Measuring Leptonic $\cancel{CP}$ (or $\cancel{T}$)

in ν Oscillation Experiments

would be a difficult and challenging task but could be very important

N. Cabbibo, PLB 72 (1978) 333
V. Barger et al, PRL 45 (1980) 2084

- Why not $\cancel{CP}$ also in lepton sector?

- Leptonic $\cancel{CP}$ $\longleftrightarrow$ Baryon assymmetry in the universe

Fukugita & Yanagida
PLB 174 (1986) 45

We need certain conditions to observe $\cancel{CP}$ in ν oscillation

- 3 (or more) flavor
- ν oscillation must be confirmed

In case of 3 ν

- Any of $\theta_{12}, \theta_{23}, \theta_{13}$ cannot be too small
- Any of $\Delta m^2_{12}, \delta m^2_{13}$ cannot be too small
- Matter Effect must be considered

Refs. of $\not{CP}$ ($\not{T}$) (incomplete)

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[2] V. Barger, K. Wisnant and R. J. N. Phillips, Phys. Rev. Lett. **45** (1980) 2084.

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[18] J. Bernabeu and M.C. Banuls, hep-ph/0003299.

[19] G. Barenboim and F. Scheck, Phys. Lett. B475 (2000) 95.

Idea

- Try to look for the situation where $\cancel{CP}$ effect is large, assuming that ν_{atm} & $\nu_{\odot}$ problem are explained by ν oscillation

- Can we do experiment in such situation?

In order to have appreciable $\cancel{CP}$ effect,

1) J must not be so small

2) $$\Delta_{12} L = 0.26 \left[\frac{\Delta m^2_{12}}{10^{-5}\,\mathrm{eV}^2}\right]\left[\frac{L}{1000\,\mathrm{km}}\right]\left[\frac{E}{100\,\mathrm{MeV}}\right]^{-1}$$

must not be so small

$\Rightarrow$ LMA MSW is the most optimistic case

If $\Delta m^2_{12} \sim$ few $\times 10^{-5}\,\mathrm{eV}^2$

$E \sim 100\,\mathrm{MeV}$ to have large $\cancel{CP}$ effect

with such low energy, we cannot use $\nu_e \to \nu_\mu, \nu_\tau$ or $\nu_\mu \to \nu_\tau$ channels but

$\boxed{\nu_\mu \to \nu_e}$ and $\boxed{\bar{\nu}_\mu \to \bar{\nu}_e}$

$$P(\nu_\mu \to \nu_e) = 4 s_{23}^2 c_{13}^2 s_{13}^2 \sin^2\left(\tfrac{1}{2}\Delta_{13} L\right)$$

$$+ c_{13}^2 \sin 2\theta_{12}^M \left[(c_{23}^2 - s_{23}^2 s_{13}^2)\sin 2\theta_{12}^M + 2 c_{23} s_{23} c_{13} \cos\delta \cos 2\theta_{12}^M\right]$$

$$\times \sin^2\left[\tfrac{1}{2}\sqrt{\left(\cos 2\theta_{12} - \tfrac{a}{\Delta_{12}} c_{13}^2\right)^2 + \sin^2 2\theta_{12}}\;\Delta_{12} L\right]$$

$$- 2 J_M(\theta_{12}^M) \sin\left[\sqrt{\left(\cos 2\theta_{12} - \tfrac{a}{\Delta_{12}} c_{13}^2\right)^2 + \sin^2 2\theta_{12}}\;\Delta_{12} L\right]$$

$$J_M(\theta_{12}^M) \equiv \cos\theta_{12}^M \sin\theta_{12}^M c_{23} s_{23} c_{13}^2 s_{13} \sin\delta$$

$\cancel{CP}$ ($\cancel{T}$) can be matter enhanced!

