4 neutrino analysis of atmospheric neutrinos

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1. Introduction

(1) sterile neutrinos

* $\nu_\odot$ $\Delta m_\odot^2 \sim 10^{-10} eV^2$ or $10^{-5} eV^2$
* ν_{atm} $\Delta m_{atm}^2 \sim 10^{-2.5} eV^2$
* ν_{LSND} $\Delta m_{LSND}^2 \sim 1\ eV^2$

To account for all the anomalies we need $\nu_{active} + \nu_s$.

(2) BBN constraints on 4ν scenarios

N. Okada - O.Y. Int. J. Mod. Phys. A12('97) 3667
Bilenky - Giunti - Grimus - Schwetz
Astropart. Phys. 11 ('99) 413

If we demand the constraint by Bugey (reactor experiment) and the Big Bang Nucleosynthesis constraint $N_\nu < 4.0$ then

$\rightarrow$ 0 (∵ Bugey)

$$U_{MNS} = \left(\begin{array}{cccc} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{array}\right) \begin{array}{c} e \\ \mu \\ \tau \\ s \end{array}$$

0 (∵ BBN $N_\nu < 4$)

$$\Longrightarrow U_{MNS} \simeq \begin{pmatrix} c_\odot & s_\odot & \epsilon & \epsilon \\ \epsilon & \epsilon & c_{atm} & s_{atm} \\ \epsilon & \epsilon & -s_{atm} & c_{atm} \\ -s_\odot & c_\odot & \epsilon & \epsilon \end{pmatrix} \begin{array}{l} |\theta_\odot| \ll 1 \\ \text{SMA MSW} \\ \theta_{atm} \simeq \frac{\pi}{4} \\ \text{maximal mixing} \end{array}$$

in this case

$\nu_\odot$: $\nu_e \leftrightarrow \nu_s$ (SMA MSW)

ν_{atm} : $\nu_\mu \leftrightarrow \nu_\tau$ (almost maximal mixing)

ν_{LSND} : $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$ from small off diagonal components

(3) 4ν scenarios w/o BBN constraints

The cosmological arguments may not be reliable very much.

If we lift the BBN constraints the remaining constraints are :

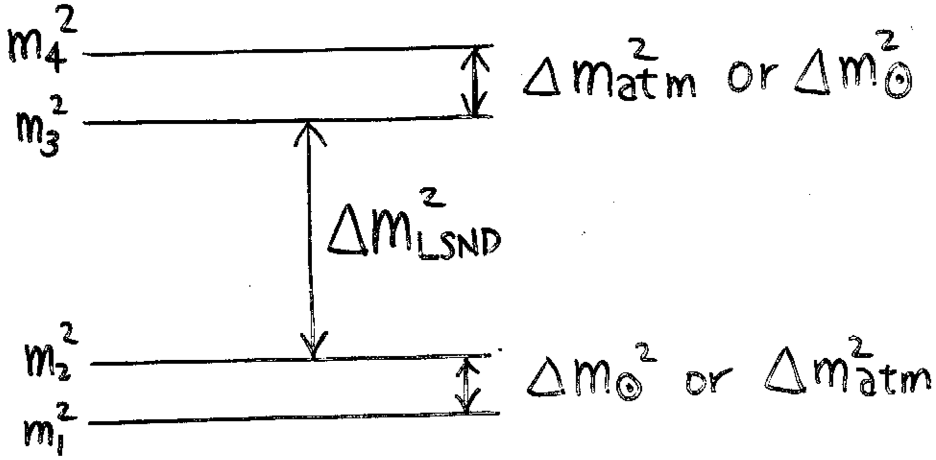
$$\left\{\begin{array}{l}\text{reactors}\\ \text{accelerators}\\ \left.\begin{array}{l}\nu_\odot \longrightarrow \text{Giunti's talk}\\ \nu_{atm} \longrightarrow \text{this talk.}\end{array}\right\}\ \text{w/ } U_{e3} = U_{e4} = 0\end{array}\right.$$

(4) $N_\nu = 4$ mixing matrix

$$U_{MNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

$U_{e3}\ U_{e4} \rightarrow 0$ (∵ Bugey)

$U_{s3}\ U_{s4} \rightarrow 0$ if BBN $N_\nu < 4$ is assumed



<u>6 mixing angles</u>

θ_{12}, θ_{13}, θ_{14}, θ_{23}, θ_{24}, θ_{34}

(if BBN $N_\nu<4$) $\theta_{23} \rightarrow 0$, $\theta_{34} \rightarrow 0$

$\theta_{12} \rightarrow \theta_\odot$

$\theta_{13}, \theta_{14} \rightarrow$ almost 0 (∵ Bugey)

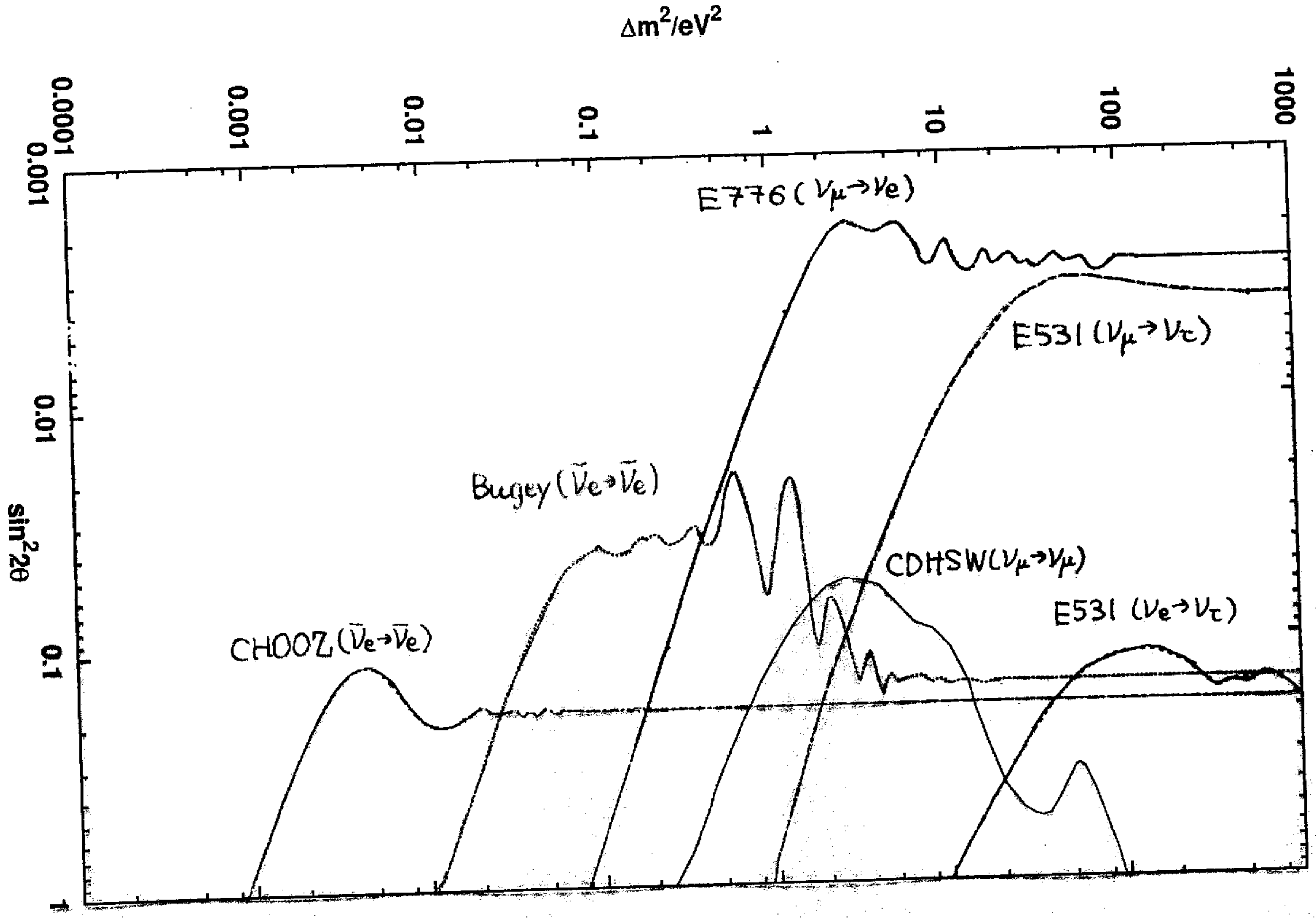
$\theta_{24} \rightarrow \theta_{atm}$

$\theta_{34} \rightarrow \left\{ \text{mixing of} \begin{pmatrix} \nu_\mu \leftrightarrow \nu_\tau \\ \& \\ \nu_\mu \leftrightarrow \nu_s \end{pmatrix} \text{in } \nu_{atm} \right\}$

$\theta_{23} \rightarrow \left\{ \text{mixing of} \begin{pmatrix} \sin^2(\Delta m^2_{atm} L/4E) \\ \& \\ \sin^2(\Delta m^2_{LSND} L/4E) \end{pmatrix} \text{in } \nu_{atm} \right\}$

<u>3 CP phases</u> δ_1, δ_2, δ_3 → decouple as $\theta_{13}, \theta_{14} \rightarrow 0$

$\underbrace{\delta_1}_{\downarrow}$

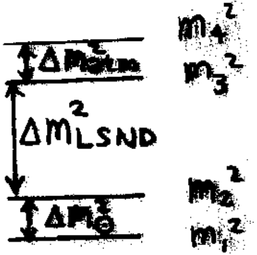

$\Delta m^2/eV^2$
0.0001
0.001
0.01
0.1
1
10
100
1000
$\sin^2 2\theta$
0.001
0.01
0.1
1
E776 ($\nu_\mu \to \nu_e$)
E531 ($\nu_\mu \to \nu_\tau$)
Bugey ($\bar{\nu}_e \to \bar{\nu}_e$)
CDHSW ($\nu_\mu \to \nu_\mu$)
E531 ($\nu_e \to \nu_\tau$)
CHOOZ ($\bar{\nu}_e \to \bar{\nu}_e$)

2. $N_\nu=4$ analysis of ν_{atm} w/o BBN constraints

O.Y.

