Extracting Matter Effects, Masses, Mixings and CP-Violation at ν-Factories

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hep-ph/9912457 with M. Lindner, S. Petcov and A. Romanino
hep-ph/0004085 with M. Lindner and P. Huber

Contents:

Neutrino Oscillation

CKM-Matrix for Three Neutrinos:

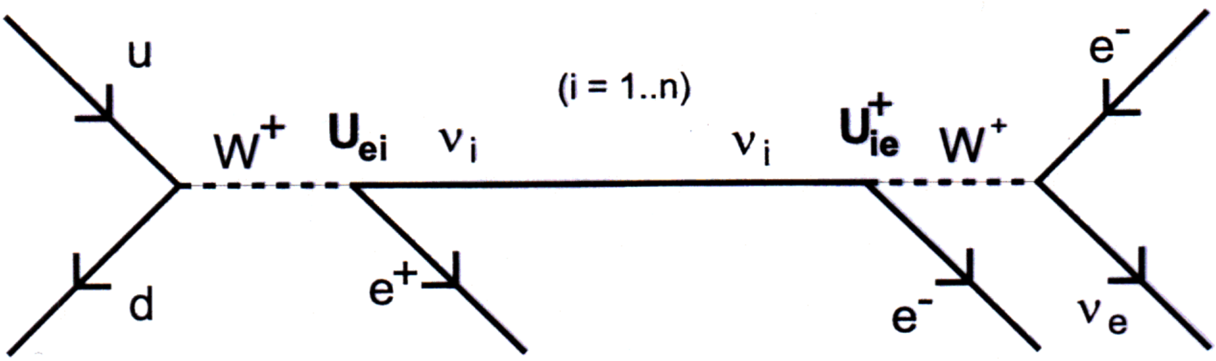
$\Rightarrow$ 3 angles + 3 phases (θ_{12}, θ_{23}, θ_{13}, δ, α_1, α_2)

$$\boxed{\mathbf{U}_{\mathrm{CKM}} := \mathbf{U} \cdot \mathbf{diag}\left(\mathbf{e}^{\mathbf{i}\alpha_1}, \mathbf{e}^{\mathbf{i}\alpha_2}, \mathbf{1}\right)}$$

α_1 and α_2 do not enter ν-oscillation!

$$\mathbf{U} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

Production, Propagation and Detection:



$p + p \rightarrow d + \nu_e + e^+$ electron scattering

$$\boxed{\text{For } E \gg m: \quad E_k = \sqrt{p^2 + m_k^2} \simeq p + \frac{m_k^2}{2p} \simeq p + \frac{m_k^2}{2E_\nu}}$$

3 Neutrino Oscillation:

Flavour $\mathrm{l} \to \mathrm{m}$**:** $P(\nu_{e_l} \to \nu_{e_m}) =$

$$= \quad \left|\langle\nu_m(t)|\nu_l(t=0)\rangle\right|^2 = \left|\langle\nu_m|Ue^{-iHt}U^\dagger|\nu_l\rangle\right|^2$$

$$= \quad \underbrace{\delta_{lm} - 4\sum_{i>j}\mathrm{Re}J_{ij}^{e_l e_m}\sin^2\Delta_{ij}}_{P_{CP}} \; \underbrace{-2\sum_{i>j}\mathrm{Im}J_{ij}^{e_l e_m}\sin 2\Delta_{ij}}_{P_{\cancel{CP}}}$$

$$\boxed{\text{Shorthands: } J_{ij}^{e_l e_m} := U_{li}U_{lj}^{*}U_{mi}^{*}U_{mj} \qquad \Delta_{ij} := \frac{\Delta m_{ij}^2 L}{4E}}$$

Two Neutrino Limit: $U = R(\theta)$, $\quad \Delta_{12} = \frac{\Delta m_{12}^2 L}{4E}$

$$\begin{aligned} P_{CP}(\nu_{e_1} \to \nu_{e_2}) &= \sin^2 2\theta \cdot \sin^2\left(\Delta m_{12}^2 L/4E\right) \\ P_{CP}(\nu_{e_i} \to \nu_{e_i}) &= 1 - \sin^2 2\theta \cdot \sin^2\left(\Delta m_{12}^2 L/4E\right) \\ P_{\cancel{CP}}(\nu_{e_1} \to \nu_{e_2}) &= P_{\cancel{CP}}(\nu_{e_i} \to \nu_{e_i}) = 0 \end{aligned}$$