J. Sato & M. Koike, hep-ph/9911258

But, here we cannot use this mechanism since we want to compare $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ (for $\cancel{T}$, it is OK)

We need to look for the condition where matter effect is small

In fact if

$$\eta \equiv \sqrt{\left(\cos 2\theta_{12} - \tfrac{a}{\Delta_{12}} c_{13}^2\right)^2 + \sin^2 2\theta_{12}}\;\Delta_{12} L$$

is small,

$$\sin 2\theta_{12}^M \sin(\eta) \sim \Delta_{12} \sin 2\theta_{12}$$

De Rujula et al NPB 547 (1999)

$\Rightarrow P(\nu_\mu \to \nu_e) \sim P(\nu_\mu \to \nu_e)$

Let us consider the matter effect

$$i\frac{d}{dt}\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix} = \left\{ U\begin{bmatrix}0&0&0\\0&\Delta_{12}&0\\0&0&\Delta_{13}\end{bmatrix}U^\dagger + \begin{bmatrix}a&0&0\\0&0&0\\0&0&0\end{bmatrix}\right\}\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix}$$

$$a = \sqrt{2}G_F N_e$$

we note that with $E \sim 100$ MeV,

$$\Delta_{13} \gg a \sim \Delta_{12}$$

$$\frac{a}{\Delta} \sim 2\left[\frac{\rho}{2.1\,g/cc}\right]\left[\frac{Y_e}{0.5}\right]\left[\frac{\Delta m^2}{10^{-5}\,eV^2}\right]^{-1}\left[\frac{E}{100\,MeV}\right]$$

- we can formulate a perturbation theory treating a and Δ_{12} as perturbations

We can rewrite the ν evolution eq. as

$$i\frac{d}{dx}\tilde{\nu} = (H_0 + H')\tilde{\nu}$$

$$\tilde{\nu} \equiv \left[e^{-i\lambda_5\theta_{13}} \Gamma_\delta^\dagger e^{-i\lambda_7\theta_{23}} \right] \nu$$

λ_i: Gell-Mann's SU(3) matrix

Kuo & Pantaleone PRD 35 (1987) 3432

$\Gamma_\delta^\dagger \equiv \mathrm{diag}[1, 1, e^{i\delta}]$

$H_0 = \mathrm{diag}[0, 0, \Delta_{13}]$

$$H' = \Delta_{12}\begin{bmatrix} s_{12}^2 & c_{12}s_{12} & 0 \\ c_{12}s_{12} & c_{12}^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} + a\begin{bmatrix} c_{13}^2 & 0 & c_{13}s_{13} \\ 0 & 0 & 0 \\ c_{13}s_{13} & 0 & s_{13}^2 \end{bmatrix}$$

Energy eigenvalues of 2x2 submatrix

$$h_{1,2} = \frac{1}{2}\left[c_{13}^2 a + \Delta_{12} \pm \sqrt{(\cos 2\theta_{12}\Delta_{12} - a c_{13})^2 + \Delta_{12}^2 \sin^2 2\theta_{12}} \right]$$

$$\sin^2 2\theta_M = \frac{\sin 2\theta_{12}}{\sqrt{(\cos 2\theta_{12} - \frac{a}{\Delta_{12}} c_{13}^2)^2 + \sin^2 2\theta_{12}}}$$

MSW resonance condition

$$c_{13}^2 \sqrt{2} G_F N_e = \Delta_{12} \cos 2\theta_{12}$$

We found that with

$$\begin{cases} \Delta m^2_{13} = 3\times10^{-3}\,eV^2, & \sin^2 2\theta_{23} = 1 \\ \Delta m^2_{12} = 2.7\times10^{-5}\,eV^2, & \sin^2 2\theta_{12} = 0.79 \\ \sin^2 2\theta_{13} = 0.1 & \\ \delta = \pi/2 & \end{cases}$$

$J \sim 0.035$

$\Delta P_{\mu e} \sim 0.06 \left(\frac{L}{500\,km}\right)$ at $E \sim 100$ MeV

Reasonably Large!

Some difficulties

1) smaller cross sections

2) lower flux due to larger beam opening angle $\Delta\theta \sim 1\,(E/100\,\text{MeV})^{-1}$ rad

