WG 2

Date : 5/22

$2^{:00} - 3^{\underline{30}}$

Speaker : M. Mangano

Title : Rare decays and BPM physics

MUON WEEK
9-5-2000

ANDRÉ DE GOUVÊA
CERN-TH

PHYSICS WITH STOPPED MUONS

A. de Gouvêa
G. Giudice
S. Lola
K. Tobe
K. Zuber

OUTLINE

- PRECISION MEASUREMENTS
- RARE MUON PROCESSES (LEPTON FLAVOUR VIOLATION
  - MOTIVATION
  - PHENOMENOLOGY
  - CURENT EXP'TAL SITUATION AND PROSPECTS
  - POLARIZED MUONS AND $\vec{\sigma}\cdot\vec{p}$
  - SOME EXPERIMENTAL REQUIREMENTS.

REFERENCE: Y. KUNO AND Y. OKADA hep-ph/9909265

MUON DECAY

$$\frac{d^2\Gamma(\mu^+ \to e^+ \nu \bar{\nu})}{dx\, d\cos\theta_e} = \frac{m_\mu}{4\pi^3} W^4 G_F^2 \sqrt{x^2 - x_0^2} \left[F_{IS}(x) + P_\mu \cos\theta_e F_{AS}(x) \right] \left[1 + P_e(x, \theta_e) \right.$$

$$W = \frac{m_\mu^2 + m_e^2}{2 m_\mu} \qquad x = \frac{E_e}{W} \qquad x_0 = \frac{m_e}{W}$$

P_μ muon polarization $\quad$ P_e projected electron pol.

$$F_{IS}(x) = x(1-x) + \frac{2}{9} \rho (4x^2 - 3x - x_0^2) + \eta\, x_0 (1-x)$$

$$F_{AS}(x) = \frac{1}{3} \xi \sqrt{x^2 - x_0^2} \left[1 - x + \frac{2}{3} \delta \left(4x - 2 + \sqrt{1 - x_0^2} \right) \right]$$

MEASUREMENT OF τ_μ

- fundamental constant G_F
- no improvements in EW fits $\qquad \frac{\Delta G_F}{G_F} = 9 \times 10^{-6}$

$\frac{\Delta M_Z}{M_Z} = 77 \times 10^{-6}$ (PDG 98) $\quad 23 \times 10^{-6}$ (LEP+SLD EWG 99)

MICHEL PARAMETERS

SM : $\rho = \frac{3}{4}$, $\eta = 0$, $\xi = 1$, $\delta = \frac{3}{4}$

$\cancel{R}$: single operator $\Rightarrow$ V-A structure

$\Rightarrow$ violation of universality

more operators : $\lambda L_L^3 L_L^1 \bar{E}_R^1 + \lambda' L_L^3 L_L^2 \bar{E}_R^2$

$$\Delta\rho = \frac{3}{16}\varepsilon^2, \; \Delta\eta = \frac{\varepsilon}{2}, \; \Delta\xi = -\frac{\varepsilon^2}{4}, \; \Delta\delta = 0 \qquad \varepsilon \equiv \frac{\lambda \lambda'}{4\sqrt{2} G_F \tilde{m}^2_{e_L^3}}$$

$$\vec{P}_e(x,\theta_e) = P_{T1}\cdot\frac{(\vec{z}\times\vec{P}_\mu)\times\vec{z}}{|(\vec{z}\times\vec{P}_\mu)\times\vec{z}|} + P_{T2}\cdot\frac{\vec{z}\times\vec{P}_\mu}{|\vec{z}\times\vec{P}_\mu|} + P_L\cdot\frac{\vec{z}}{|\vec{z}|}, \tag{35}$$

where $\vec{z}$ is the direction of the $e^\pm$ momentum, and $\vec{P}_\mu$ is the muon spin polarization. P_L, P_{T1}, and P_{T2} are, respectively, the $e^\pm$ polarization component parallel to the $e^\pm$ momentum direction, that transverse to the $e^\pm$ momentum within the decay plane, and that transverse to the $e^\pm$ momentum and normal to the decay plane. A non-zero value of the triple T-odd correction, P_{T2}, would imply violation of time-reversal invariance. They are given by

$$P_{T1}(x,\theta_e) = \frac{P_\mu \sin\theta_e F_{T1}(x)}{F_{IS}(x) \pm P_\mu\cos\theta_e F_{AS}(x)}, \tag{36}$$

$$P_{T2}(x,\theta_e) = \frac{P_\mu \sin\theta_e F_{T2}(x)}{F_{IS}(x) \pm P_\mu\cos\theta_e F_{AS}(x)}, \tag{37}$$

$$P_L(x,\theta_e) = \frac{\pm F_{IP}(x) + P_\mu\cos\theta_e F_{AP}(x)}{F_{IS}(x) \pm P_\mu\cos\theta_e F_{AS}(x)}, \tag{38}$$

where the $\pm$ sign corresponds to $\mu^\pm$ decays, and

$$F_{T1}(x) = \frac{1}{12}\left\{-2\left[\xi'' + 12(\rho - \frac{3}{4})\right](1-x)x_0 - 3\eta(x^2 - x_0^2) + \eta''(-3x^2 + 4x - x_0^2)\right\}, \tag{39}$$

$$F_{T2}(x) = \frac{1}{3}\sqrt{x^2 - x_0^2}\left\{3\frac{\alpha'}{A}(1-x) + 2\frac{\beta'}{A}\sqrt{1-x_0^2}\right\}, \rightarrow \text{T-VIOLATING} \tag{40}$$

$$F_{IP}(x) = \frac{1}{54}\sqrt{x^2 - x_0^2}\left\{9\xi'(-2x + 2 + \sqrt{1-x_0^2}) + 4\xi(\delta - \frac{3}{4})(4x - 4 + \sqrt{1-x_0^2})\right\}, \tag{41}$$

$$F_{AP}(x) = \frac{1}{6}\left\{\xi''(2x^2 - x - x_0^2) + 4(\rho - \frac{3}{4})(4x^2 - 3x - x_0^2) + 2\eta''(1-x)x_0\right\}. \tag{42}$$

where ξ', ξ'', η'', (α'/A), and (β'/A) are newly defined Michel parameters (Kinoshita and Sirlin, 1957b; Fetscher and Gerber, 1998). In the SM, $\xi' = \xi'' = 1$ and $\eta'' = (\alpha'/A) = (\beta'/A) = 0$.