Terminology:

Appearance Experiments: $P(\nu_{e_1} \to \nu_{e_2})$

Disappearance Experiments: $P(\nu_{e_i} \to \nu_{e_i})$

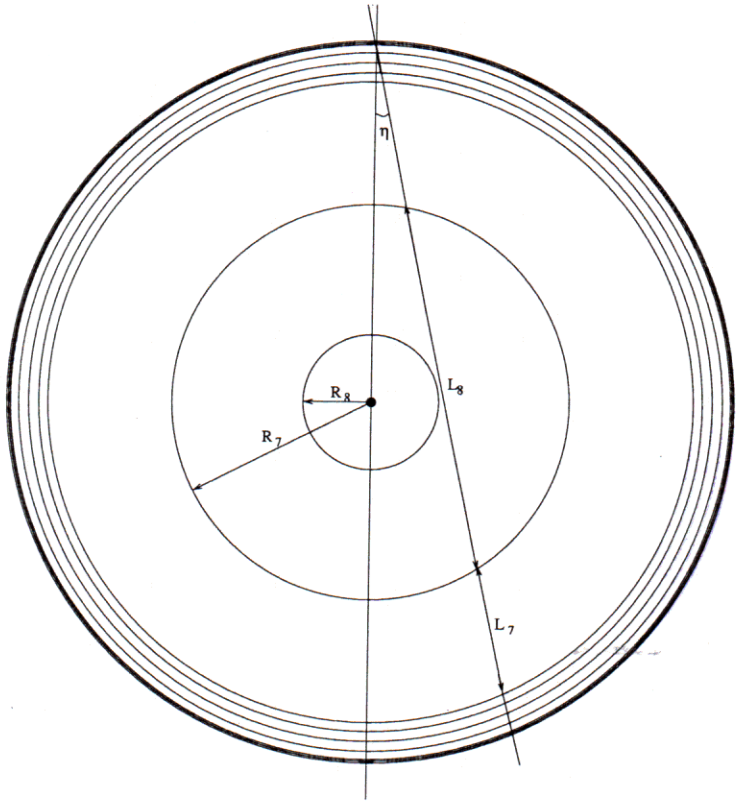
Possible Signals

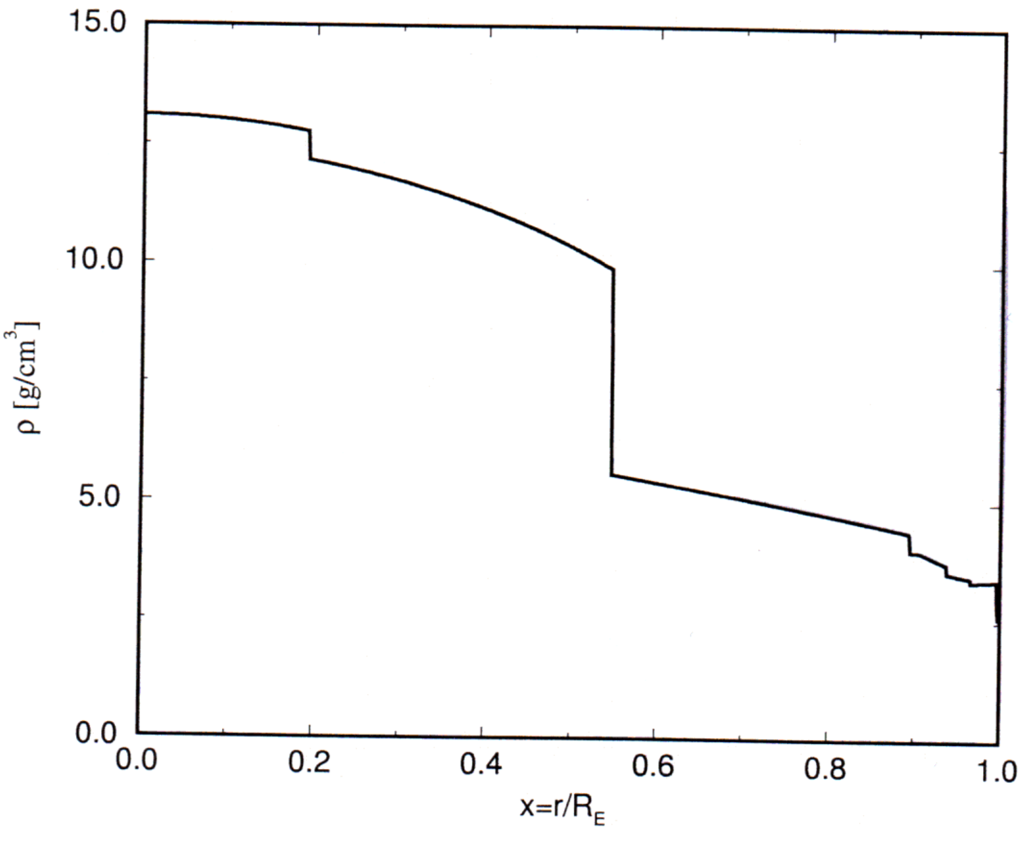
Beam $\Rightarrow$ Oscillation $\Rightarrow$ Detector $\Rightarrow$ Muons