we should try to keep L shorter

but cannot be so short to have large ~~CP~~

$L \sim 250 - 500$ Km seems preferable

$\cancel{CP}$ in vacuum

Let us assum 3 ν flavor

Let us first consider ν oscillation in vacuum

Under the assumption,

$$\Delta m^2_{12} = \Delta m^2_{\odot} \ll \Delta m^2_{13} = \Delta m^2_{atm},$$

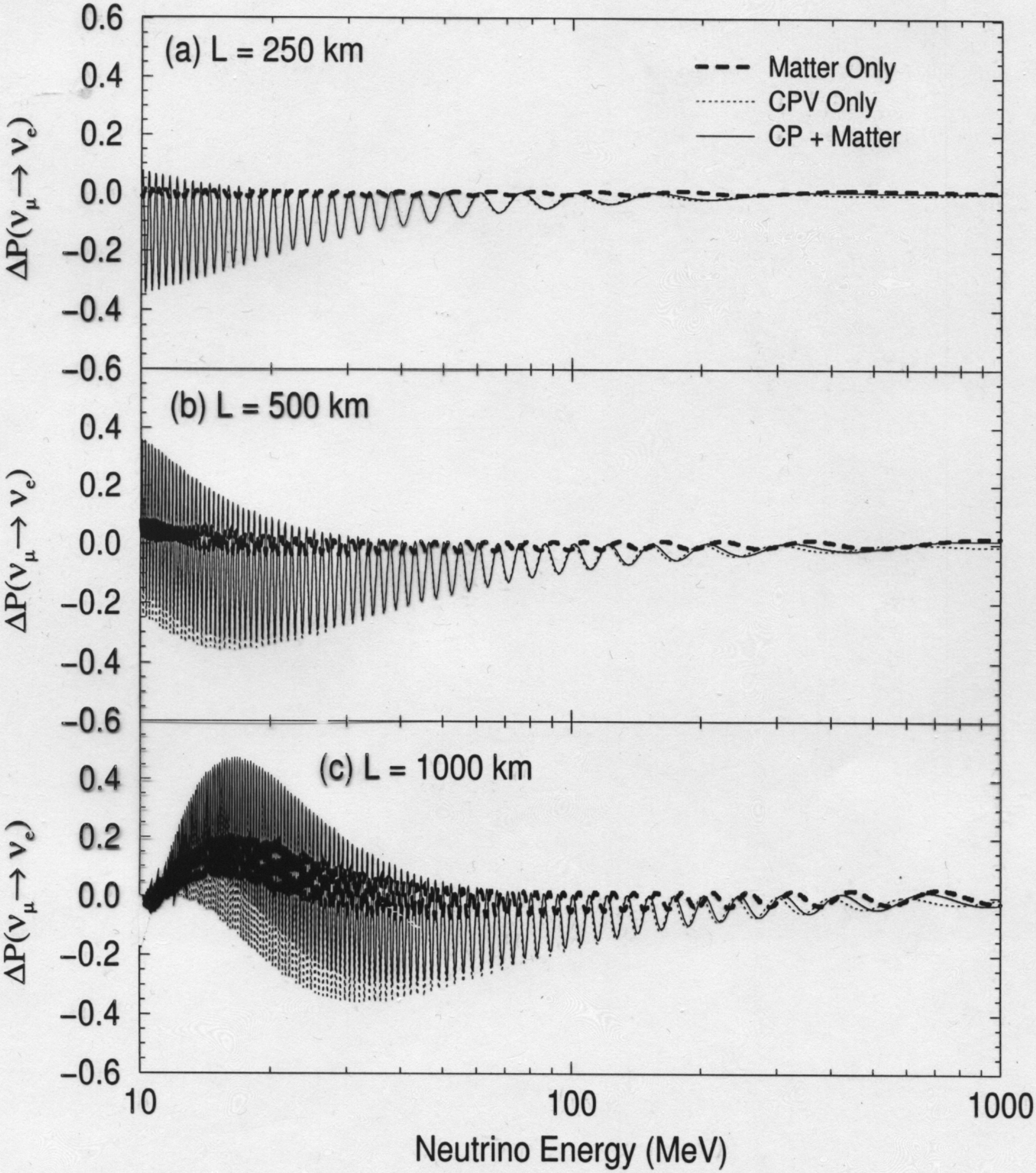
$$P(\nu_\beta \to \nu_\alpha) = 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2 \sin^2\left(\frac{\Delta_{13} L}{2}\right)$$
$$- 4\mathrm{Re}\left[U_{\alpha 1} U^*_{\alpha 2} U^*_{\beta 1} U_{\beta 2}\right] \sin^2\left[\frac{\Delta_{12} L}{2}\right]$$
$$- 2J \sin(\Delta_{12} L)\left[1 - \cos(\Delta_{13} L)\right]$$
$$+ 4J \sin(\Delta_{13} L) \sin^2(\Delta_{12} L)$$

where,

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij}}{2E} = \frac{m_j^2 - m_i^2}{2E}$$

$$U_{MNS} = \begin{bmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} e^{i\delta} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} e^{i\delta} \end{bmatrix}$$

