Linear Analysis of non-Isosceles Absorbers

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Introduction

- Wedge-shaped absorbers in conjunction with dispersion are used to create longitudinal-transverse coupling.
- Generally people consider both faces having the same angle to the vertical.
  - To linear order, energy loss depends only on vertical displacement.
- For theoretical treatment, consider absorbers with arbitrary face angles
  - First, consider both tilts in the same plane.
  - Later, consider arbitrary tilts.
  - Find energy loss to linear order in dispacement and angles
  - Really just find length of orbit inside absorber
- Of course, for simulation you just do the right thing…

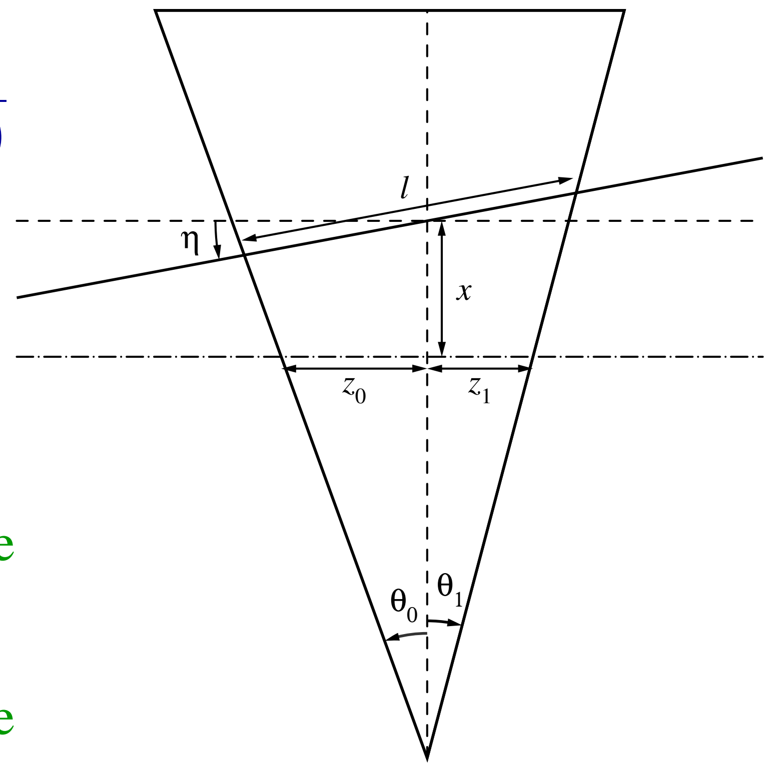
Tilts in Same Plane

- Full expression for path length ($L = z_1 + z_2$):

$$l = \left(\frac{L}{\tan\theta_0 + \tan\theta_1} + x\right)\frac{\cos\eta\sin(\theta_0+\theta_1)}{\cos(\eta-\theta_0)\cos(\eta-\theta_1)}$$

- To linear order in x, η:

$$l = L + x(\tan\theta_0 + \tan\theta_1) + \eta L(\tan\theta_1 - \tan\theta_0)$$



  - Length depends linearly on incoming angle if face angles different
  - Parallel but tilted faces give linear dependence only on angle

General Face Angles

- Two absorber faces described by planes through p_k and unit normal u_k:

$$(x - p_0) \cdot u_0 = 0 \qquad (x - p_1) \cdot u_1 = 0$$

  - Points faces pass through

$$p_0 = (0, 0, -z_0) \qquad p_1 = (0, 0, z_1)$$

  - Unit normals: tilt by θ_k, rotate by ϕ_k

$$u_0 = (\sin\theta_0 \cos\phi_0, \sin\theta_0 \sin\phi_0, \cos\theta_0) \quad u_1 = (-\sin\theta_1 \cos\phi_1, -\sin\theta_1 \sin\phi_1, \cos\theta_1)$$