$$
\boxed{\mu^-} \Longrightarrow
\left\{
\begin{array}{l}
\overline{\nu}_e \xrightarrow{oscillation}
\left\{
\begin{array}{lll}
\overline{\nu}_e \Rightarrow e^+ & & n_{\mu^-}(e^+) \\
\overline{\nu}_\mu \Rightarrow \boxed{\mu^+} & & n_{\mu^-}(\mu^+) \\
\overline{\nu}_\tau \Rightarrow \tau^+ & &
\end{array}
\right. \\
\\
\nu_\mu \xrightarrow{oscillation}
\left\{
\begin{array}{lll}
\nu_e \Rightarrow e^- & & \\
\nu_\mu \Rightarrow \boxed{\mu^-} & & n_{\mu^-}(\mu^-) \\
\nu_\tau \Rightarrow \tau^- & &
\end{array}
\right.
\end{array}
\right.
$$

$$
\boxed{\mu^+} \Longrightarrow
\left\{
\begin{array}{l}
\nu_e \xrightarrow{oscillation}
\left\{
\begin{array}{lll}
\nu_e \Rightarrow e^- & & n_{\mu^+}(e^-) \\
\nu_\mu \Rightarrow \boxed{\mu^-} & & n_{\mu^+}(\mu^-) \\
\nu_\tau \Rightarrow \tau^- & &
\end{array}
\right. \\
\\
\overline{\nu}_\mu \xrightarrow{oscillation}
\left\{
\begin{array}{lll}
\overline{\nu}_e \Rightarrow e^+ & & \\
\overline{\nu}_\mu \Rightarrow \boxed{\mu^+} & & n_{\mu^+}(\mu^+) \\
\overline{\nu}_\tau \Rightarrow \tau^+ & &
\end{array}
\right.
\end{array}
\right.
$$