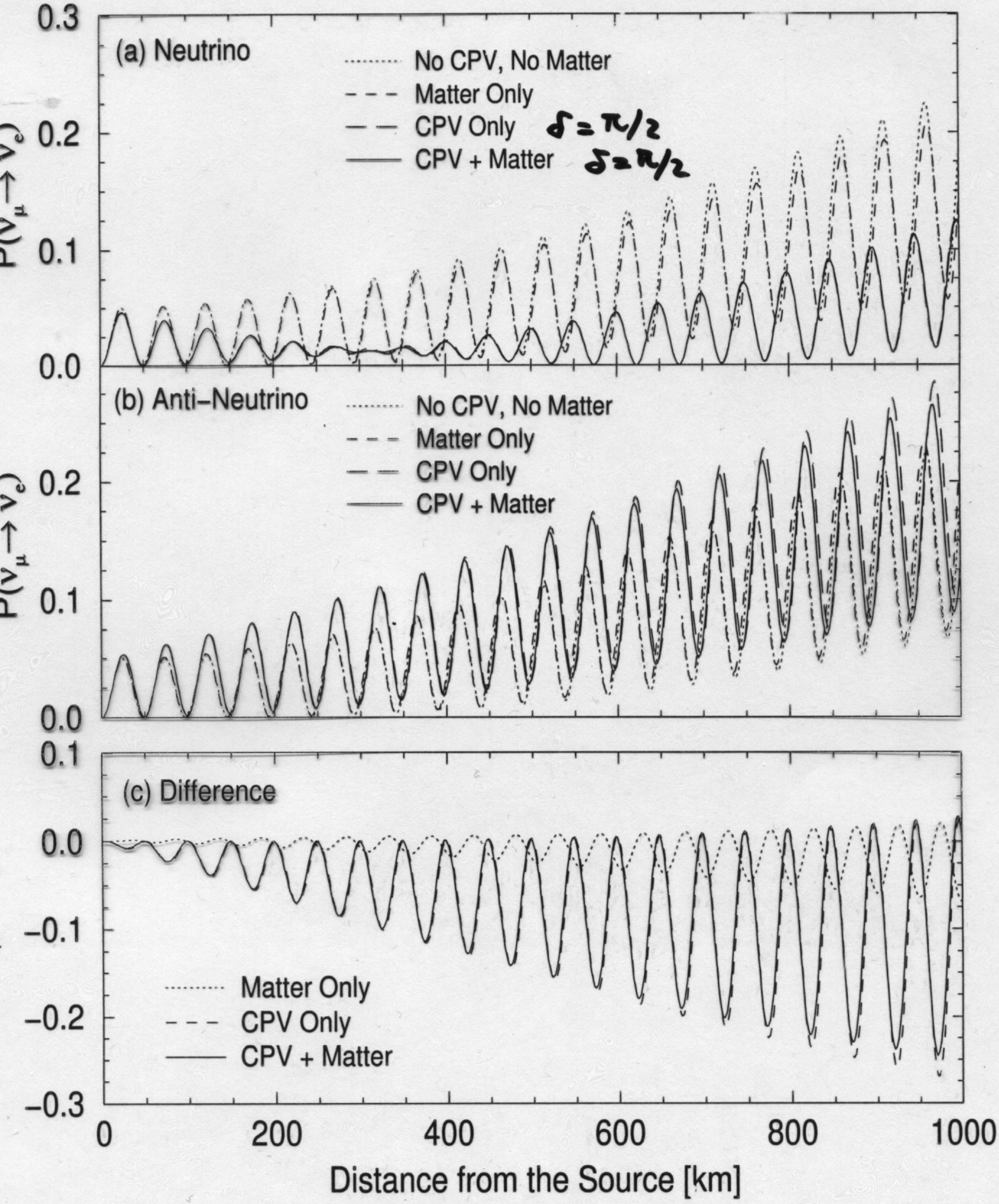
$$J \equiv \mathrm{Im}\left[U_{\alpha i} U^*_{\alpha j} U^*_{\beta i} U_{\beta j}\right]$$