$$U\,\mathrm{diag}(-\Delta E_{21},0,\Delta E_{32},\Delta E_{32}+\Delta E_{43})U^{-1}+\mathrm{diag}(A_{cc},0,0,-A_{NC})$$

$\Delta E_{jk}\equiv\frac{\Delta m_{jk}^2}{2E}$, $A_{cc}\equiv\sqrt{2}\,G_F N_e$, $A_{NC}\equiv\frac{1}{\sqrt{2}}G_F N_n$



$$U=R_{34}(\tfrac{\pi}{2}-\theta_{34})R_{24}(\theta_{24})R_{23}(\tfrac{\pi}{2})e^{i\delta_1\lambda_3}R_{23}(\theta_{23})e^{-i\delta_1\lambda_3}$$
$$\times\boxed{\xcancel{e^{i\delta_3\lambda_{15}}R_{14}(\theta_{14})e^{-i\delta_3\lambda_{15}}}}\boxed{\xcancel{e^{i\delta_2\lambda_8}R_{13}(\theta_{13})e^{-i\delta_2\lambda_8}}}\boxed{\xcancel{R_{12}(\theta_{12})}}$$

reactor (Bugey) $\Rightarrow\theta_{13}=\theta_{14}=0$

$\Delta E_{21}=\Delta E_\odot\to 0\Rightarrow\theta_{12}$ decouples

Thus

$$U=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -c_{24}s_{23}e^{i\delta_1} & c_{23}c_{24} & s_{24}\\ 0 & -c_{23}s_{34}+c_{34}s_{23}s_{24}e^{i\delta_1} & -c_{23}c_{34}s_{24}-s_{23}s_{34}e^{-i\delta_1} & c_{24}c_{34}\\ 0 & c_{23}c_{34}+s_{23}s_{24}s_{34}e^{i\delta_1} & -c_{23}s_{34}s_{24}+s_{23}c_{34}e^{-i\delta_1} & c_{24}s_{34}\end{pmatrix}$$

<u>w/ BBN $N_\nu<4$</u>
$$\begin{cases}|\theta_{23}|,|\theta_{34}|\ll 1\\ \theta_{24}=\theta_{atm}\simeq\frac{\pi}{4}\end{cases}$$

<u>w/o BBN $N_\nu<4$</u> $\theta_{24},\theta_{23},\theta_{34},\delta_1$: can be of $O(1)$

$N_\nu=4$ analysis of $\nu_\odot$ Giunti-Gonzalez-Garcia-Peña-Gray

$$C_s\equiv|U_{s1}|^2+|U_{s2}|^2=|c_{23}c_{34}+s_{23}s_{24}s_{34}e^{i\delta_1}|^2=\text{"}c_{23}^2c_{24}^2\text{"}$$

$C_s=0$ $\nu_\odot$: $\nu_e\leftrightarrow\nu_{active}$ only

$C_s=1$ $\nu_\odot$: $\nu_e\leftrightarrow\nu_s$ only

$$\nu_\odot\begin{cases}0\le C_s\le 0.2 & : \text{SMA, VO, LMA}\\ 0.2\le C_s\le 0.4 & : \text{SMA, VO}\\ 0.4\le C_s\le 1 & : \text{SMA}\end{cases}$$

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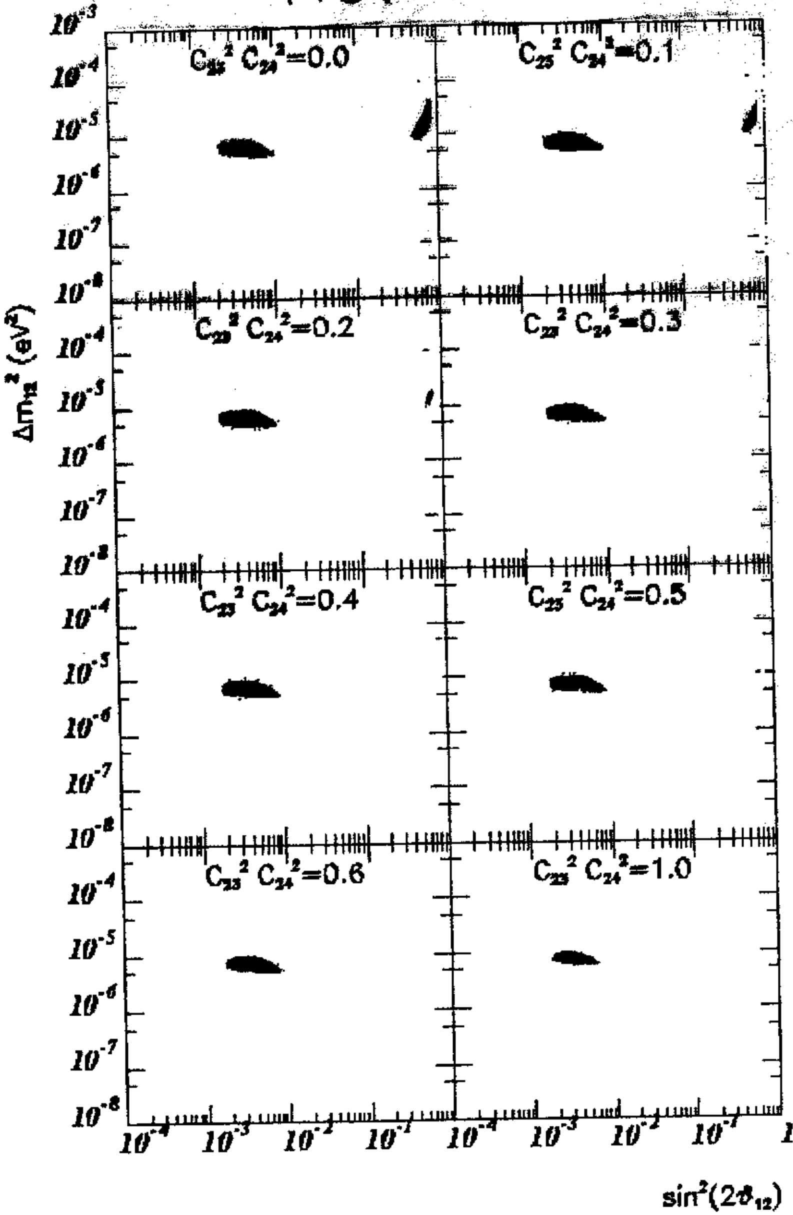
$N_\nu = 4$ analysis of $\nu_\odot$ Giunti-Gonzalez-Garcia-Peña-Gray '00

MSW



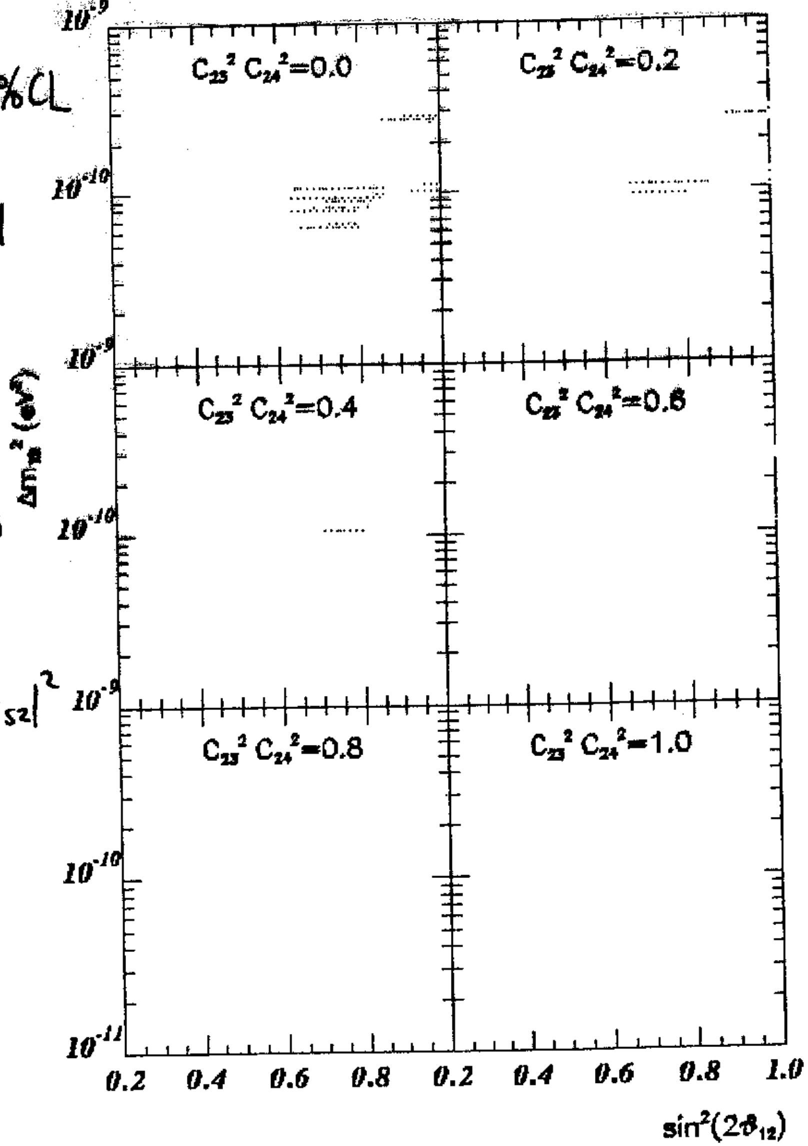
only 90%CL region is displayed

"$C_{23}^2 C_{24}^2$"

$= C_s$

$\equiv |U_{s1}|^2 + |U_{s2}|^2$

vacuum



NB LSND ('95) $\bar{\nu}_\mu \to \bar{\nu}_e$ $0.2\,\mathrm{eV}^2 \lesssim \Delta m^2_{LSND} \lesssim 2.5\,\mathrm{eV}^2$

CDHSW ('84) $\nu_\mu \to \nu_\mu$ $0.3\,\mathrm{eV}^2 \lesssim \Delta m^2_{CDHSW} \lesssim 90\,\mathrm{eV}^2$

disappearance (negative)

If $\Delta m^2_{LSND} \geq 0.4\,\mathrm{eV}^2$ and $1 - P(\nu_\mu \to \nu_\mu)\Big|_{\Delta m^2_{32} = \Delta m^2_{LSND}} > 0.3$

then the scheme doesn't work.