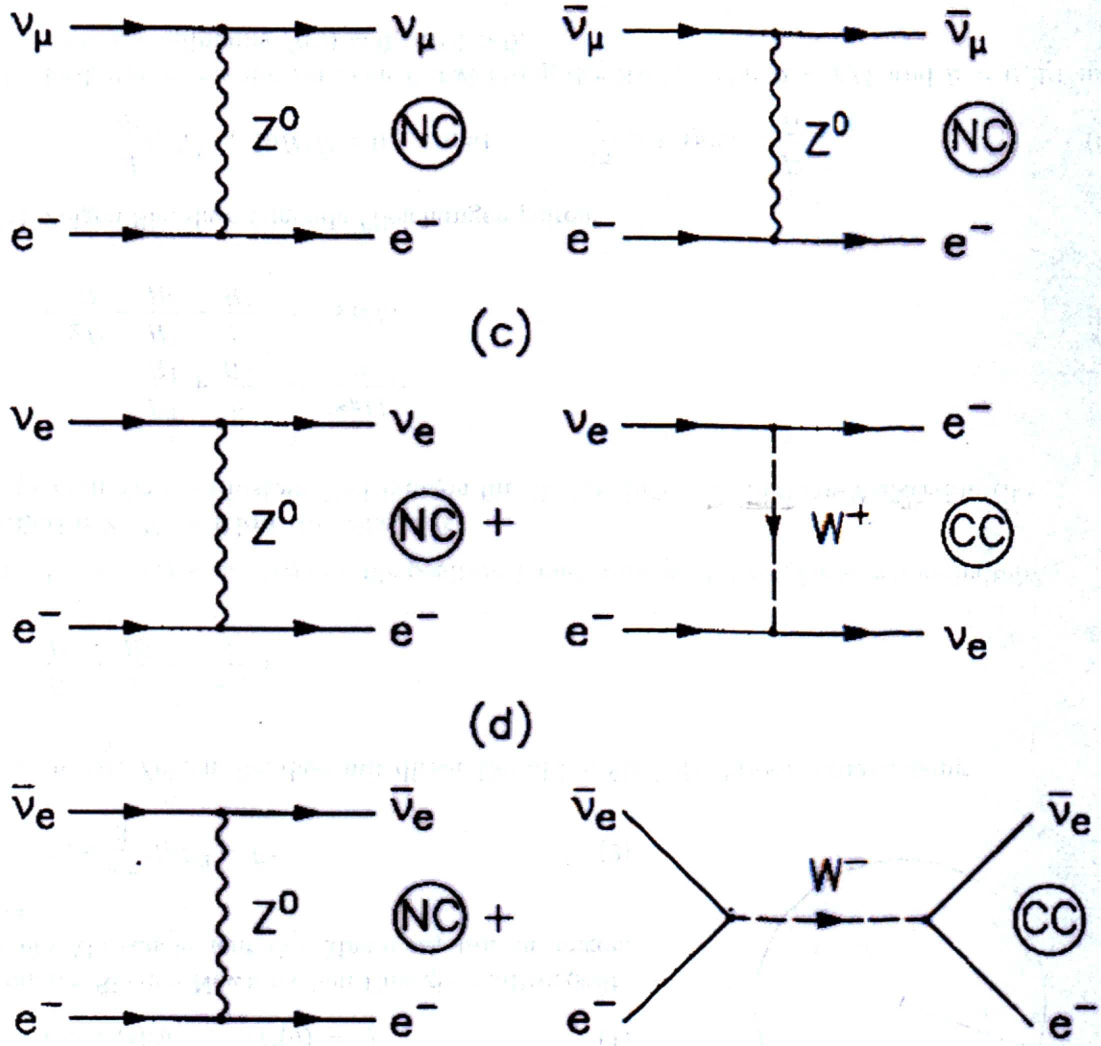
Neutrino Beams crossing Earth





The MSW-Effect

(Mikheyev-Smirnov-Wolfenstein Effect)



$$\boxed{\mathcal{L}_{NC} = \text{flavour universal}}$$

$$\boxed{\mathcal{L}_{CC} = \sqrt{2} G_F n_e}$$

Matter Effects and MSW Resonance

Hamiltonian in flavour basis:

$$\mathcal{H} = \frac{1}{2E}\left[U\begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}U^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]$$

where $A = \pm\frac{2\sqrt{2}G_F Y \rho E_\nu}{m_n}$

Re-diagonalization $\Rightarrow$ parameter mapping

$$(\theta_{ij}, \delta, \Delta m_{12}^2, \Delta m_{23}^2) \to (\theta'_{ij}, \delta', (\Delta m_{12}^2)', (\Delta m_{23}^2)')$$