- Incoming beam described as line through x_0 with unit tangent t

$$x = x_0 + \lambda t$$

  - In terms of particle coordinates at origin:

$$x_0 = (x, y, 0) \qquad t = (p_x/p, p_y/p, \sqrt{p^2 - p_x^2 - p_y^2}/p)$$

- Path length is

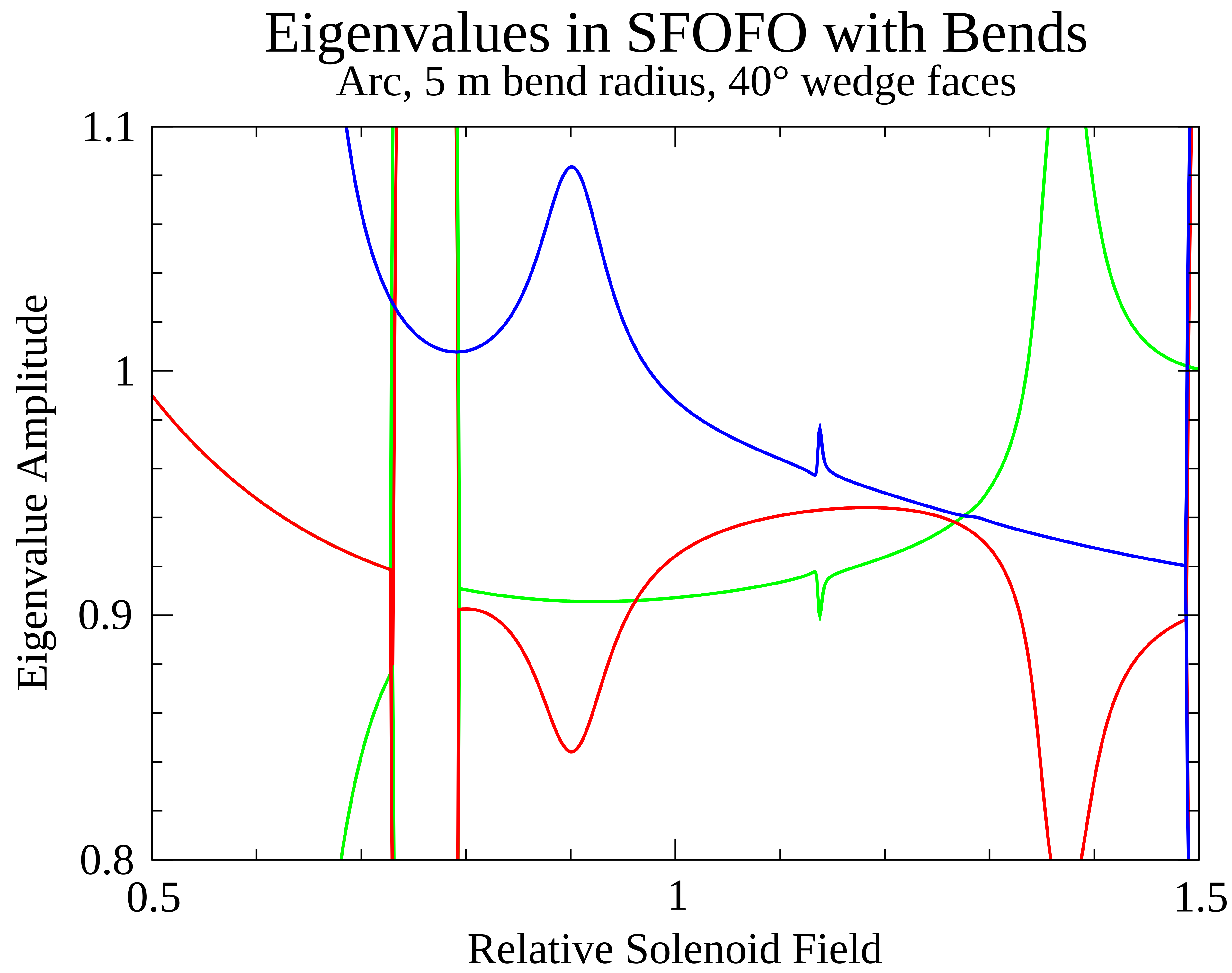
$$\frac{(p_1 - x_0) \cdot u_1}{t \cdot u_1} - \frac{(p_0 - x_0) \cdot u_0}{t \cdot u_0}$$