CURRENT EXP'TAL SITUATION

TABLE V. Experimental values of some of the Michel decay parameters.

Michel parameter	SM value	Experimental value	Sensitive observables
ρ	3/4	0.7518 ± 0.0026	F_{IS}
η	0	-0.007 ± 0.013	F_{IS} and P_{T1}
δ	3/4	0.7486 ± 0.0038	F_{AS} and P_L
ξ	1	1.0027 ± 0.0084	$F_{AS}^\dagger$ and P_L
ξ'	1	1.00 ± 0.04	P_L
ξ''	1	0.65 ± 0.36	P_L

† Only the product of ξP_μ is measured.

– ISOTROPIC PART OF DECAY

– [illegible]

– FROM ELECTRON POLARIZATION

$$\vec{P}_e(x,\theta_e) = P_{T1}\cdot\frac{(\vec{z}\times\vec{P}_\mu)\times\vec{z}}{|(\vec{z}\times\vec{P}_\mu)\times\vec{z}|} + P_{T2}\cdot\frac{\vec{z}\times\vec{P}_\mu}{|\vec{z}\times\vec{P}_\mu|} + P_L\cdot\frac{\vec{z}}{|\vec{z}|}, \tag{35}$$

where $\vec{z}$ is the direction of the $e^\pm$ momentum, and $\vec{P}_\mu$ is the muon spin polarization. P_L, P_{T1}, and P_{T2} are, respectively, the $e^\pm$ polarization component parallel to the $e^\pm$ momentum direction, that transverse to the $e^\pm$ momentum within the decay plane, and that transverse to the $e^\pm$ momentum and normal to the decay plane. A non-zero value of the triple T-odd correction, P_{T2}, would imply violation of time-reversal invariance. They are given by

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$$F_{IP}(x) = \frac{1}{54}\sqrt{x^2-x_0^2}\left\{9\xi'(-2x+2+\sqrt{1-x_0^2}) + 4\xi(\delta-\frac{3}{4})(4x-4+\sqrt{1-x_0^2})\right\}, \tag{41}$$

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ξ''	1	0.65 ± 0.36	P_L

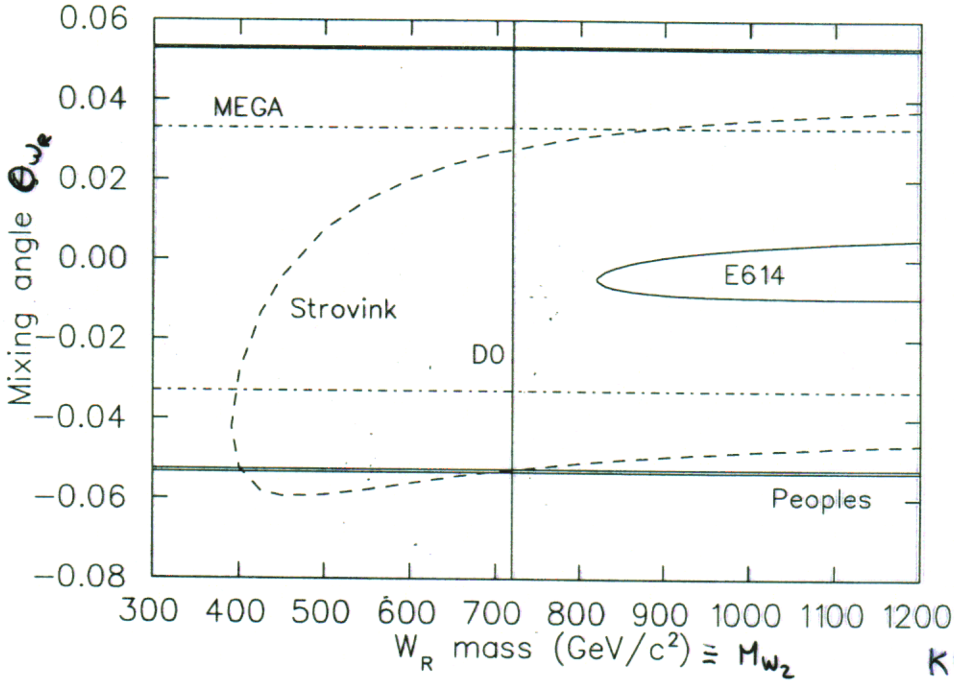
† Only the product of ξP_μ is measured.

– ISOTROPIC PART OF DECAY
– ANISOTROPIC PART OF DECAY ($P_\mu \neq 0$)
– FROM ELECTRON POLARIZATION

LEFT-RIGHT MODELS

$$\Delta\rho = -\frac{3}{2}\theta_{W_R}^2 \qquad \Delta\xi = -2\theta_{W_R}^2 - 2\left(\frac{m_{W_1}}{m_{W_2}}\right)^4$$

θ_{W_R} mixing angle $W_L - W_R$



KUNO-OKADA

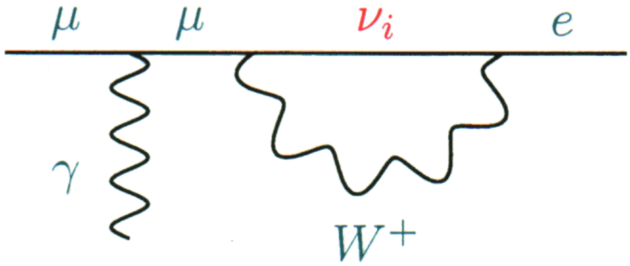
FIG. 3. Constraints on the mass of W_R vs. its mixing angle (ζ) in the manifest left-right symmetric model. The experimental constraints of "Strovink", "Peoples", "MEGA" and "D0" are from Jodidio, *et al.* (1986), Derenzo (1969), the MEGA experiment (unpublished), and Abachi, *et al.* (1996), respectively. The aimed goal for E614 is also shown (provided by D.R.Gill).