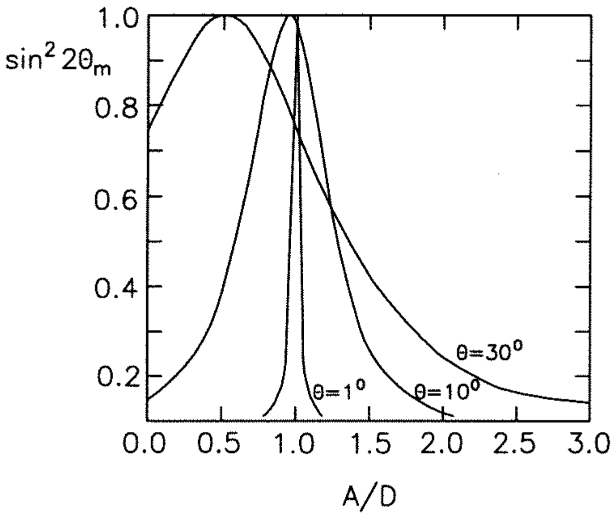
Different mappings for **neutrinos** and **antineutrinos**

Two Neutrino Case

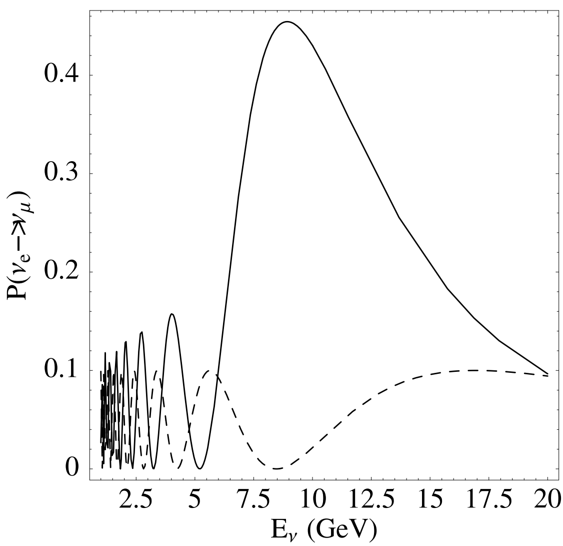
$$\Delta m^{2\prime} = \Delta m^2 \sqrt{\left(\frac{A}{\Delta m^2} - \cos 2\theta\right)^2 + \sin^2 2\theta}$$

$$\sin^2 2\theta' = \frac{\sin^2 2\theta}{\left(\frac{A}{\Delta m^2} - \cos 2\theta\right)^2 + \sin^2 2\theta}$$

Resonance at $\frac{A}{\Delta m^2} = \cos 2\theta \approx 1$



$< \rho >= 2.8\frac{\mathrm{g}}{\mathrm{cm}^3}$ (Mantle) $\Rightarrow$ $\boxed{E_{\mathrm{res}} \approx 15 \text{ GeV}}$



MSW Effect with 3 Neutrinos

$$\boxed{\text{Approximation } \Delta m_{12}^2 = 0}$$

$$U = R_{23}(\theta_{23}) \cdot R_{13}(\theta_{13}) \cdot R_{12}(\theta_{12})$$

$$\mathcal{H} = \frac{1}{2E} R_{23} \left[R_{13} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta m^2 \end{pmatrix} R_{13}^T + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] R_{23}^T$$

→ Reduction to 2 neutrino matter effect case

→ Mixing angle θ_{23} and θ_{12} unaffected

$$\boxed{\begin{aligned} \sin^2 2\theta'_{13} &= \frac{\sin^2 2\theta_{13}}{C_\pm^2} \\ \Delta m_{31,m}^2 &= \Delta m^2 C_\pm \\ \Delta m_{32,m}^2 &= \frac{\Delta m^2\ (C_\pm + 1) + A}{2} \\ \Delta m_{21,m}^2 &= \frac{\Delta m^2\ (C_\pm - 1) - A}{2} \end{aligned}}$$

with $C_\pm$ from 2 neutrino matter effect calculation

$$\boxed{C_\pm^2 = \left(\frac{A}{\Delta m^2} - \cos 2\theta\right)^2 + \sin^2 2\theta}$$