(a) L = 250 km
Matter Only
CPV Only
CP + Matter
(b) L = 500 km
(c) L = 1000 km
$\Delta P(\nu_\mu \to \nu_e)$
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
10
100
1000
Neutrino Energy (MeV)

$\Delta M^2_{13} = 3\times10^{-3}\,eV^2$, $\sin^2 2\Theta_{23} = 1$

$\Delta M^2_{12} = 2.7\times10^{-5}\,eV^2$. $\sin^2 2\Theta_{12} = 0.79$

$\sin^2 2\Theta_{13} = 0.1$, $E = 60\,MeV$



Let us assume only 3 ν flavor

observational indications of ν mass and mixing

- Atmospheric ν observations

$\nu_\mu - \nu_\tau$ oscillation can explain data

$\nu_\mu - \nu_s$ disfavored by SK

$\nu_\mu - \nu_e$ must be small

$$\begin{cases} \Delta m^2_{23} \equiv \Delta m^2_{atm} \sim (2-6)\times 10^{-3}\,\mathrm{eV}^2 \\ \sin^2 2\theta_{23} \equiv \sin^2 2\theta_{atm} \sim 0.9 - 1.0 \end{cases}$$

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- Solar ν observations

- LMA MSW $\begin{cases} \Delta m^2_{12} \sim (0.2 - 2.0) \times 10^{-4}\,eV^2 \\ \sin^2 2\theta_{12} \sim 0.65 - 0.96 \end{cases}$

- SMA MSW $\begin{cases} \Delta m^2_{12} \sim (4.0 - 10) \times 10^{-6}\,eV^2 \\ \sin^2 2\theta_{12} \sim 0.001 - 0.01 \end{cases}$

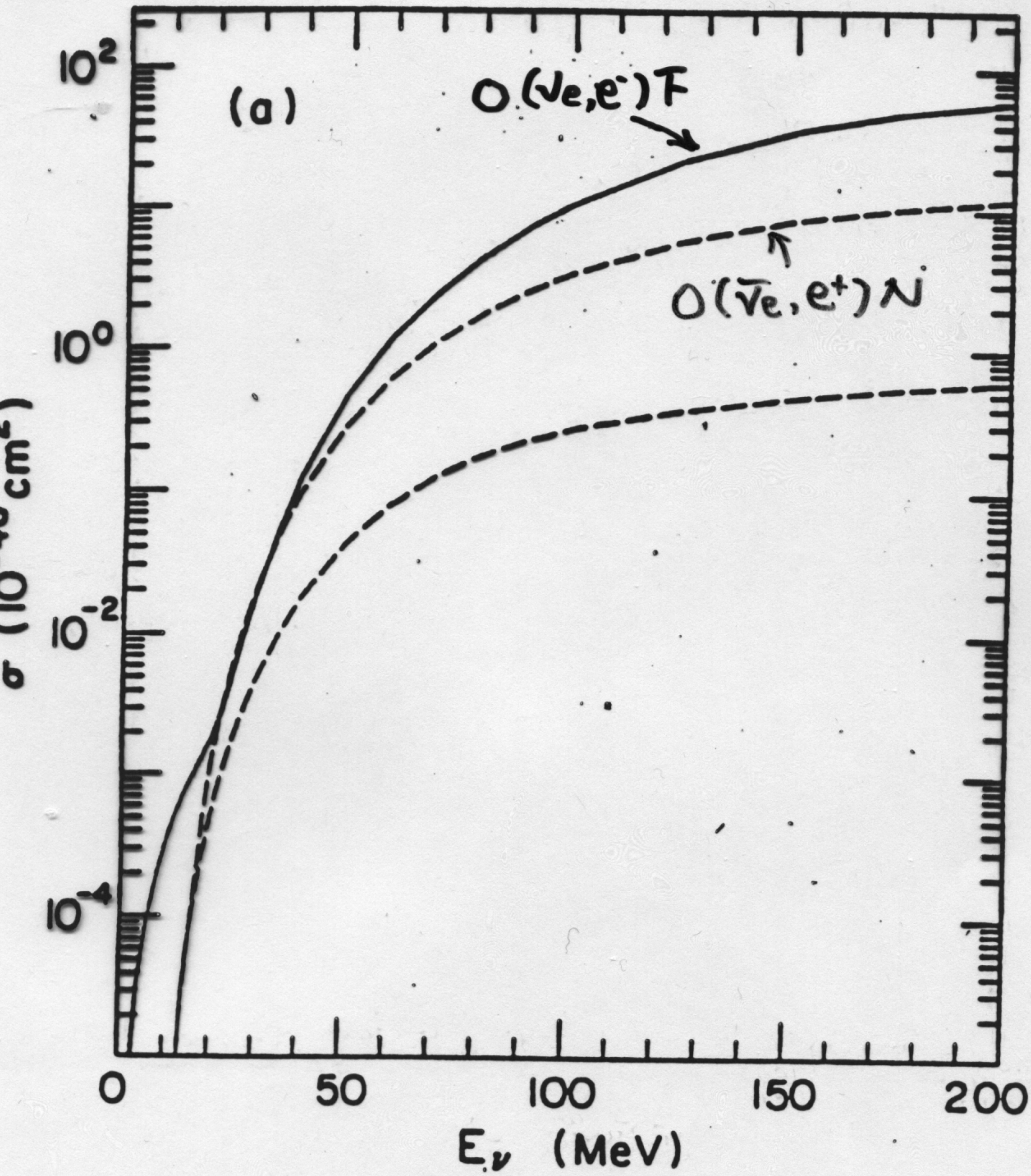
- Low MSW $\begin{cases} \Delta m^2_{12} \sim (0.5 - 2.0) \times 10^{-7}\,eV^2 \\ \sin^2 2\theta_{12} \sim 0.9 - 1.0 \end{cases}$