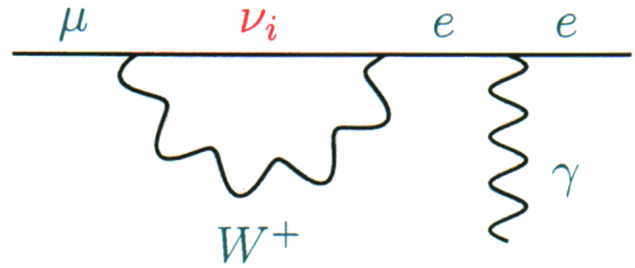
E614 with 10^9 μ's

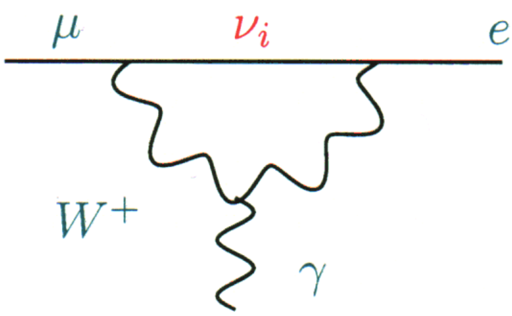
precision $\Delta\rho < 1\times10^{-4}$, $\Delta\delta < 3\times10^{-4}$, $\Delta(P_\mu\xi) < 2\times10^{-4}$

(TRIUMF)

1. $\mu \to e\gamma$ in the SM with $m_{\nu_i} \neq 0$







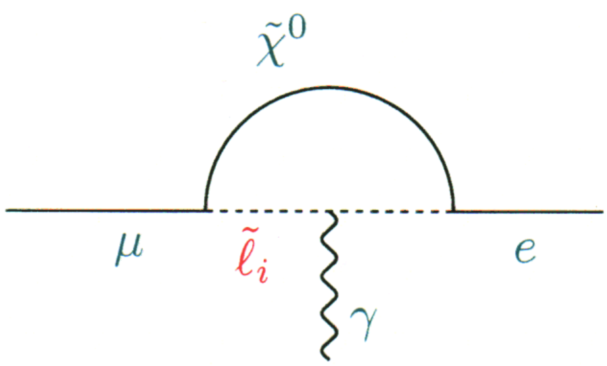
$$\nu_i = \nu_\mu \cos\theta + \nu_e \sin\theta$$

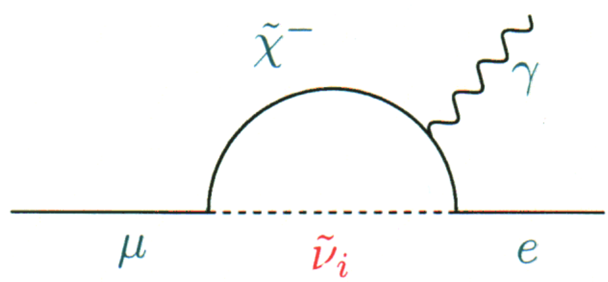
$$\Gamma \propto \left(\frac{m_2^2 - m_1^2}{m_W^2}\right)^2 \sin^2\theta \cos^2\theta$$

Neutrino data: $BR \leq 10^{-50}$!!

2. LFV in minimal SUSY

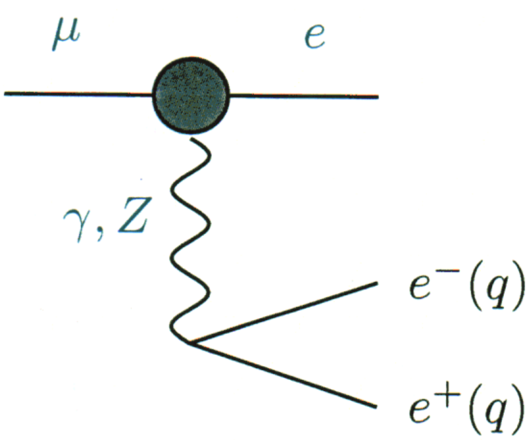
For $\tilde{\mu}$-$\tilde{e}$ ($\tilde{\nu}_\mu$-$\tilde{\nu}_e$) mixing:





The fermion in the loop is now a neutralino/chargino instead of a neutrino ($m_{\tilde{\chi}^0}, m_{\tilde{\chi}^\pm} \gg m_\nu \Rightarrow$ large rates)

3. $\mu \to 3e$ et μ -e conversion on nuclei



$+$ ...(suppressed box-diagr.)

If R-violation, tree-level $\mu \to 3e$ and $\mu - e$ conversion !

4. Massive neutrinos and SUSY

Even if:

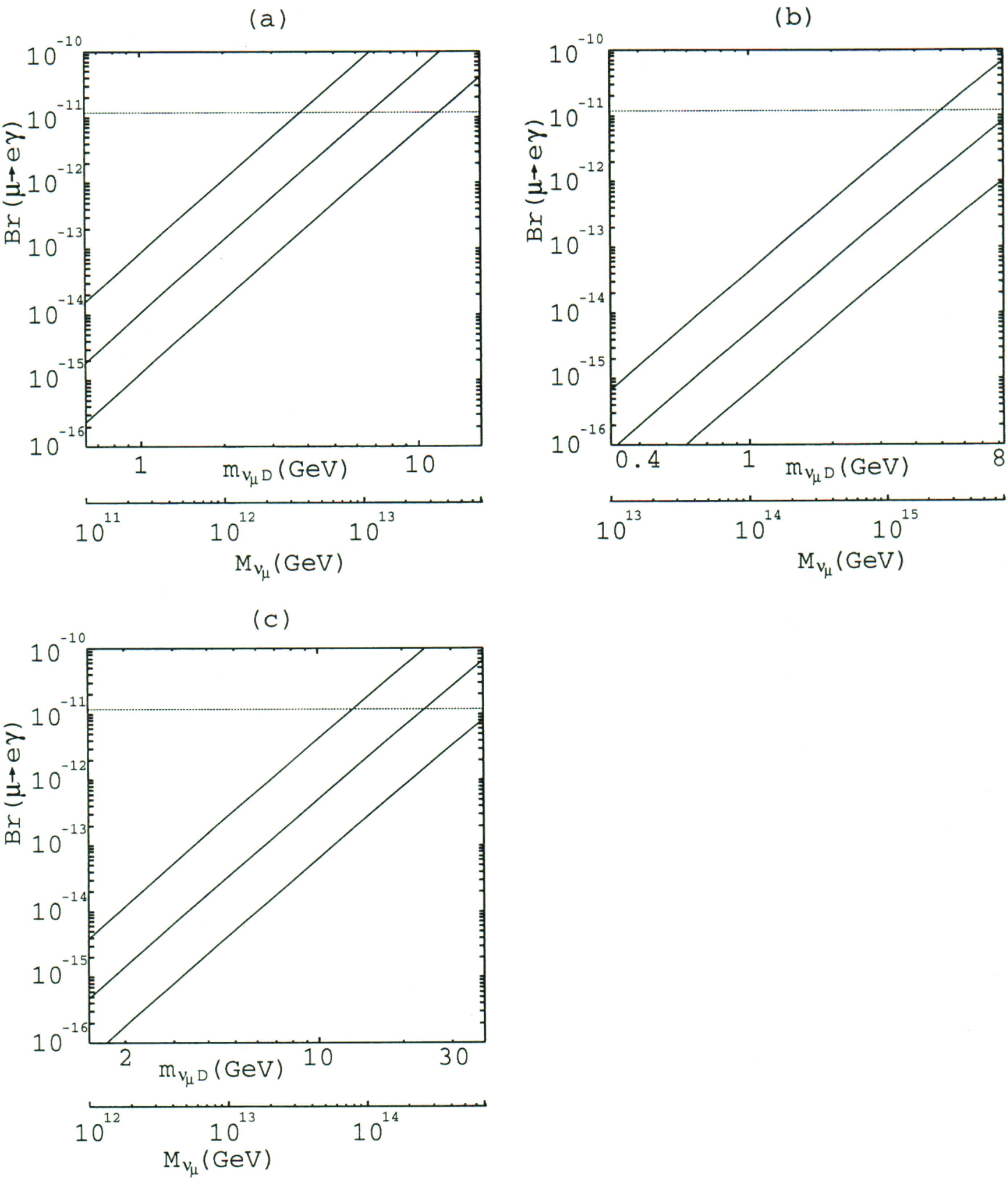
$$M_{\mathrm{GUT}} : m_{\tilde{\ell},\tilde{\nu}} \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathrm{RGEs} \longrightarrow \begin{pmatrix} 1 & \star & \star \\ \star & 1 & \star \\ \star & \star & 1 \end{pmatrix}$$

Corrections in the basis where $(\lambda_\ell^\dagger \lambda_\ell)_i^j$ is diagonal, ie:

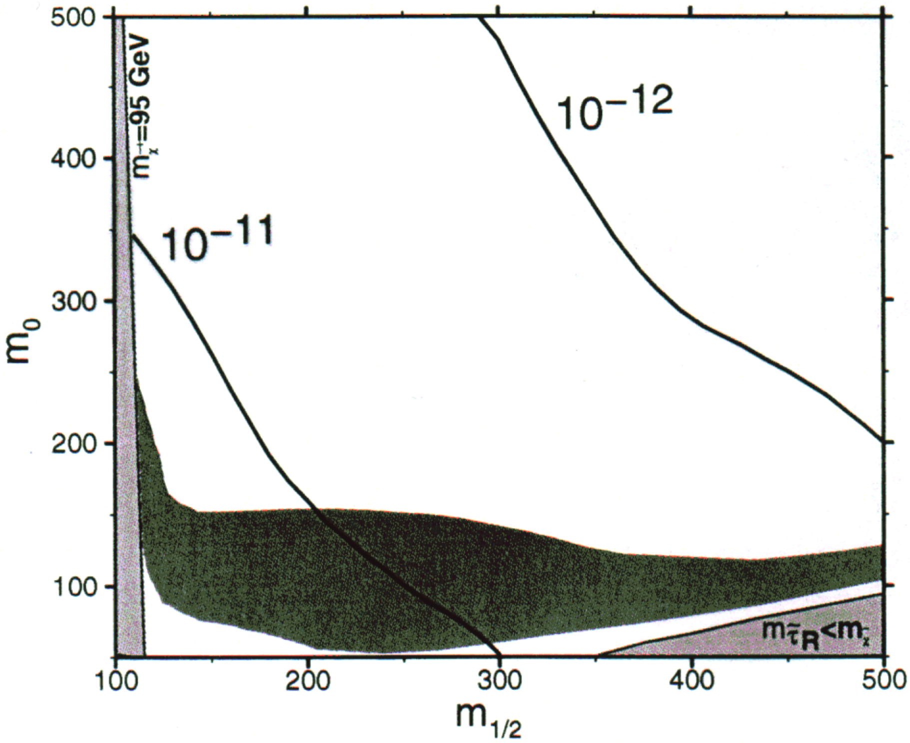
$$\delta m^2_{\tilde{\ell}} \propto \frac{1}{16\pi} \ln \frac{M_{\mathrm{GUT}}}{M_N} \underbrace{\lambda_\nu^\dagger \lambda_\nu}_{\text{Dirac mass}} m^2_{\mathrm{SUSY}}$$

(And similar corrections for $\delta m_{\tilde{\nu}}$)