MSW Effects in VLBL Neutrino Oscillation:

$$
\begin{aligned}
P(\nu_e \leftrightarrow \nu_\mu) &= \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2(\Delta_{31}) \\
P(\nu_\mu \leftrightarrow \nu_\mu) &= 1 - \sin^2 2\theta_{23}\sin^2\theta_{13}\sin^2(\Delta_{21}) \\
&\quad - \sin^4\theta_{23}\sin^2 2\theta_{13}\sin^2(\Delta_{31}) \\
&\quad - \sin^2 2\theta_{23}\cos^2\theta_{13}\sin^2(\Delta_{32}) \\
P(\nu_\tau \leftrightarrow \nu_\mu) &= \sin^2 2\theta_{23}\Big[\sin^2\theta_{13}\sin^2(\Delta_{21}) \\
&\quad - \sin^2 2\theta_{13}\sin^2(\Delta_{31}) \\
&\quad + \cos^2\theta_{13}\sin^2(\Delta_{32})\Big]
\end{aligned}
$$

where parameters must be taken in matter $\Rightarrow$ e.g.

$\nu_e \rightarrow \nu_\mu$ Transition Probability in Matter:

$$
P_E^{3\nu}(\nu_e \rightarrow \nu_\mu, \overline{\nu}_e \rightarrow \overline{\nu}_\mu) = \sin^2\theta_{23}\sin^2 2\theta_{13}\left(\frac{\sin(\Delta_{31}C_\pm)}{C_\pm}\right)^2
$$

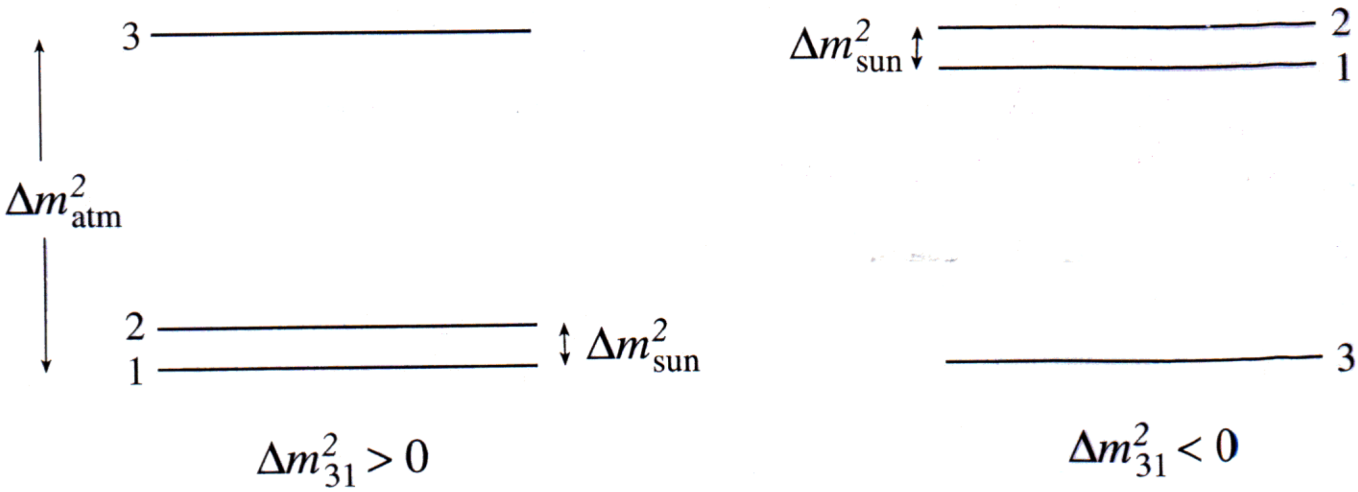
where $C_\pm^2 = \left(1 \mp \frac{2EV}{\Delta m_{31}^2}\right)^2 \pm 4\frac{2EV}{\Delta m_{31}^2}\sin^2\theta_{13}$

with MSW "potential" $V = \sqrt{2}G_F\overline{N}_e^{man}$

The Sign of Δm^2_{31} via Matter Effects:

Vacuum oscillation insensitive to sign of Δm^2

$\Rightarrow$ **Ambiguities in mass ordering schemes**



$\Rightarrow$ **Sign of Δm^2 via matter effects** $\Leftrightarrow$ $\simeq$ **resonance**

$$\sin^2 2\theta_{13,m} = \frac{\sin^2 2\theta_{13}}{\left(\frac{2EV}{m_3^2 - m_1^2} - \cos 2\theta_{13}\right)^2 + \sin^2 2\theta_{13}}$$

