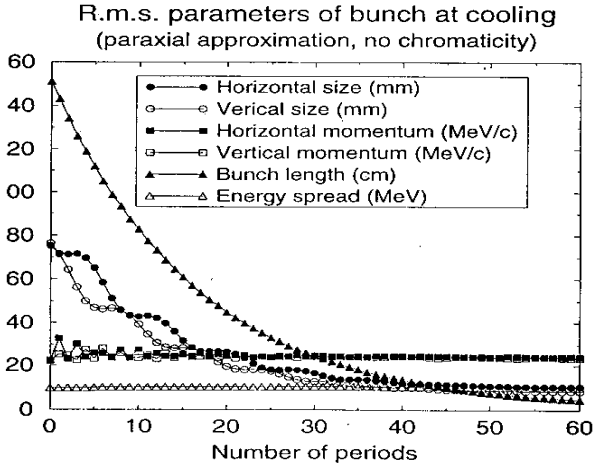
Longitudinal phase space



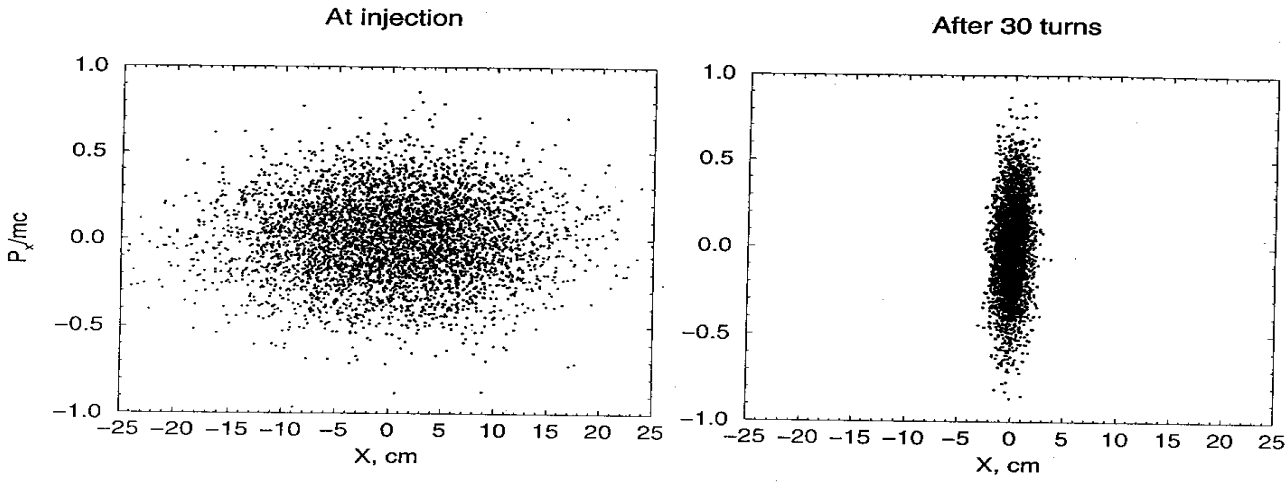
A7. Cooling simulation (paraxial approximation)

Chromaticity and to an less degree nonlinearity cause an increase of transverse emittance. The following results are obtained in linear approximation:

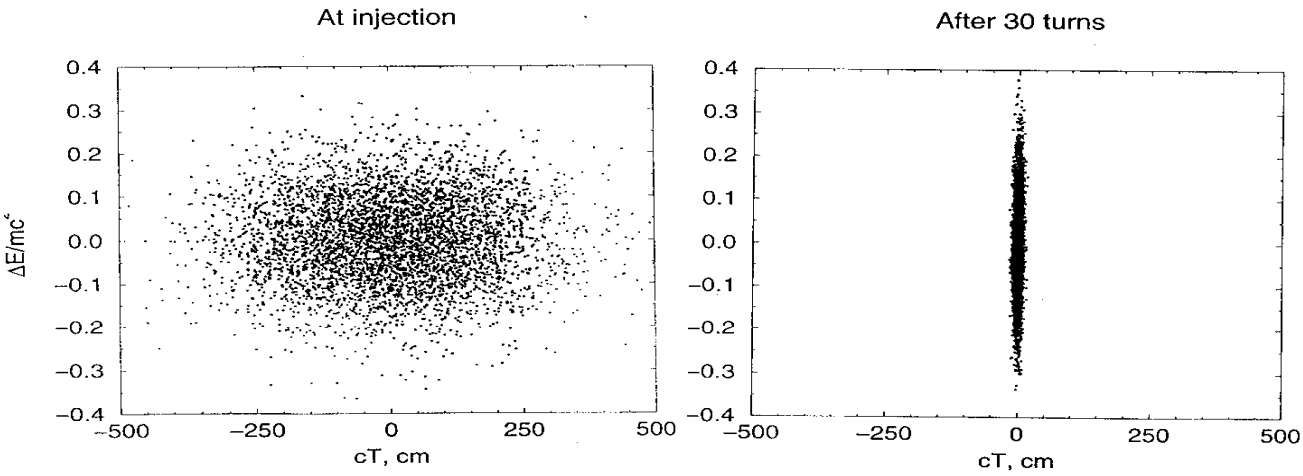




Transverse phase space



Longitudinal phase space

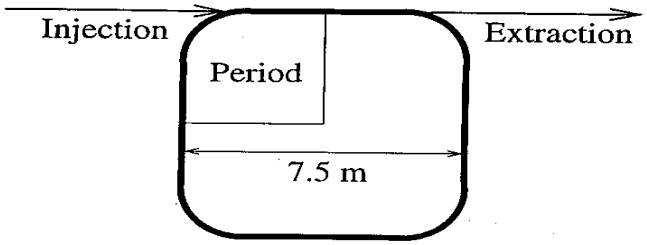


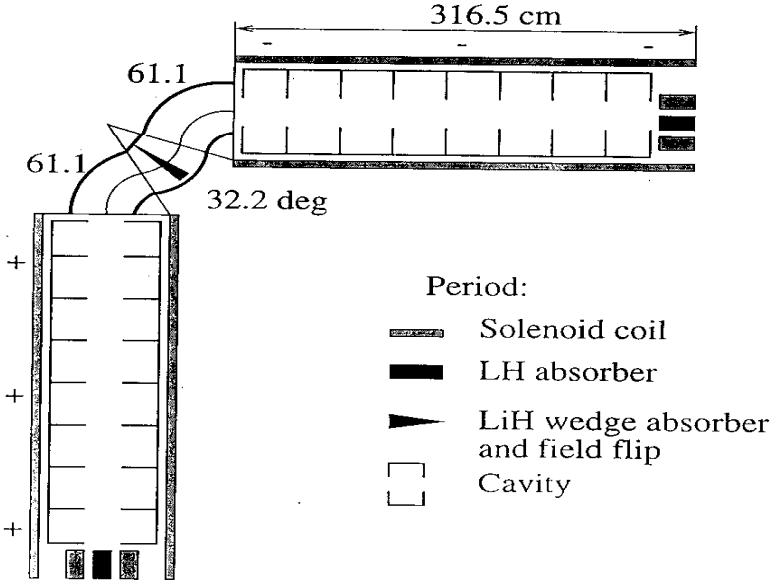
A8. Summary; effect of different factors on the cooling and emittance exchange (beam emittance and transmission after 30 tutns)

	ε_x (mm)	ε_y (mm)	ε_z (mm)	ε_6 (mm^3)	Trans.(%)
Injected beam	12	12	150	21600	100
Paraxial approximation	2.6	2.0	5.0	26	55 (decay)
+ Chromaticity	5.3	4.2	5.8	129	45
+ Transverse nolinearity	6.1	4.6	10	280	40
+ Longitudinal nonlinearity	5.7	4.7	44	1180	36

Part B: High frequency ring cooler (RF = 201.25 MHz, multibunch cooling)