INFO: The larger the μ-e lepton mixing and the neutrino masses, the larger the rates for $\mu \to e\gamma$

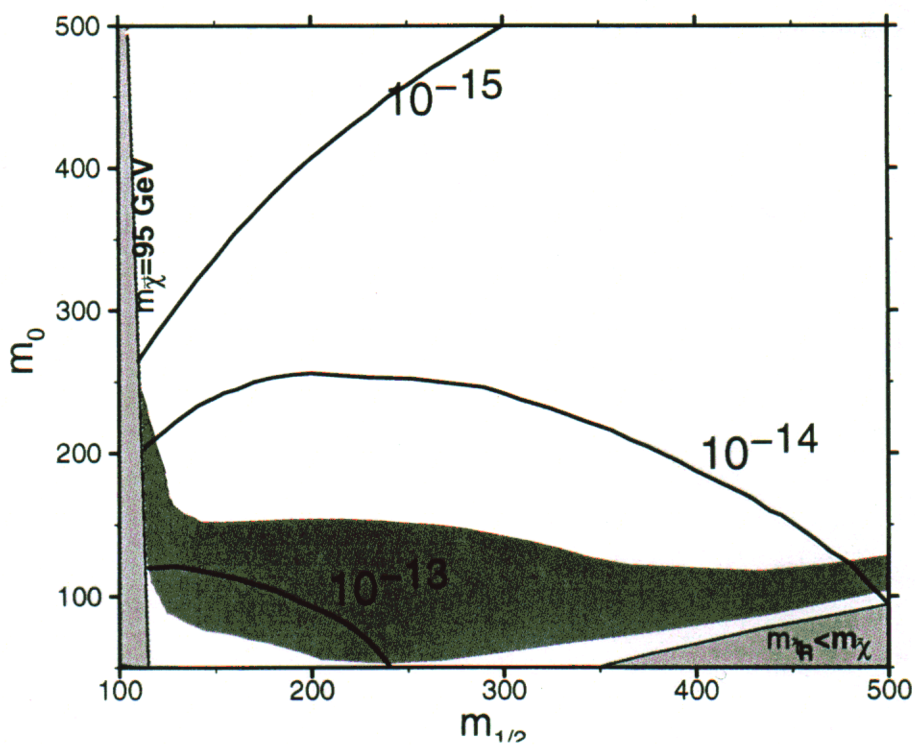


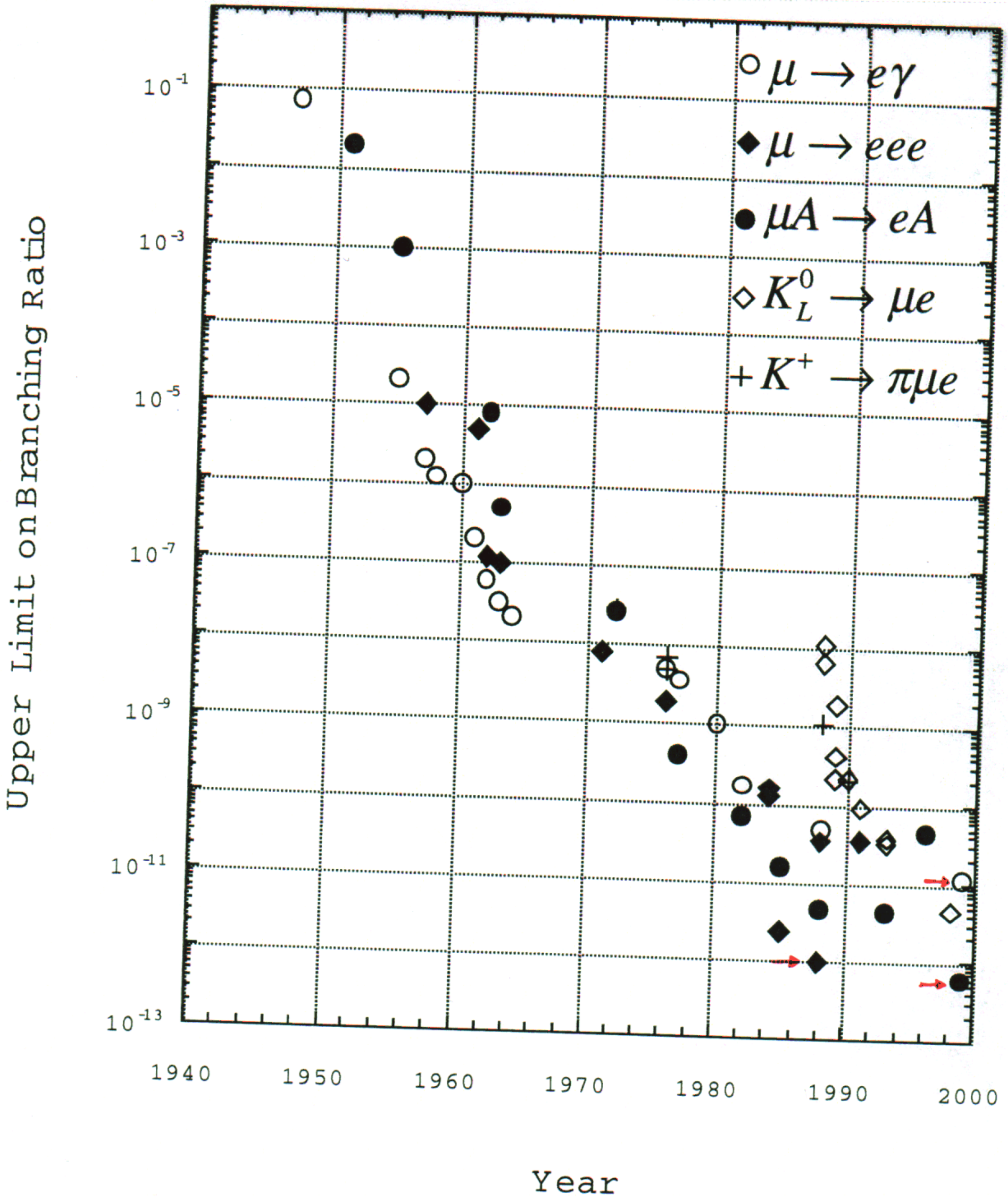
a: large angle MSW b: vacuum oscillations
c: small angle MSW
(Hisano and Nomura, hep-ph/0004061)