- To linear order,

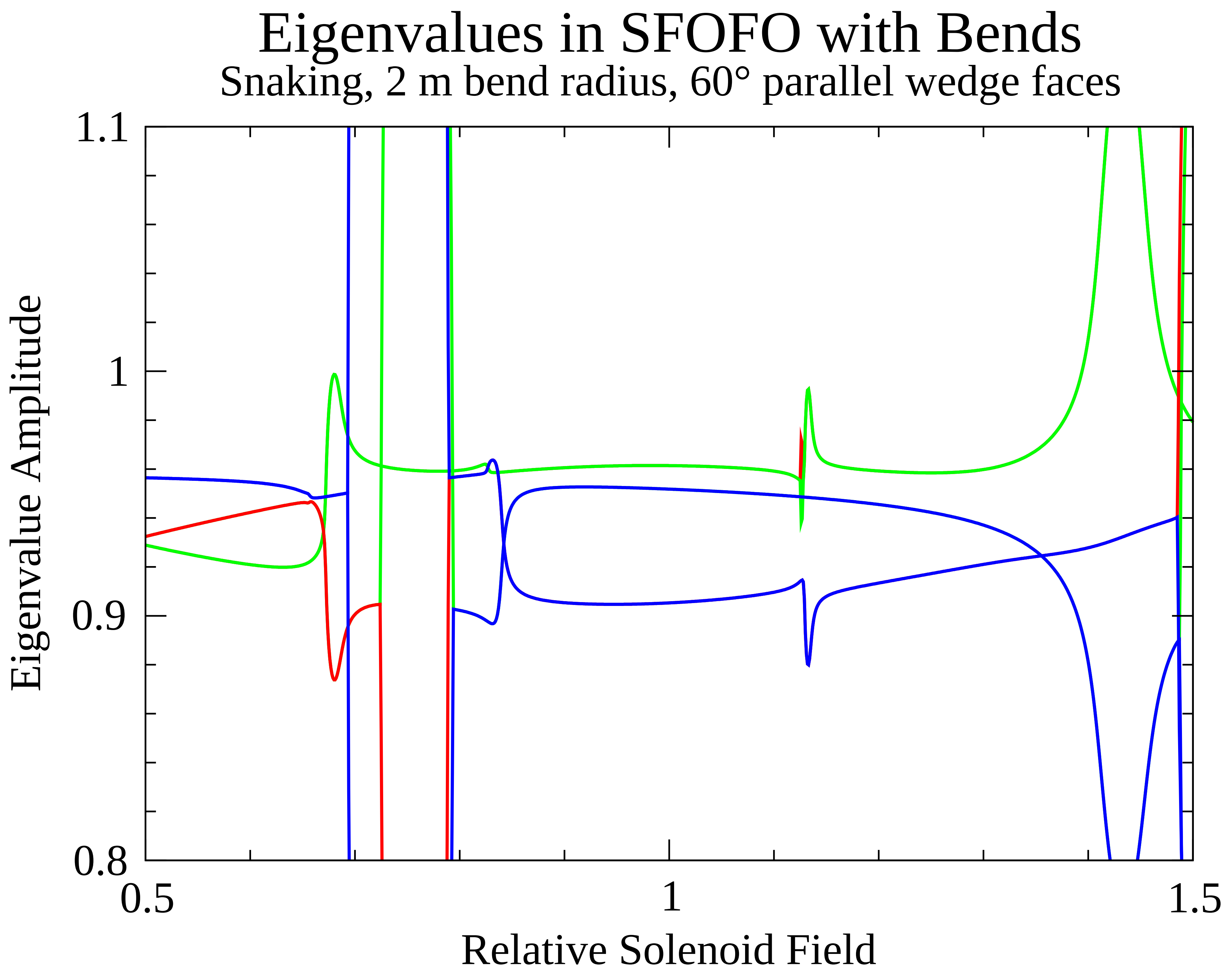
$$z_0 + z_1 + x(\tan\theta_0 \cos\phi_0 + \tan\theta_1 \cos\phi_1) + y(\tan\theta_0 \sin\phi_0 + \tan\theta_1 \sin\phi_1)$$
$$+ p_x/p(z_1 \tan\theta_1 \cos\phi_1 - z_0 \tan\theta_0 \cos\phi_0) + p_y/p(z_1 \tan\theta_1 \sin\phi_1 - z_0 \tan\theta_0 \sin\phi_0)$$

Possible Uses

- Parallel faced absorber in region with angular dispersion
- Straight cooling channel with emittance exchange
  - Get very high Q value
    - Climb up RF as longitudinal cools
    - Reduce β-function as transverse cools
  - Possibly better multiple scattering (no position dispersion in absorber)
- More tilt required to get maximal exchange when compared with wedge
- Parallel faced has better looking behavior over energy range



Eigenvalues in SFOFO with Bends
Arc, 5 m bend radius, 40° wedge faces
1.1
1
0.9
0.8
0.5
1
1.5
Eigenvalue Amplitude
Relative Solenoid Field


Eigenvalues in SFOFO with Bends
Snaking, 2 m bend radius, 60° parallel wedge faces
1.1
1
0.9
0.8
0.5
1
1.5
Eigenvalue Amplitude
Relative Solenoid Field