B1. Schematic of the cooler

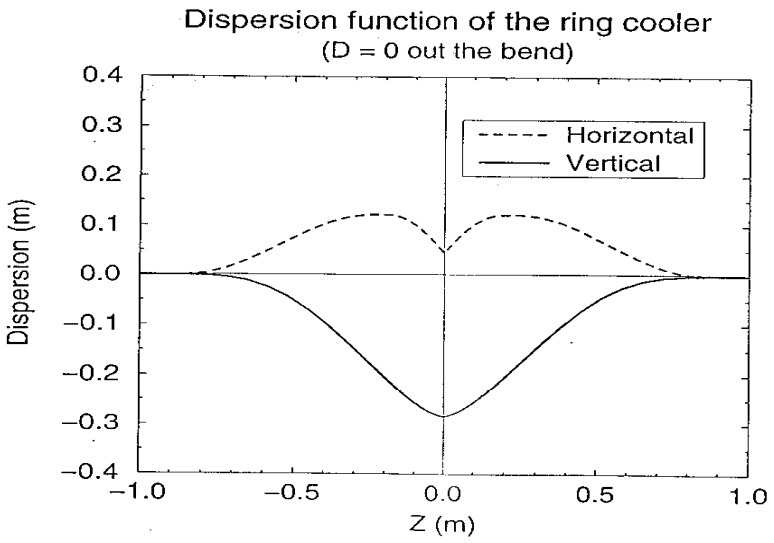




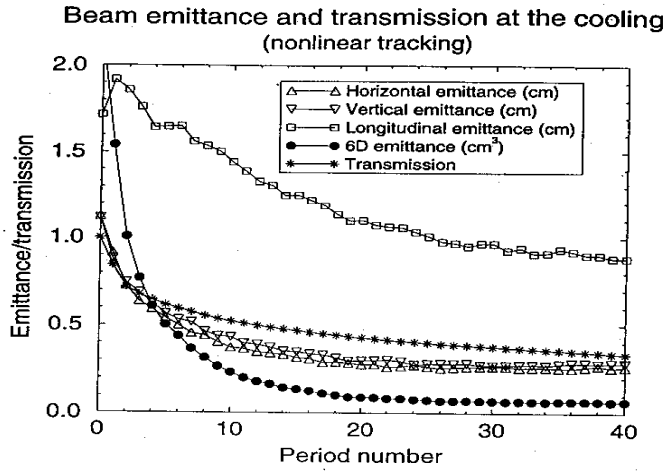
B2. List of parameters

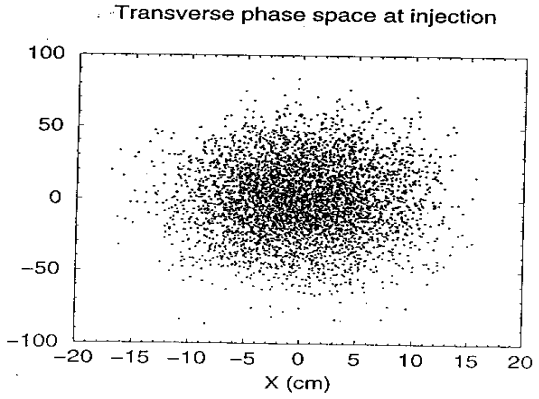
1.	Circumference:	32.19 m
2.	Kinetic energy of muons:	86.1 – 124.5 MeV
3.	Revolution frequency:	8.05 MHz
4.	Bending radius:	63.66 cm
5.	Bending magnets:	B = ± 0.957 T, field index 0.5
6.	Bent solenoids:	B = ± 3.042 T
7.	Central solenoids:	B = ± 3.329 T, L = 3.165 cm
8.	Accelerating gradient	15 MeV/m
9.	RF harmonic number:	25
10.	Main absorber	LH, 120 cm
11.	Wedge absorber:	LiH, dE/dy = 1 MeV/cm
12.	Injected beam:	$\sigma_X = \sigma_Y = 5$ cm $\sigma_{Px} = \sigma_P = 25$ MeV/c $\sigma_Z =$ 10 cm $\sigma_E = 16$ MeV
13.	β - function	36.6 cm at central energy

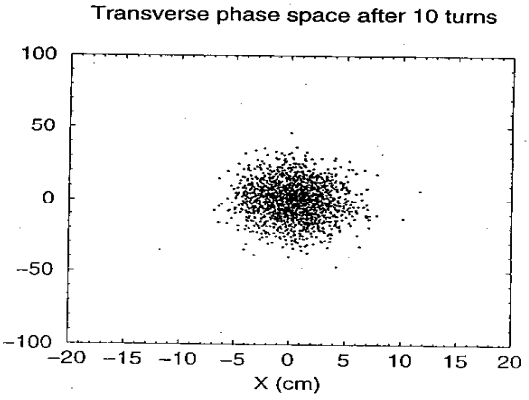
B3. Dispersion function (D = 0 out the bend)