Predictions for $\mu \to e\gamma$ and $\mu - e$ conversion for the small angle MSW and $m_{\nu_3} \gg m_{\nu_{1,2}}$

(Ellis et al., hep-ph/9911459)





PSI	$\mu \to e\gamma$	10^{-14}
SINDRUM II, PSI	$\mu \to 3\ e$	?
MECO, BNL	$\mu^- Ti \to e^- Ti$	10^{-16}

PRISM $\mu \to e\gamma : \ 10^{-15}, \ \mu^- Ti \to e^- Ti : \ 10^{-18}$

PHENOMENOLOGY

MODEL INDEPENDENT LAGRANGIANS (DIM-5 AND 6 OPERATORS)

$$\mathcal{L}_{\mu\to e\gamma} = -\frac{4G_F m_\mu}{\sqrt{2}}\left[A_R\, \bar{\mu}_R \sigma^{\mu\nu} e_L F_{\mu\nu} + A_L\, \bar{\mu}_L \sigma^{\mu\nu} e_R F_{\mu\nu} + \text{H.c.}\right]$$

$$B(\mu^+\to e^+\gamma) = 384\pi^2\left(|A_R|^2 + |A_L|^2\right)$$

$$\mathcal{L}_{\mu\to eee} = \mathcal{L}_{\mu\to e\gamma} - \frac{4G_F}{\sqrt{2}}\Big[g_1 (\bar{\mu}_R e_L)(\bar{e}_R e_L) + g_2 (\bar{\mu}_L e_R)(\bar{e}_L e_R) +$$

$$+ g_3 (\bar{\mu}_R \gamma_\mu e_R)(\bar{e}_R \gamma^\mu e_R) + g_4 (\bar{\mu}_L \gamma_\mu e_L)(\bar{e}_L \gamma^\mu e_L) +$$

$$+ g_5 (\bar{\mu}_R \gamma_\mu e_R)(\bar{e}_L \gamma^\mu e_L) + g_6 (\bar{\mu}_L \gamma_\mu e_L)\, \bar{e}_R \gamma^\mu e_R) + \text{H.c.}$$

$$B(\mu^+\to e^+e^+e^-) = 2(C_1+C_2) + (C_3+C_4) + 16(C_7+C_8)$$

$$+ 8(C_9+C_{10}) + 32\left[\ln\left(\frac{m_\mu^2}{m_e^2}\right) - \frac{11}{4}\right](C_5+C_6)$$

WHERE

$$C_1 = \frac{|g_1|^2}{16} + |g_3|^2 \; ; \qquad C_2 = \frac{|g_2|^2}{16} + |g_4|^2$$

$$C_3 = |g_5|^2 \; ; \quad C_4 = |g_6|^2 \; ; \quad C_5 = |eA_R|^2 \; ; \quad C_6 = |eA_L|^2$$

$$C_7 = \mathrm{Re}(eA_R g_4^*) \; ; \quad C_8 = \mathrm{Re}(eA_L g_3^*) \; ; \quad C_9 = \mathrm{Re}(eA_R g_6^*)$$

$$C_{10} = \mathrm{Re}(eA_L g_5^*) \qquad C_{11} = \mathrm{Im}(eA_R g_4^* + eA_L g_3^*)$$

$$C_{12} = \mathrm{Im}(eA_R g_6^* + eA_L g_5^*)$$

POLARIZED MUONS

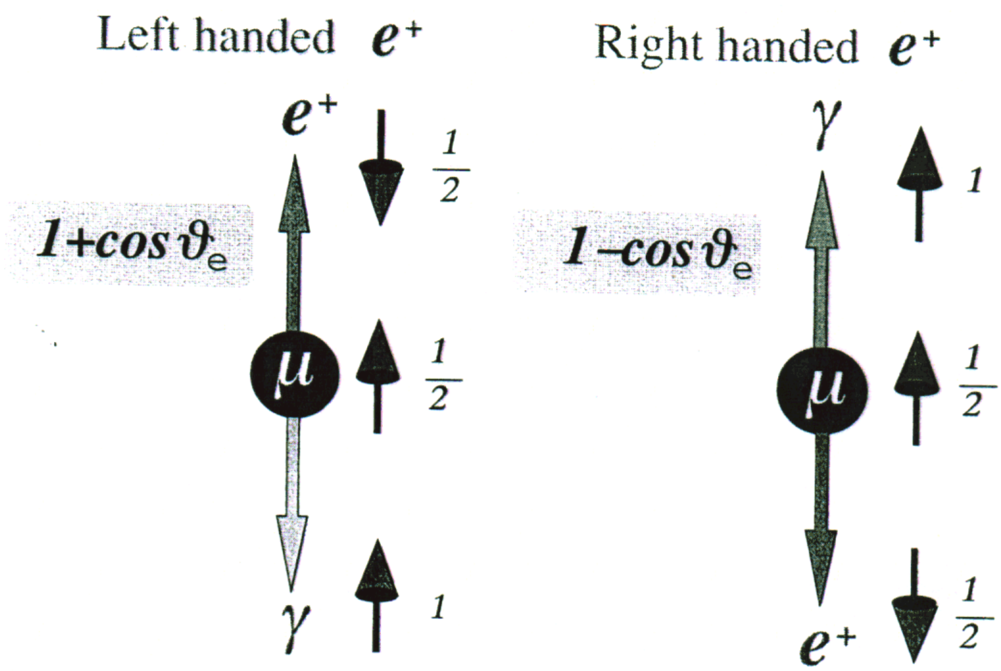


FIG. 19. Angular distribution of e^+ in polarized $\mu^+ \to e^+\gamma$ decay.

$$\frac{dB(\mu^+ \to e^+\gamma)}{d(\cos\theta_e)} = 192\pi^2 \left[|A_R|^2 (1 - P_\mu \cos\theta_e) + |A_L|^2 (1 + P_\mu \cos\theta_e) \right]$$

WAY OF SEPARATING MODELS!

e.g. SU(5) SUSY GUT: ONLY $A_L \neq 0$

(ONLY $\mu^+ \to e^+_L \gamma$ OCCURS)

SUSY WITH RIGHT-HANDED NEUTRINO (NO GUT)

: ONLY $A_R \neq 0$

POLARIZED MUONS ALSO HELP BACKGROUND SUBTRACTION !