$\Rightarrow \Delta m^2_{LSND} = 0.3\,\mathrm{eV}^2$ as reference value here.

results of analysis of ν_{atm}

$$\chi^2 = \chi^2(\text{SK contained}) + \chi^2(\text{SK upward through going } \mu)$$

best fit : $\Delta m^2_{atm} = 1.0 \times 10^{-3}\,\mathrm{eV}^2$

$\delta_1 = 0$, $\theta_{24} = 40°$, $\theta_{34} = 15°$, $\theta_{23} = 20°$

$\chi^2_{min} = 44$ (d.o.f. = 45)

NB the point $\theta_{24} = \frac{\pi}{4}$, $\theta_{34} = \frac{\pi}{2}$, $\theta_{23} = \frac{\pi}{6}$ is not pure $\nu_\mu \leftrightarrow \nu_s$:

$$P(\nu_\mu \to \nu_\tau) = \frac{3}{8} \sin^2\left(\frac{\Delta m^2_{LSND} L}{4E}\right) \simeq \frac{3}{16}$$

$$P(\nu_\mu \to \nu_s) = \frac{1}{16} \sin^2\left(\frac{\Delta m^2_{LSND} L}{4E}\right) + \frac{3}{4} \sin^2\left(\frac{\Delta m^2_{atm} L}{4E}\right)$$

$$\simeq \frac{1}{32} + \frac{3}{4} \sin^2\left(\frac{\Delta m^2_{atm} L}{4E}\right)$$

NB $\theta_{24} = 30°$ ($\sin^2 2\theta_{24} = 0.75$) is allowed @ 90% CL only for $\theta_{23} \sim 20°$.

(cf. De Rújula - Gavela - Hernandez hep-ph/0001124
$N_\nu = 3$ "The atmospheric ν anomaly w/o maximal mixing?")

in our case

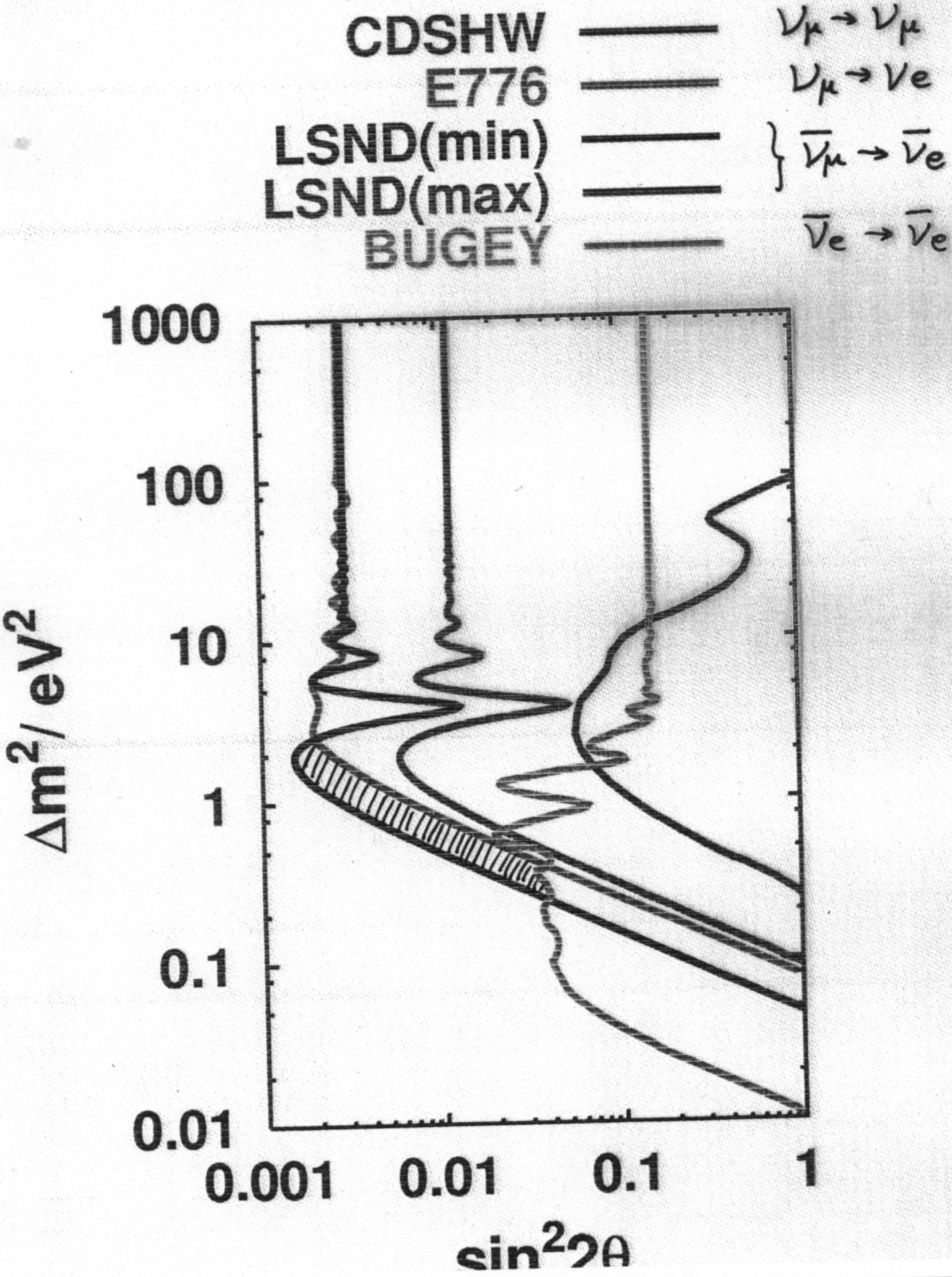
$$|\Delta E_{32}| \gg |\Delta E_{43}| \simeq |A_{NC}|$$

$$(0, (-\Delta E_{32}, \underbrace{0, 0}_{\substack{\text{degeneracy}\\ \text{in unperturbed}\\ \text{system}}})) \xrightarrow[\substack{\uparrow\\ \text{matter}\\ \text{effect}}]{} (0, (-\Delta E_{32} + \epsilon, \epsilon, \epsilon))$$

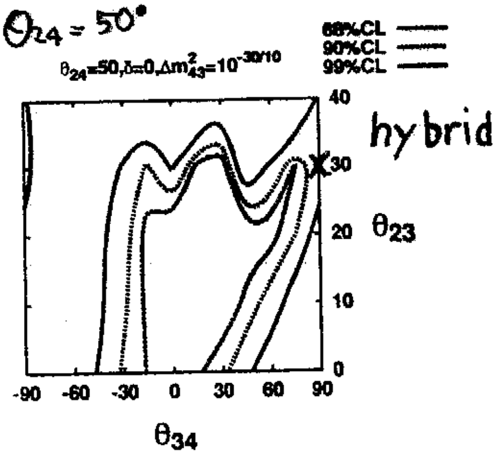
$$U_M \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$\theta \sim O(1)$

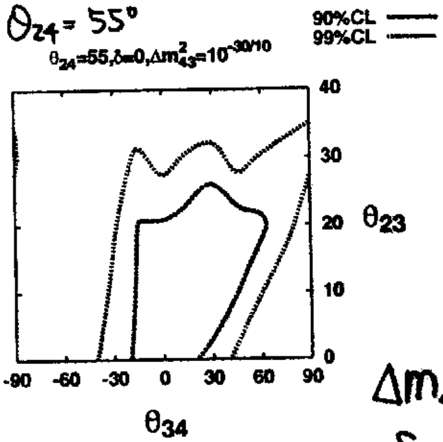
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$$: no strong constraint on U


CDSHW $\nu_\mu \to \nu_\mu$
E776 $\nu_\mu \to \nu_e$
LSND(min)
LSND(max)
} $\bar{\nu}_\mu \to \bar{\nu}_e$
BUGEY $\bar{\nu}_e \to \bar{\nu}_e$
1000
100
10
1
0.1
0.01
$\Delta m^2 / eV^2$
0.001
0.01
0.1
1
$\sin^2 2\theta$

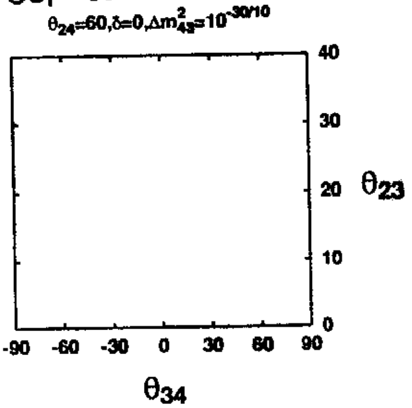
$\theta_{24} = 50°$



$\theta_{24} = 55°$



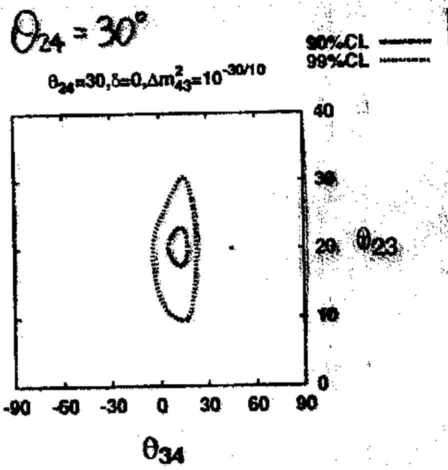
$\theta_{24} = 60°$



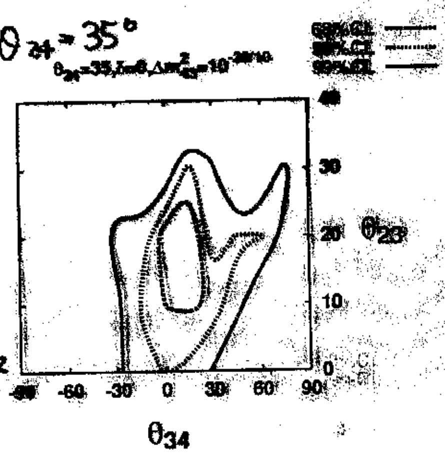
$\Delta m^2_{43} = 1 \times 10^{-3}\ eV^2$