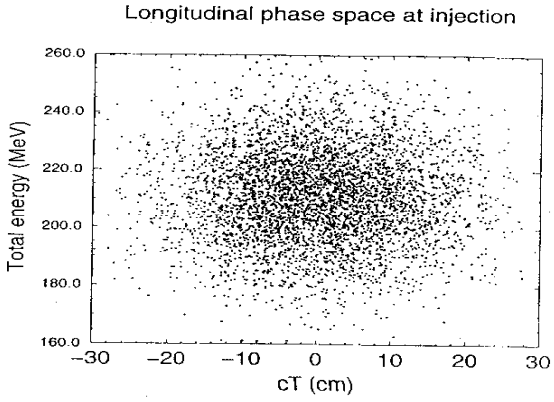


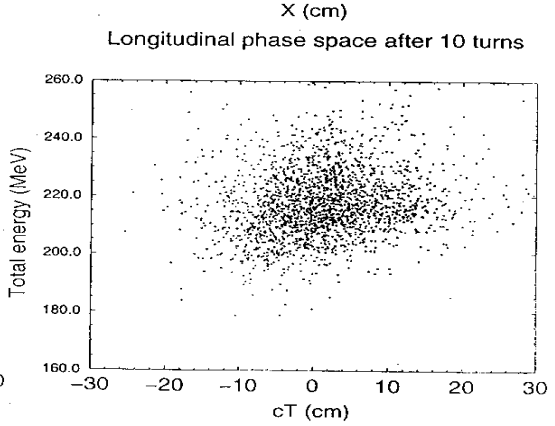
B4. Cooling simulation (nonlinear tracking)











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