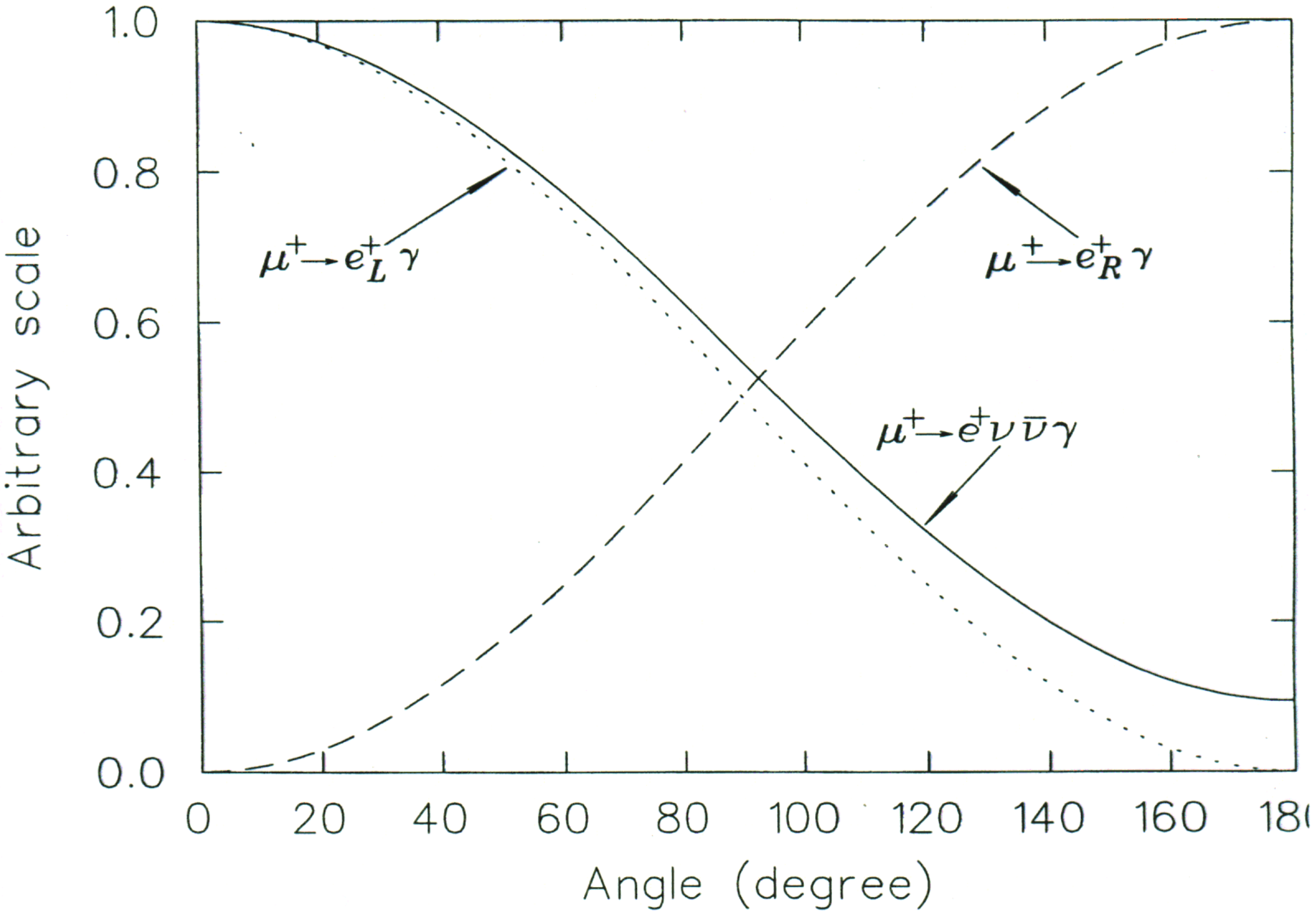


FIG. 24. Angular distribution of the physics background from the $\mu^+ \to e^+\nu_e\overline{\nu}_\mu\gamma$ decay from polarized muons (a solid line). $\mu^+ \to e^+_L\gamma$ (a dotted line) and $\mu^+ \to e^+_R\gamma$ (a dashed line) decays are also shown (after Kuno and Okada, (1996)).