$\delta_1 = 0$

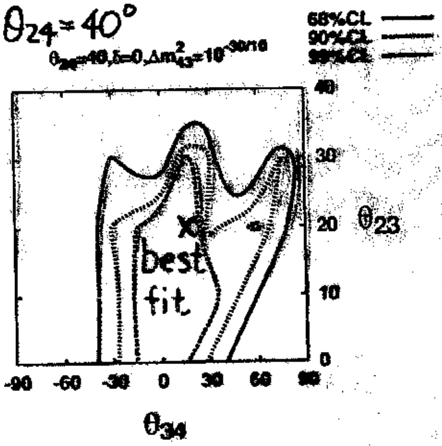
$\theta_{24} = 30°$



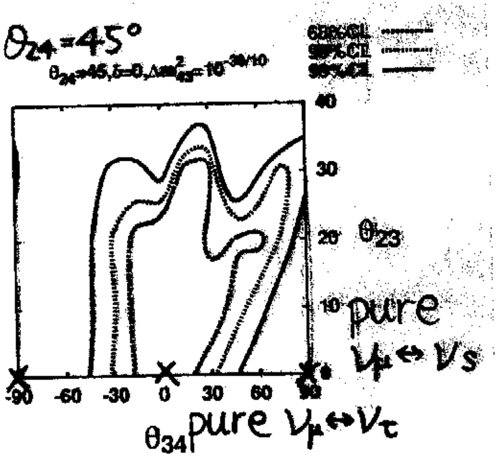
$\theta_{24} = 35°$

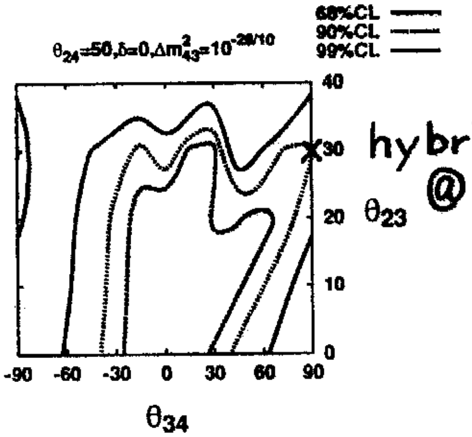


$\theta_{24} = 40°$

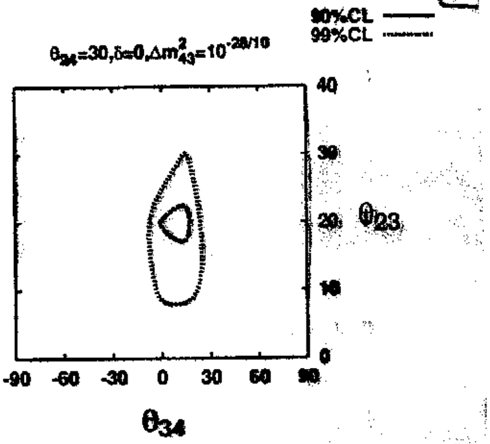


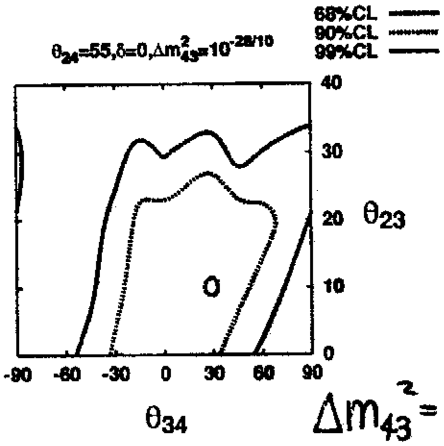
$\theta_{24} = 45°$

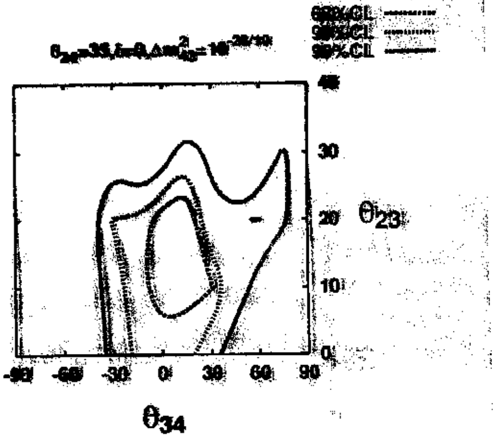




hybrid
@90%CL

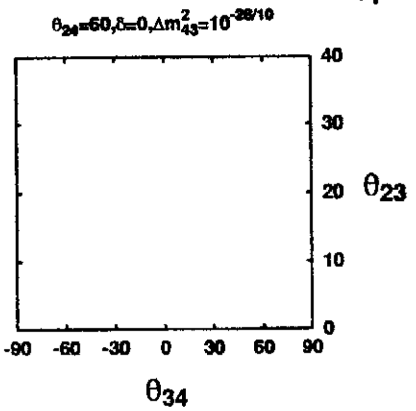


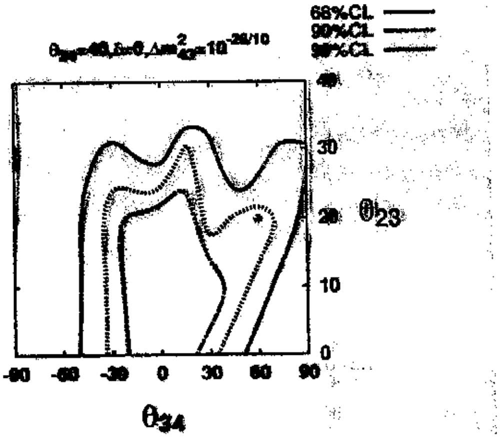


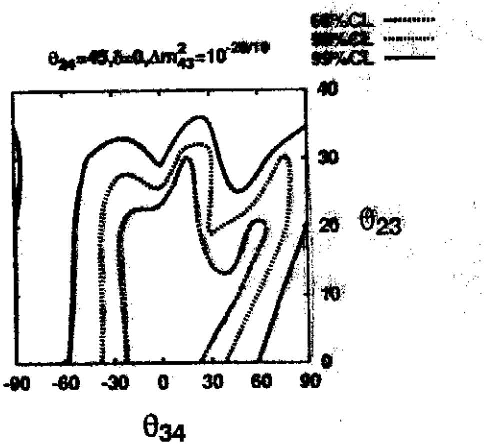


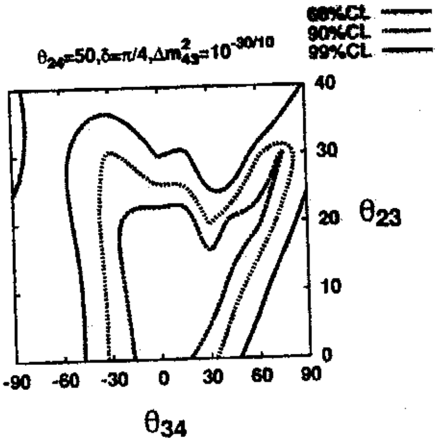
$\Delta m^2_{43} = 1.6 \times 10^{-3} eV^2$

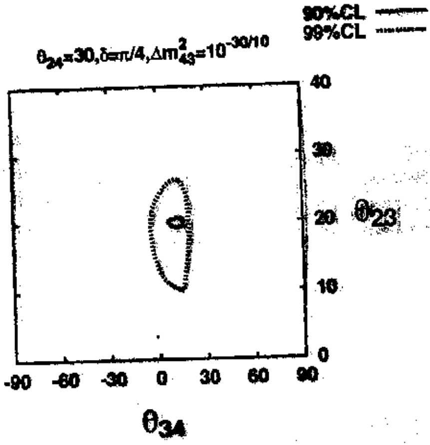
$\delta_1 = 0$

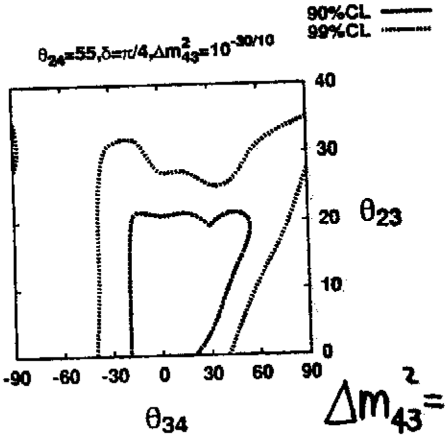


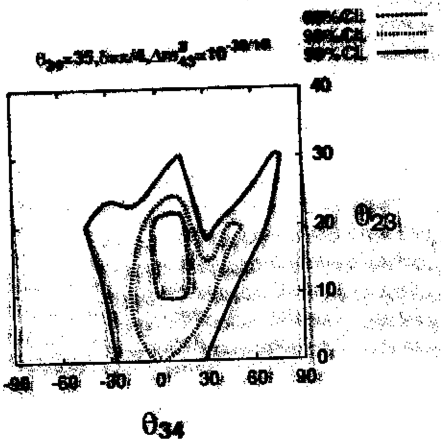






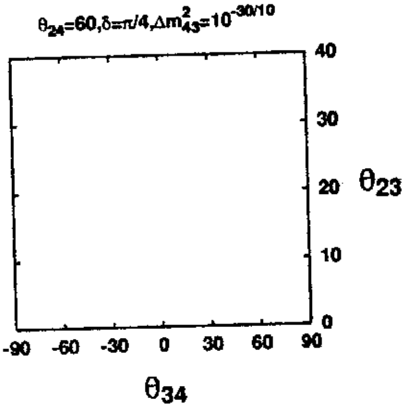


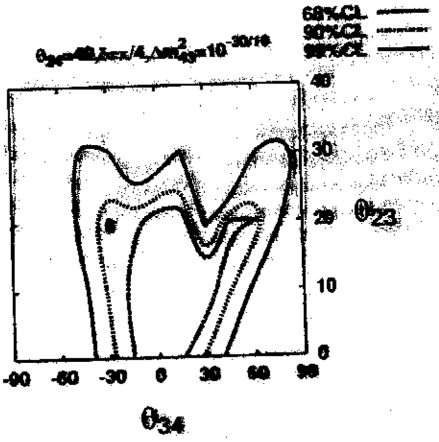


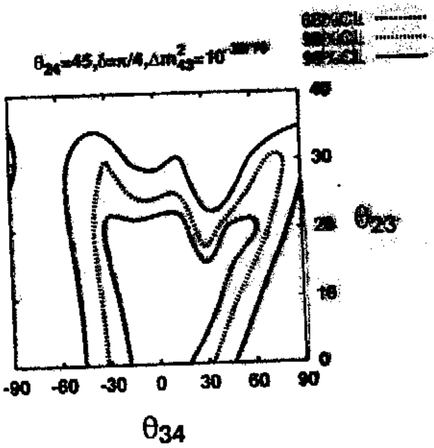


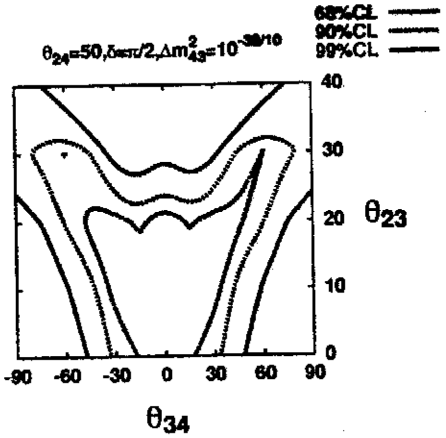
$\Delta m_{43}^2 = 1.0\times10^{-3} eV^2$