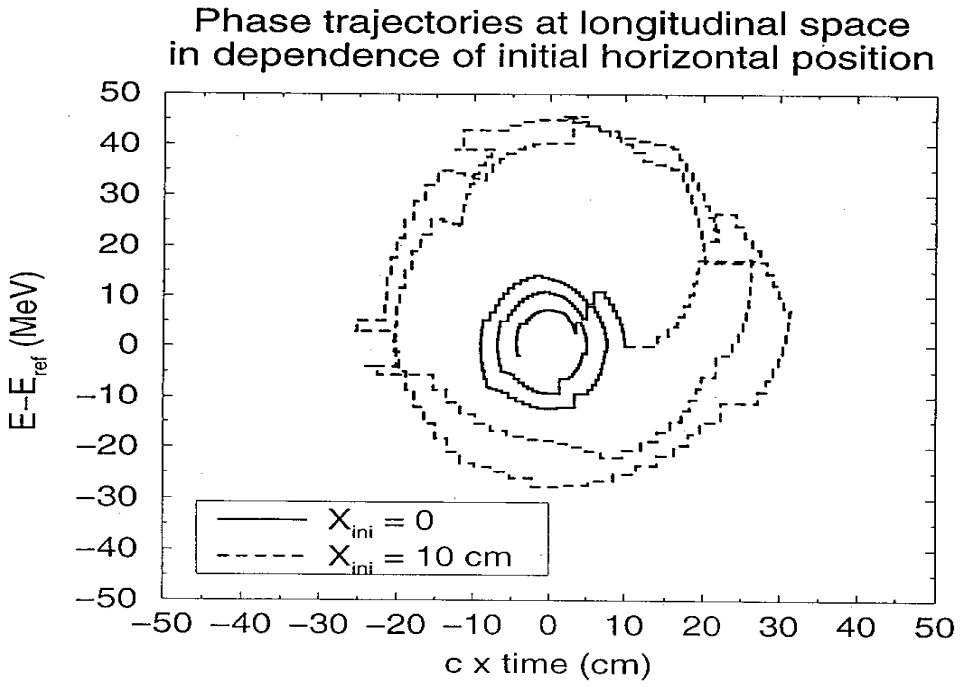
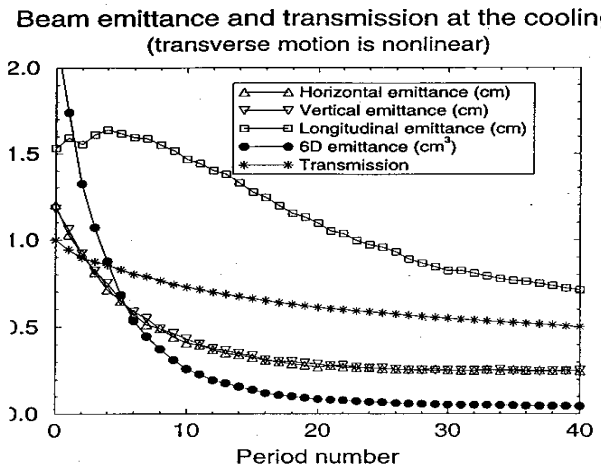
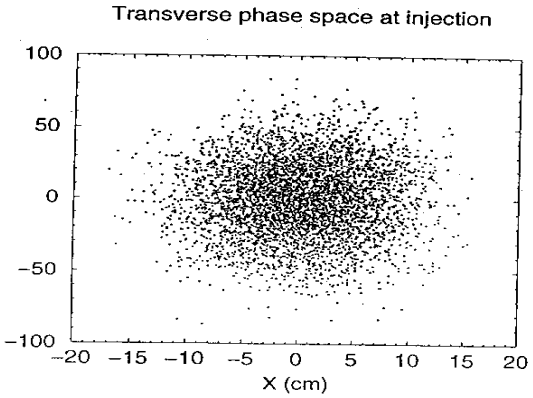
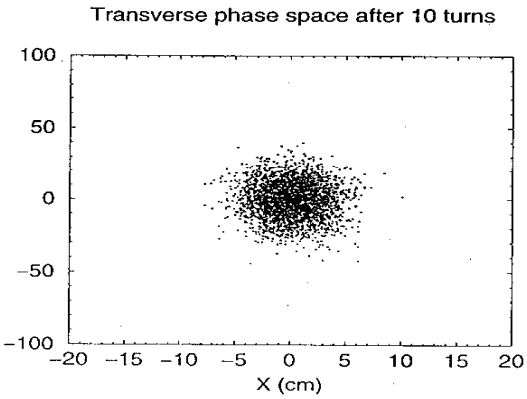
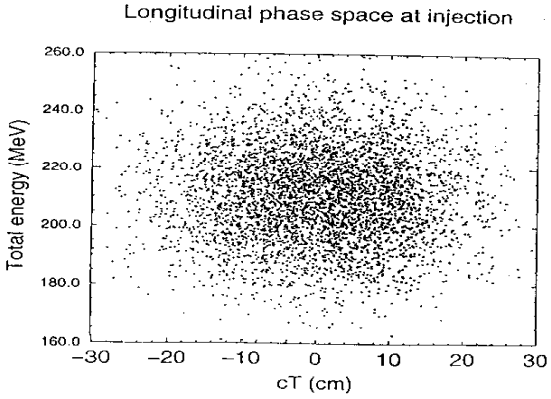