$\mu^+ \to e^+ e^- e^+$ AND CP- VIOLATION

$$\mu^+(\vec{\sigma}_\mu) \to e^+(\vec{p}_1)\, e^+(\vec{p}_2)\, e^-(\vec{p}_3)$$

T ODD COMBINATION : $\vec{\sigma}_\mu \cdot (\vec{p}_1 \times \vec{p}_2)$

OBSERVABLE $\propto$ events with $\phi \in [0, \pi]$ - events with $\phi \in [\pi, 2\pi]$

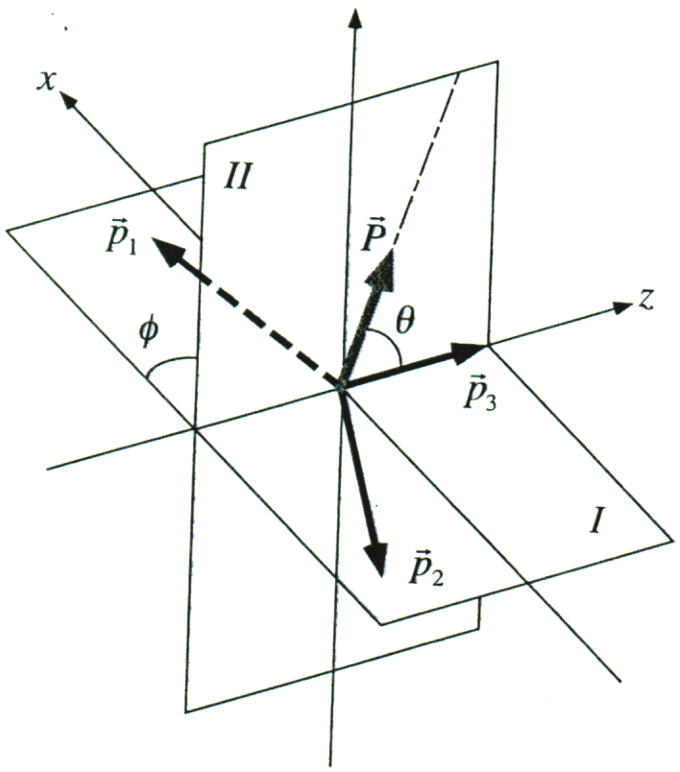


FIG. 27. Kinematics of the $\mu^+ \to e^+e^+e^-$ decay in the muon center-of-mass system, in which $\vec{p}_1$, $\vec{p}_2$ are the momentum vectors of the two e^+s and $\vec{p}_3$ is that of the e^-, respectively. The plane-I is the decay plane on which $\vec{p}_1$, $\vec{p}_2$, and $\vec{p}_3$ lie. The plane-II is the plane in which the muon polarization vectors, $\vec{P}$ and $\vec{p}_3$, are located (after Okada, *et al.*, (1999)).

$$A_T(\mu^+ \to e^+e^+e^-) = \frac{64}{35}\,\frac{(3C_{11} - 2C_{12})}{B(\mu^+ \to e^+e^+e^-)}$$ ← INTERFERENCE BETWEEN ON-SHELL PHOTON AND 4 FERMION INTERACTION

CHALLENGE $A_T^{MAX} \lesssim 15\%$ (MODEL INDEPENDENT ?)

SOME REQUIREMENTS ON THE BEAM

- LOW-ENERGY MUONS e.g. PRISM ($T_\mu = 20$ MeV)
- SMALL MOMENTUM SPREAD e.g. PRISM ($\Delta T = .5-1$ MeV

FOR $\mu \to e\gamma$; $\mu \to eee$, IN ORDER TO GET RID OF ACCIDENTAL BACKGROUND ONE WOULD LIKE TO HAVE A CONTINUOUS MUON SOURCE

i.e. HIGH REPETITION RATE, SAY 10 TO 100 MHz *

FOR $\mu - e$ CONVERSION AND $\mu^+ e^- \leftrightarrow \mu^- e^+$ A PULSED BEAM IS MORE APPROPRIATE

i.e. SLOW REPETITION RATE, SAY FEW 100 kHz + HIGH BEAM EXTINCTION BETWEEN PULSES *

* FROM PRISM TECHNOTE No. 9 BY Y. KUNO

http://psux1.kek.jp/~prism

COMMENTS:

IF THE DIM-5 OPERATORS DOMINATE ALL PROCESSES

$$\frac{B(\mu^+ \to e^+e^+e^-)}{B(\mu^+ \to e^+\gamma)} = 6\times10^{-3}$$

(TRUE OF MANY SUSY MODELS)

$$\frac{R(\mu \to e \text{ in Ti})}{B(\mu \to e\gamma)} = 5\times10^{-3}$$

HOWEVER, IN DIM-6 OPERATORS DOMINATE IT IS POSSIBLE THAT $\mu \to e\gamma$ YIELDS NO OBSERVABLE RATE, WHILE $\mu \to eee$ AND $\mu - e$ CONVERSION YIELD "LARGE" SIGNALS !

FURTHERMORE, IT IS ALSO POSSIBLE THAT $\mu \to eee$ GIVES A SIGNAL WHILE $\mu - e$ CONVERSION DOES NOT.

e.g. SUSY $\not{R}_p$ WITH NON NEGLIGIBE LLE COUP.

$$W \supset f_{131}\, L^1L^3E^1 + f_{231}\, L^2L^3E^1$$

$$\frac{B(\mu \to 3e)}{B(\mu \to e\gamma)} = 10^4 \qquad (m_{\tilde{\nu}_3} = m_{\tilde{e}_R} = 100\text{ GeV})$$

$$\frac{B(\mu \to 3e)}{R(\mu \to e \text{ in Ti})} = 10^2 \qquad (\;''\;)$$

EXTENDING $\mu \to eee$ REACH IS IMPORTANT!

OTHER POSSIBILITIES

- $\mu^+e^- \leftrightarrow \mu^-e^+$ OSCILLATIONS
- $\mu^+ \to e^+ X$, X is "HEAVY" SCALAR
- $\mu \to e^+ \nu X$ "KARMEN ANOMALY"
- IMPROVE μ $g-2$ MEASUREMENT
- ...

CONCLUSIONS

PHYSICS WITH A LARGE # OF STOPPED MUONS IS VERY EXCITING

THERE IS PLENTY OF MOTIVATION FROM BOTH THE S.M. PRECISION MEASUREMENT POINT OF VIEW AND FROM THE P.O.V. OF NEW PHYSICS

LEPTON FLAVOUR VIOLATION IS A NICE, CLEAN WINDOW TO NEW PHYSICS, AND WE SHOULD EXPLORE AS MANY CHANNELS AS POSSIBLE

i.e.

ONE SHOULD NOT "DISMISS" $\mu^+ \to e^+e^-e^+$