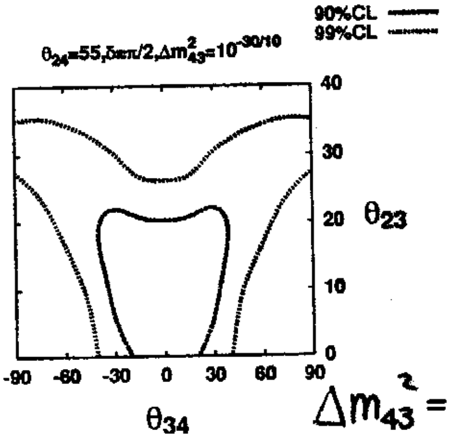
$\delta_1 = \frac{\pi}{4}$

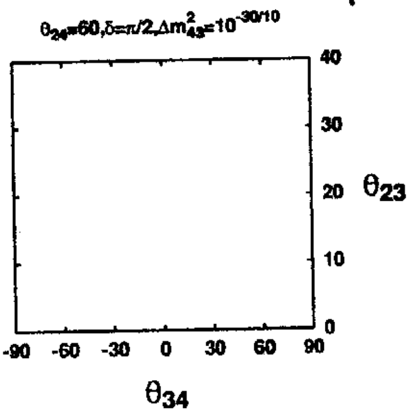






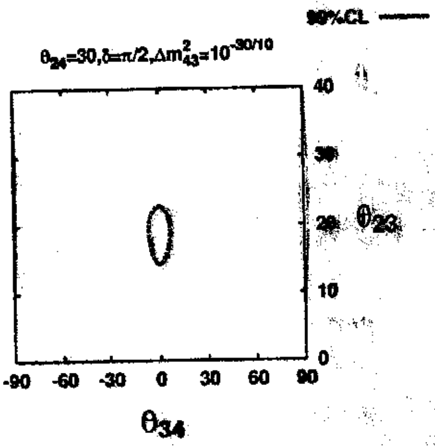


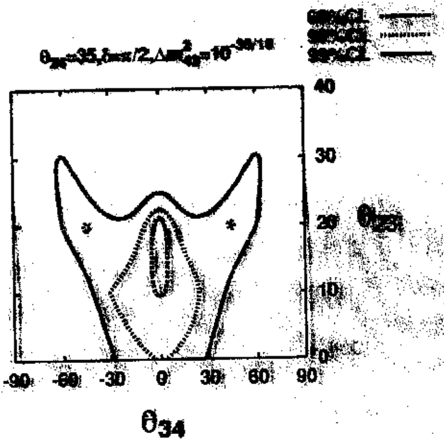


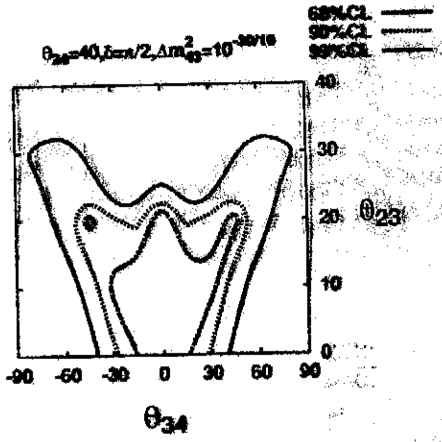


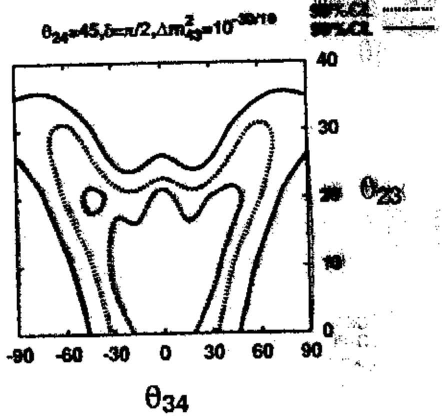
$$\Delta m_{43}^2 = 1.0 \times 10^{-3} eV^2$$