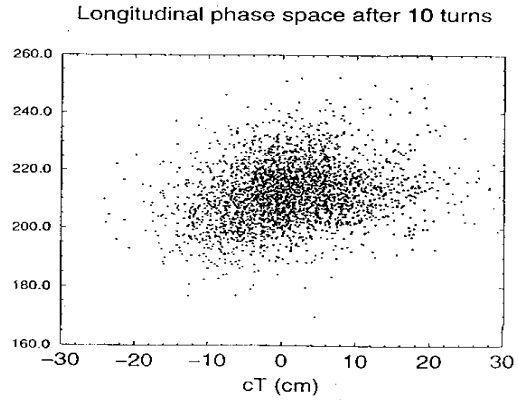
B5. Cooling simulation
(longitudinal motion in paraxial approximation)



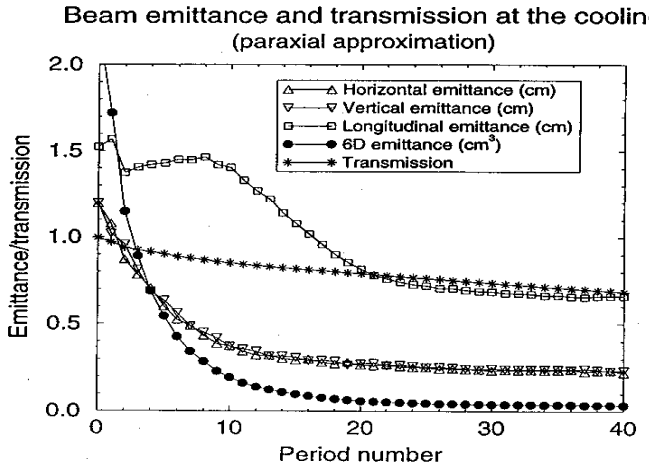


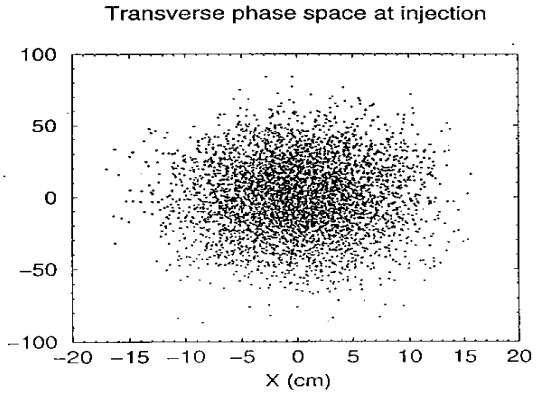


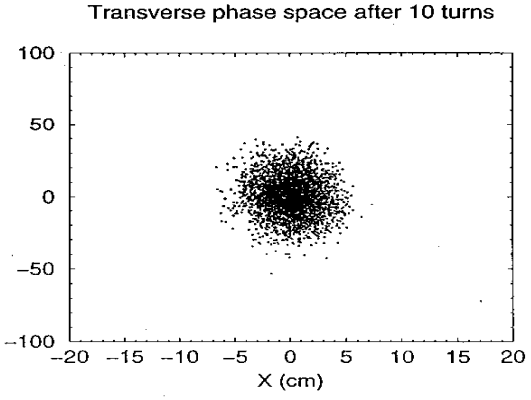


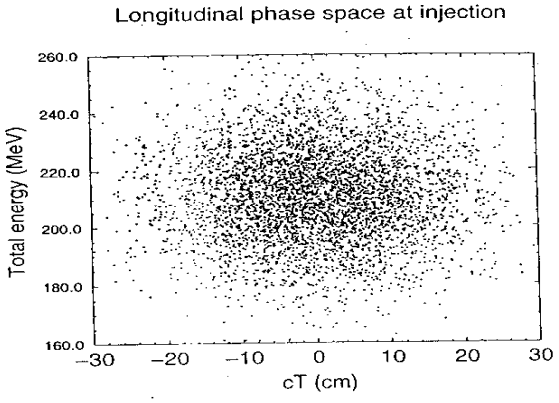


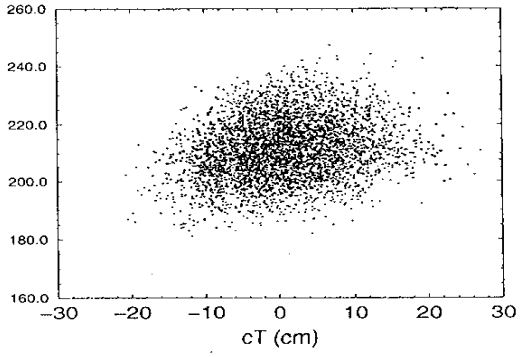
B6. Cooling simulation (paraxial approximation)











B7. Summary; effect of different factors on the cooling and emittance exchange (beam emittance and transmission after 10 tutns)

	ε_x (mm)	ε_y (mm)	ε_z (mm)	ε_6 (mm^3)	Trans.(%)
Injected beam	12	12	15	2160	100
Paraxial approximation	2.5	2.3	6.6	38	68
+ Transverse nolinearity	2.5	2.5	7.1	44	50
+ Longitudinal nonlinearity	2.6	2.8	8.8	64	33

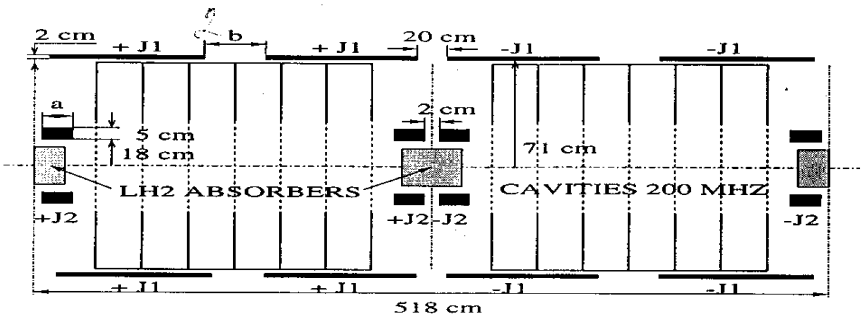


Figure 3: Schematic of the cooling channel cell.