- Vacuum Oscillation (VO) $\begin{cases} \Delta m^2_{12} \sim (0.5 - 5) \times 10^{-10}\,eV^2 \\ \sin^2 2\theta_{12} \sim 0.7 - 1.0 \end{cases}$

- We do not consider LSND ...

Total cross section off natural oxygen

Haxton. PRD 36 (1987) 2283



How to measure?

Let us consider very high intensity $\sim 10^{22}$ POT ($100 \times$ K2K) ν_μ beam (may be possible at JHF)

Let us also consider supermassive, $\sim$ Mton water Chrenkov detector of SK type

For $E \sim 100$ MeV, relevant reactions are

$$\nu_e O \to e^- F \,,\ \sigma \sim 10^{-39}\,\mathrm{cm}^2 \text{ @ } 100\,\mathrm{MeV}$$

$$N^{\nu}_{\mathrm{expected}} \sim 10^4 \left[\frac{L}{250\,\mathrm{km}}\right]^{-2} \left[\frac{V}{\mathrm{Mton}}\right] \left[\frac{\mathrm{POT}}{10^{22}}\right] P(\nu_\mu \to \nu_e)$$

$\sim$ few $\times 100$ events for $P \sim$ few %

For $\bar{\nu}_\mu \to \bar{\nu}_e$

$$\bar{\nu}_e p \to e^+ n \,,\ \sigma \sim 10^{-39}\,\mathrm{cm}^2 \text{ @ } 100\,\mathrm{MeV}$$

$$\bar{\nu}_e O \to e^+ N \,,\ \sigma \sim \tfrac{1}{3} 10^{-39}\,\mathrm{cm}^2 \text{ @ } 100\,\mathrm{MeV}$$

$$N^{\bar{\nu}}_{\mathrm{expected}} \sim 2 N^{\nu}_{\mathrm{expected}}$$

$\Delta P \sim$ few % seems to be detected

Conclusions

- In 3 ν flavor framework, assuming mass and mixing parameters which can explain atmospheric ν data by $\nu_\mu - \nu_\tau$ maximal oscillation and solar ν data by LMA MSW solution,

$$\Delta P = P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)$$

canbe $\sim$ few% for $E \sim 100$ MeV

$L \sim$ few 100 Km, $\delta = \frac{\pi}{2}$, $\sin^2 2\theta_{13} \sim 0.1$

with small matter effect

- With high intensity $\sim 100\times$ K2K (10^{22} POT) ν_μ ($\bar{\nu}_\mu$) beam and with a Mton water Cherenkov detector, measuring $\cancel{CP}$ seems to be feasible.

- To use ν beam from a muon storage ring, we need to identify ν_e, $\bar{\nu}_e$ separately, eg. by identifing n a la SNO for $\bar{\nu}_e p \to n + e^+$.