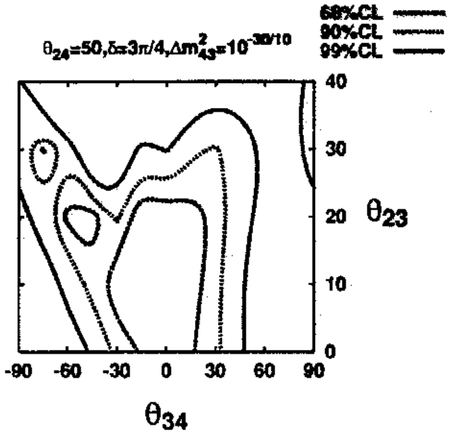
$$\delta_1 = \frac{\pi}{2}$$

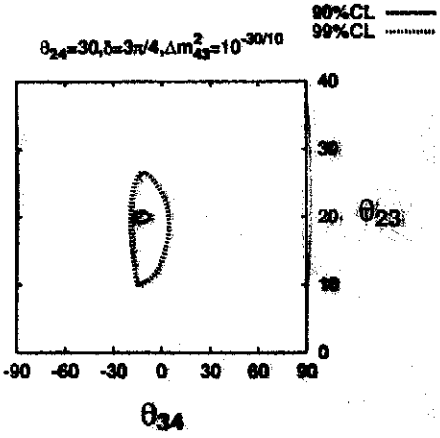


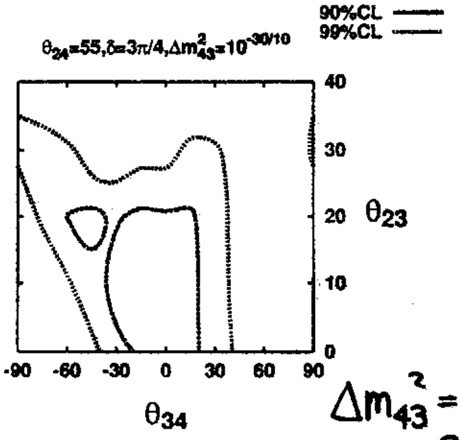


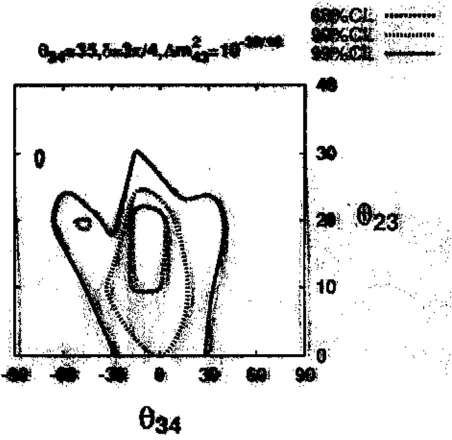





θ_{24}=50,δ=3π/4,Δm^2_{43}=$10^{-30/10}$

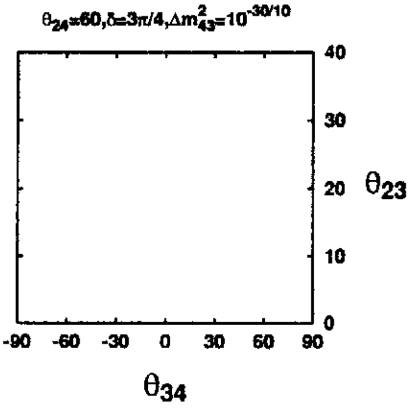

θ_{24}=30,δ=3π/4,Δm^2_{43}=$10^{-30/10}$

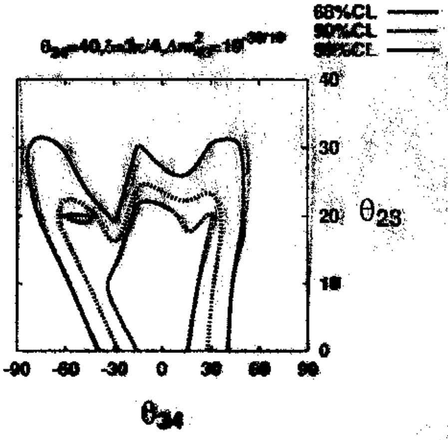

θ_{24}=55,δ=3π/4,Δm^2_{43}=$10^{-30/10}$

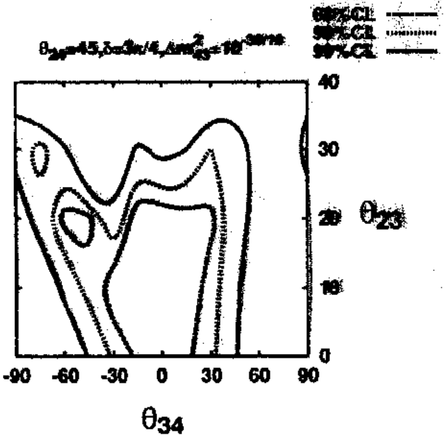

θ_{24}=35,δ=3π/4,Δm^2_{43}=$10^{-30/10}$

$\Delta m^2_{43} = 1.0 \times 10^{-3} eV^2$

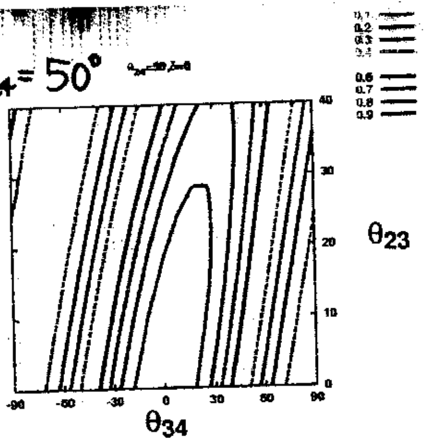
$\delta_1 = \frac{3}{4}\pi$


θ_{24}=60,δ=3π/4,Δm^2_{43}=$10^{-30/10}$

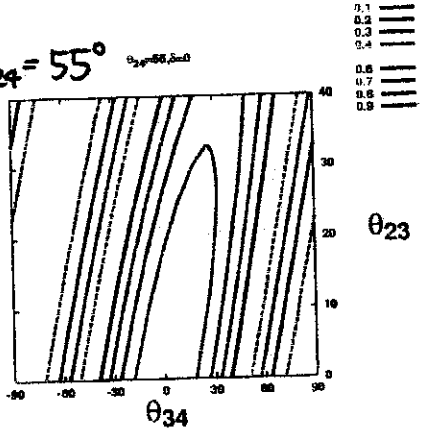

θ_{24}=40,δ=3π/4,Δm^2_{43}=$10^{-30/10}$


θ_{24}=45,δ=3π/4,Δm^2_{43}=$10^{-30/10}$

$\theta_{24} = 50°$



$\theta_{24} = 55°$



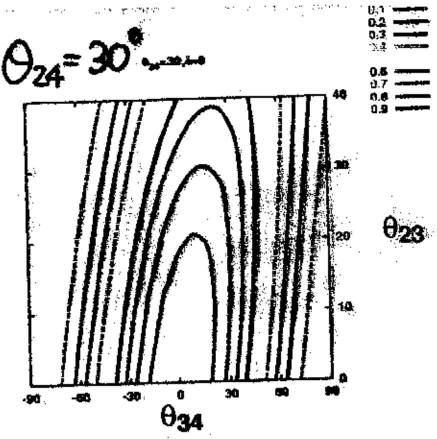
$\delta_1 = 0$

C_s
0.1 ▬
0.2 ▬
0.3 ▬
0.4 ▬

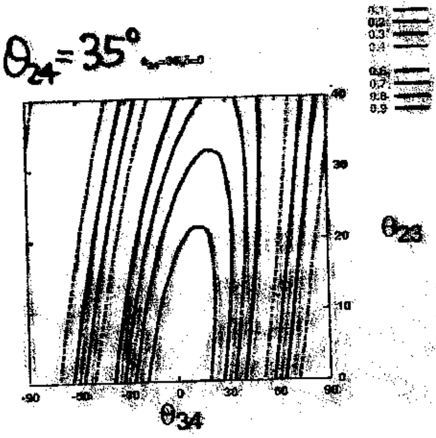
0.6 ▬
0.7 ▬
0.8 ▬
0.9 ▬

$$C_s \equiv |U_{s1}|^2 + |U_{s2}|^2$$

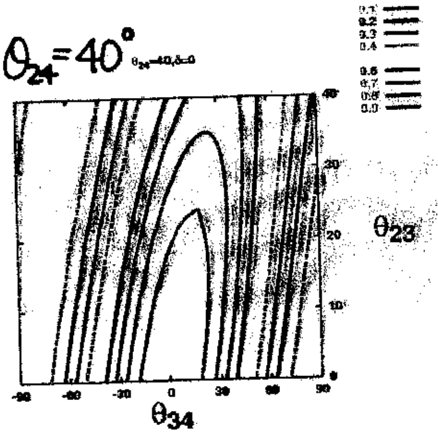
$\theta_{24} = 30°$



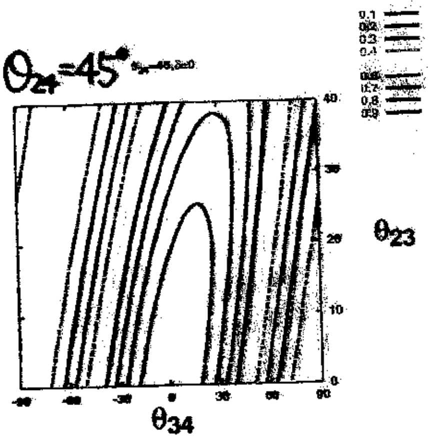
$\theta_{24} = 35°$

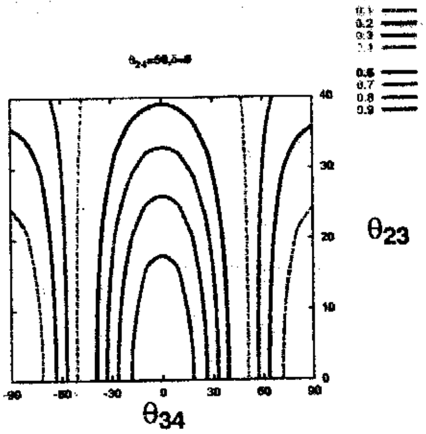


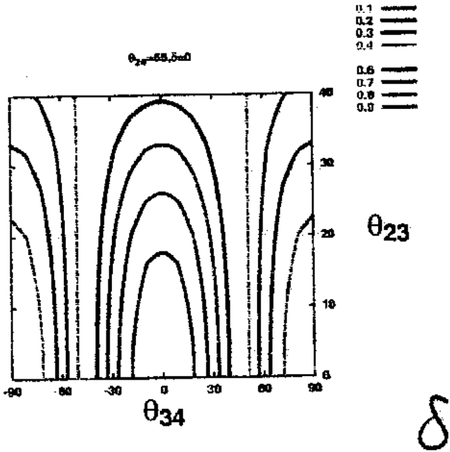
$\theta_{24} = 40°$



$\theta_{24} = 45°$



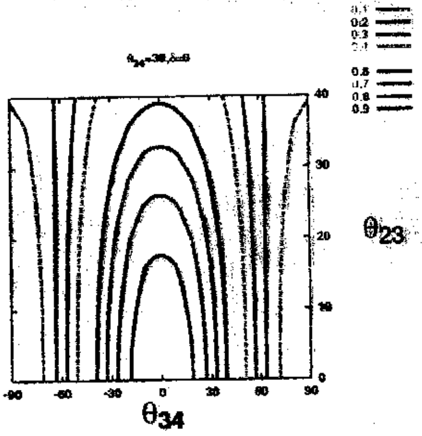

θ_{24}=50,δ=0

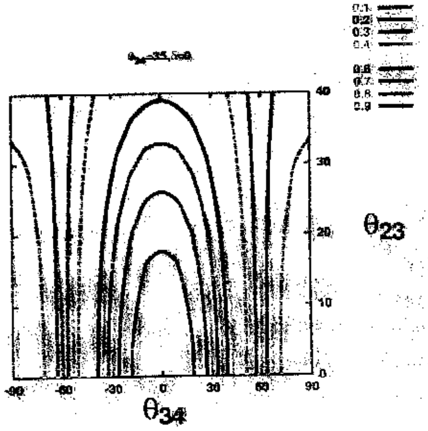

θ_{24}=55,δ=0

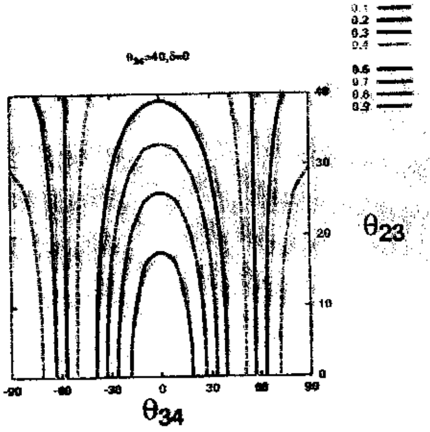
$\delta_1 = \frac{\pi}{2}$

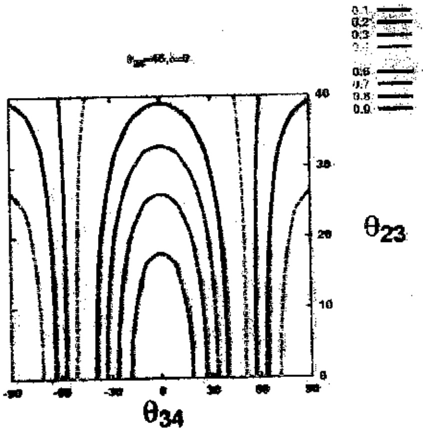
C_s	
0.1	▬
0.2	▬
0.3	▬
0.4	▬
0.6	▬
0.7	▬
0.8	▬
0.9	▬

$C_s = |U_{s1}|^2 + |U_{s2}|^2$

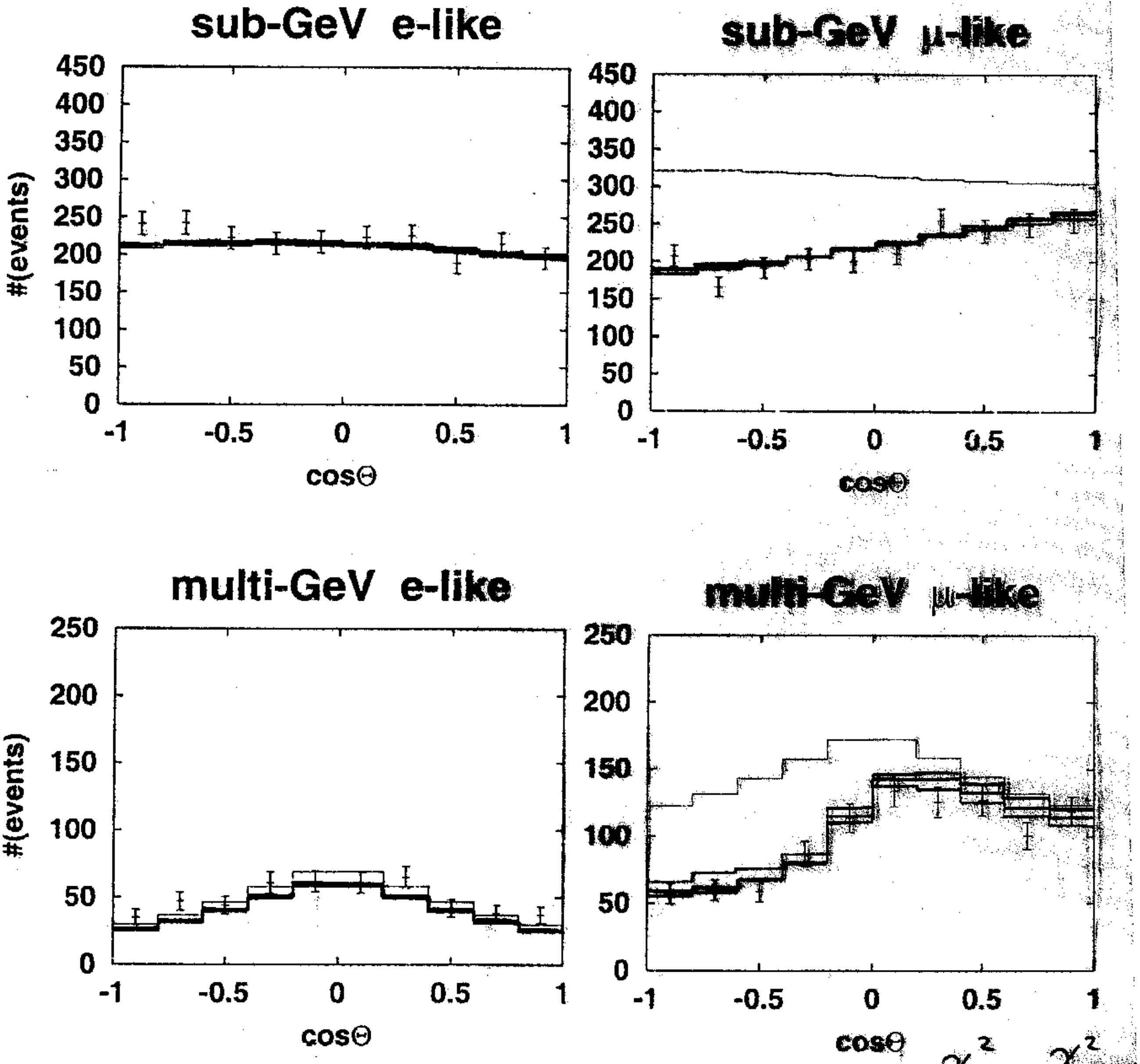

θ_{24}=30,δ=0


θ_{24}=35,δ=0


θ_{24}=40,δ=0


θ_{24}=45,δ=0

contained events

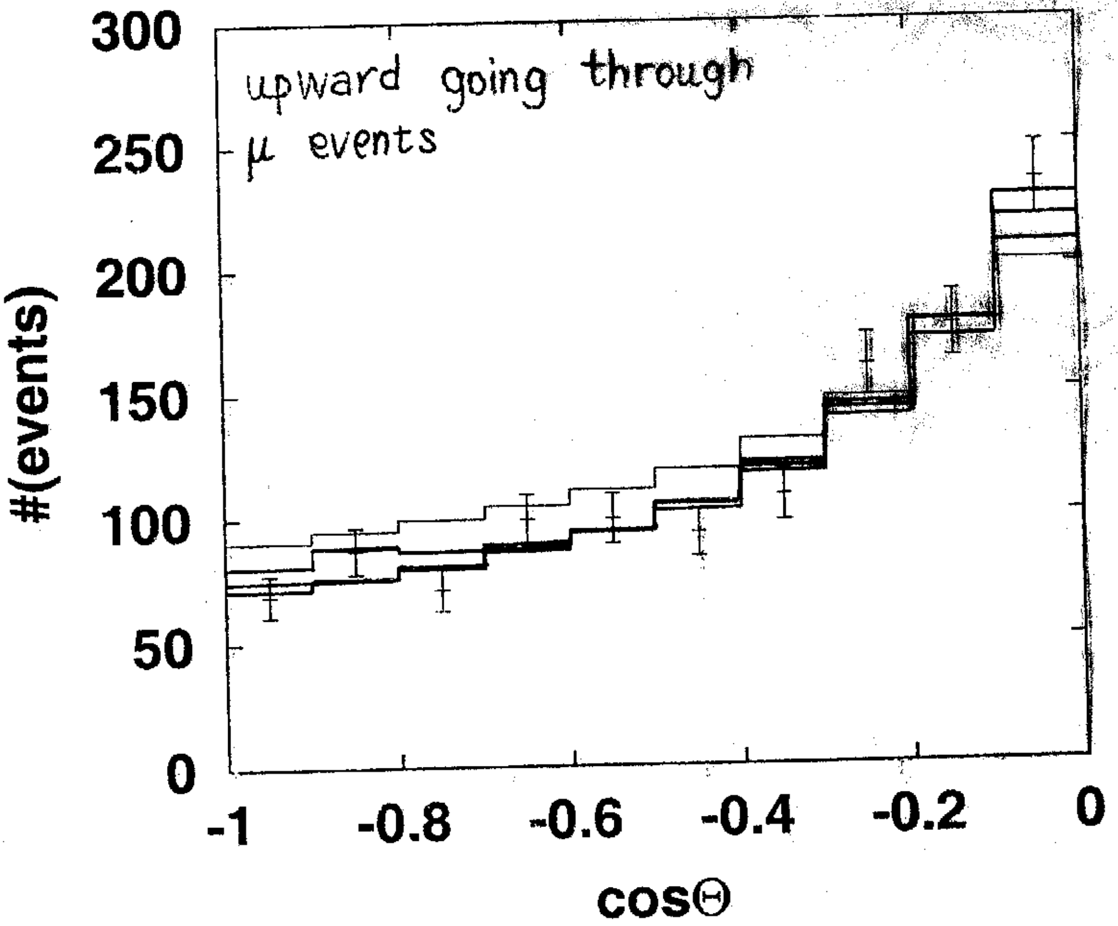


	χ^2_{sub}	χ^2_{multi}
$\Delta m^2_{43}=10^{-29/10}eV^2, \theta_{24}=50^\circ, \theta_{34}=90^\circ, \theta_{23}=30^\circ$	19.0	17.0
$\Delta m^2_{43}=10^{-30/10}eV^2, \theta_{24}=40^\circ, \theta_{34}=15^\circ, \theta_{23}=20^\circ$	19.0	14.1
$\Delta m^2_{43}=10^{-26/10}eV^2, \theta_{24}=45^\circ, \theta_{34}=0, \theta_{23}=0$	19.1	19.7
no oscillation		

$\chi^2_{total} = 48.8$ (best fit among pure $\nu_\mu \leftrightarrow \nu_\tau$)
$\chi^2_{total} = 44.7$ (best fit)
$\chi^2_{total} = 51.0$ (allowed at 90 % cL)

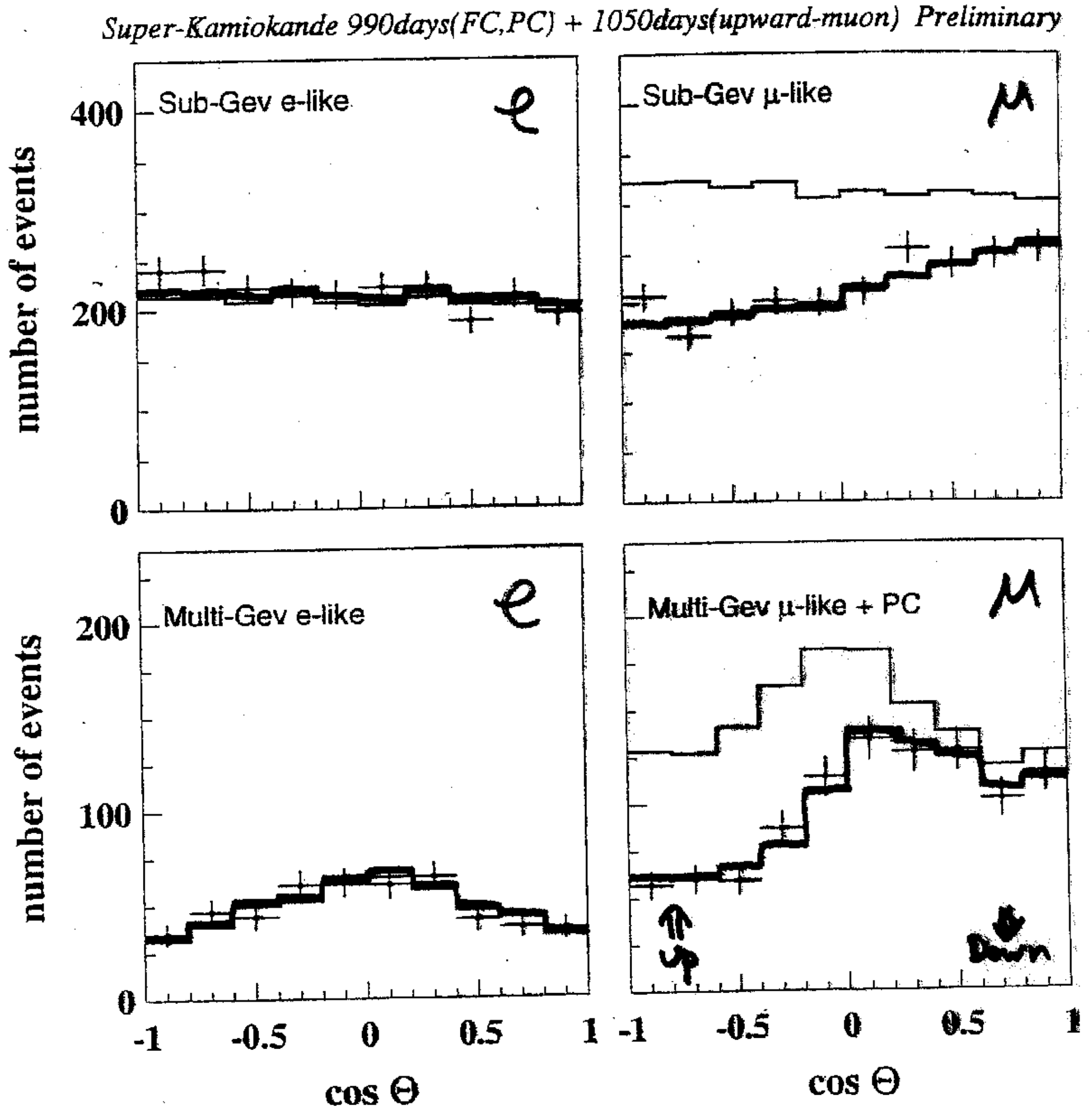
no oscillation

$\Delta m^2_{43}=10^{-29/10}eV^2,\theta_{24}=50^\circ,\theta_{34}=90^\circ,\theta_{23}=30^\circ$ $\chi^2_{up}=15.0$
$\Delta m^2_{43}=10^{-30/10}eV^2,\theta_{24}=40^\circ,\theta_{34}=15^\circ,\theta_{23}=20^\circ$ $\chi^2_{up}=11.6$
$\Delta m^2_{43}=10^{-26/10}eV^2,\theta_{24}=45^\circ,\theta_{34}=0,\theta_{23}=0$ $\chi^2_{up}=10.0$

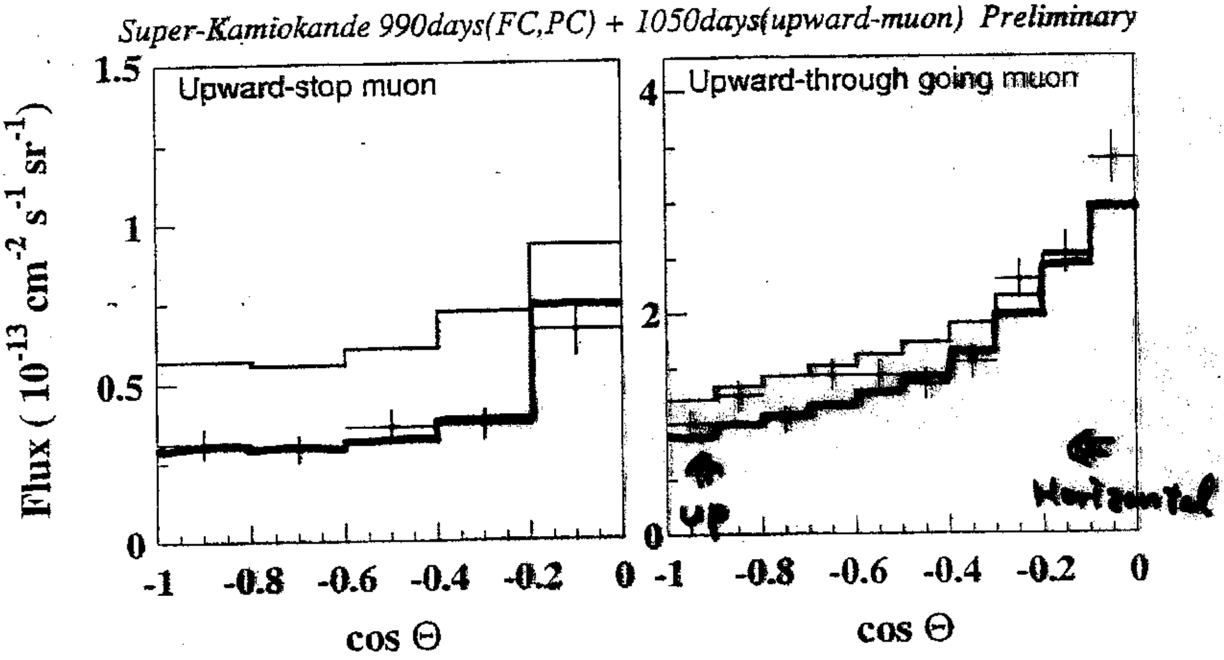


Zenith angle

$\nu_\mu \rightarrow \nu_\tau \left(\begin{array}{l} \Delta m^2 = 2.8 \times 10^{-3} eV^2 \\ \sin^2 2\Theta = 1 \end{array} \right)$



Super-Kamiokande 990days(FC,PC) + 1050days(upward-muon) Preliminary



χ^2 (best fit) = 61.1/82 d.o.f

$(\sin^2 2\theta, \Delta m^2) = (1.00, 2.8 \times 10^{-3} eV^2)$ (including unphysical region)

χ^2 (no osc.) = 232.5/84 d.o.f

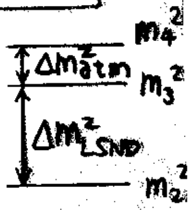
3. Discussions

(1) CP violation

Since $\theta_{12} = \theta_{\odot}\big|_{SMA} \ll 1$, effectively

$$U_{MNS} \sim \begin{pmatrix} 1 & \begin{matrix} 0 & \qquad\qquad\qquad 0 & \qquad\qquad\qquad 0 \end{matrix} \\ \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} & \boxed{\begin{matrix} -C_{atm}S_{23}e^{i\delta_1} & C_{23}C_{atm} & S_{atm} \\ -C_{23}S_{34}+C_{34}S_{23}S_{atm}e^{i\delta_1} & -C_{23}C_{34}S_{atm}-S_{23}S_{34}e^{-i\delta_1} & C_{atm}C_{34} \\ C_{23}C_{34}+S_{23}S_{34}S_{atm}e^{i\delta_1} & -C_{23}S_{34}S_{atm}-S_{23}C_{34}e^{-i\delta_1} & C_{atm}S_{34} \end{matrix}} \end{pmatrix}$$

↳ full mixing between $\begin{pmatrix} \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix}$ & $\begin{pmatrix} \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$



$$\cancel{CP} \quad P(\nu_\mu \to \nu_\tau) - P(\bar{\nu}_\mu \to \bar{\nu}_\tau)$$

$$\simeq \frac{1}{8} \underbrace{\boxed{C_{atm} \sin 2\theta_{atm} \sin 2\theta_{23} \sin 2\theta_{34}}}_{\text{could be } O(1)} \sin\left(\frac{\Delta m^2_{atm} L}{2E}\right) \sin\delta_1$$

measurement of $\cancel{CP}$ in this channel may be a nice test of

- $N_\nu = 4$ scenario
- BBN constraint $N_\nu < 4$
- LSND

although this is off the main idea of ν factories.

(2) Fate of sterile ν scenarios

"standard"

$$U_{MNS} \sim \begin{pmatrix} C_{\odot} & S_{\odot} & 0 & 0 \\ 0 & 0 & C_{atm} & S_{atm} \\ 0 & 0 & -S_{atm} & C_{atm} \\ -S_{\odot} & C_{\odot} & 0 & 0 \end{pmatrix} \quad \theta_{\odot} : \text{SMA MSW}$$

consistent with
- $\nu_{\odot}$ $\nu_e \leftrightarrow \nu_s$ SMA
- ν_{atm} $\nu_\mu \leftrightarrow \nu_\tau$
- ν_{LSND}
- BBN $N_\nu < 4$

This will die if SNO & SK prove that $\nu_{\odot}$ is $\nu_e \leftrightarrow \nu_{active}$

more general

$$U_{MNS} \sim \begin{pmatrix} C_{\odot} & S_{\odot} & 0 & 0 \\ S_{\odot} C_{atm} S_{23} & -C_{\odot} C_{atm} S_{23} e^{i\delta_1} & C_{atm} C_{23} & S_{atm} \\ S_{\odot} w & -C_{\odot} w & U_{\tau 3} & C_{atm} C_{34} \\ -S_{\odot} z & C_{\odot} z & U_{s3} & C_{atm} S_{34} \end{pmatrix}$$

$w \equiv C_{23} S_{34} - C_{34} S_{23} S_{atm} e^{i\delta_1}$, $z \equiv C_{23} C_{34} - S_{34} S_{23} S_{atm} e^{i\delta_1}$

$U_{\tau 3} \equiv -C_{23} C_{34} S_{atm} - S_{23} S_{34} e^{-i\delta_1}$, $U_{s3} \equiv -C_{23} S_{34} S_{atm} - S_{23} C_{34} e^{-i\delta_1}$

consistent with
- $\nu_{\odot}$ mixtures of $\nu_e \leftrightarrow \nu_{active}$, $\nu_e \leftrightarrow \nu_s$
- ν_{atm} mixtures of $\nu_\mu \leftrightarrow \nu_\tau$, $\nu_\mu \leftrightarrow \nu_s$
- ν_{LSND}

not consistent with BBN $N_\nu < 4$

If we forget about BBN constraint, this scenario won't die until it is shown that $\nu_{\odot}$ is almost of $\nu_e \leftrightarrow \nu_{active}$ and high statistics of upward going μ requires $\theta_{23} \simeq 0$, or that ν_{LSND} is wrong.

Maybe they'll survive for a couple more years!

4. Conclusions

4 ν scenario w/o BBN constraint ($N_\nu<4$)

$\longrightarrow$ ν_{atm} is accounted for wide range of parameters :

$$30^\circ \lesssim \theta_{24} \equiv \theta_{atm} \lesssim 55^\circ$$

$$6\times 10^{-4}\,\mathrm{eV}^2 \lesssim \Delta m^2_{43} \lesssim 7\times 10^{-3}\,\mathrm{eV}^2$$

$$-45^\circ \lesssim \theta_{34} \lesssim 90^\circ$$

$$0 \leq \theta_{23} \lesssim 30^\circ$$

$$0 \leq \delta_1 \leq 2\pi$$

The parameter region near pure $\nu_\mu \leftrightarrow \nu_s$ is excluded. $\Longleftrightarrow$ $\left\{\begin{array}{l}\text{consistent with}\\ \text{SK analysis}\end{array}\right\}$

$$1 \geq C_s \equiv |U_{s1}|^2 + |U_{s2}|^2 \gtrsim 0.2$$

$\uparrow$

VO, LMA sol. may be possible