Barger, Geer, Raja and Whisnant: hep-ph/9911524
Freund, Lindner, Petcov and A. Romanino: hep-ph/9912457
Cervera, Donini, Gavela, Gomez Cadenas,
Hernandez, Mena, Rigolin: hep-ph/0002108

Extracting Parameters from Data

$\Rightarrow$ Event Rates including Beam and Detector

$$n_{\mu^\pm}(...) = N_{\mu^\pm} N_{\mathrm{kT}} \frac{10^9 N_A}{m_\mu^2 \pi} \frac{E_\mu^3}{L^2} \int_{E_{\min}}^{E_\mu} f_{...} P_E^{3\nu}(...) \, (dE/E_\mu)$$

where

- $P_E^{3\nu}(...)$ - 3-ν oscillation probabilities in matter
- $N_{\mu^\pm}$ - number of useful muon decays (flux)
- N_{kT} - number of kilotons in the detector (typically 10kt)
- $10^9 N_A$ - nucleons/kiloton

Spectrum, X-section and Detection Efficiency enter via f_i:

$$f_{\nu_e \nu_\mu}(E) = g_{\nu_e}(E/E_\mu)(\sigma_{\nu_\mu}(E)/E_\mu)\epsilon_{\mu^-}(E)$$
$$f_{\overline{\nu}_e \overline{\nu}_\mu}(E) = ...$$
$$f_{\nu_\mu \nu_\mu}(E) = ...$$
$$f_{\overline{\nu}_\mu \overline{\nu}_\mu}(E) = ...$$

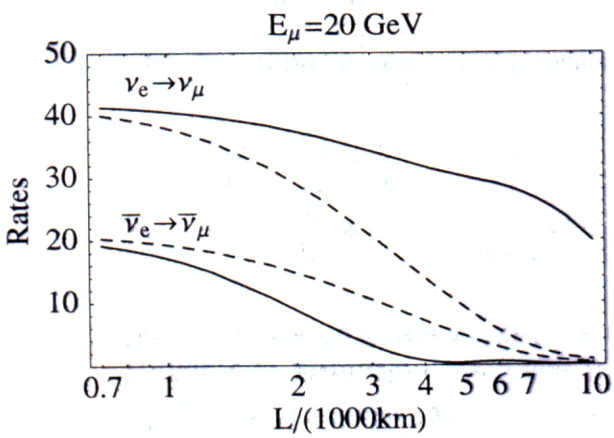
X-sections: $\sigma_{\nu_\mu}(E) = 2 \cdot \sigma_{\overline{\nu}_\mu}(E) = 0.67 \cdot 10^{-38} E \, \mathrm{cm}^2/\mathrm{GeV}$

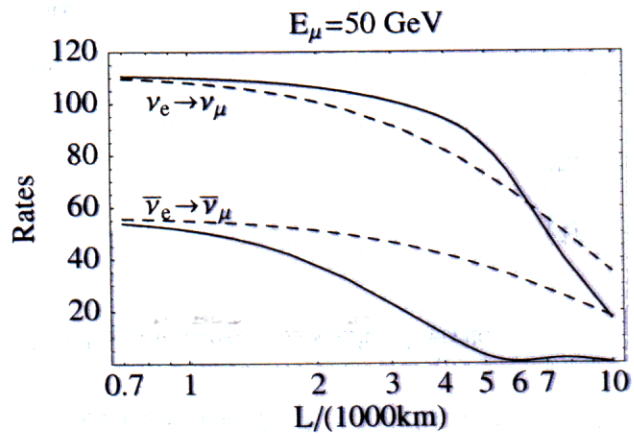
Efficiency: $\epsilon_{\mu^-}(E) = \epsilon_{\mu^+}(E) = 0.5$ for $E > E_{\min} \simeq 4$ GeV

Matter Effects on Event Rates:

Dashed lines: Oscillation w/o matter. Solid lines: Matter

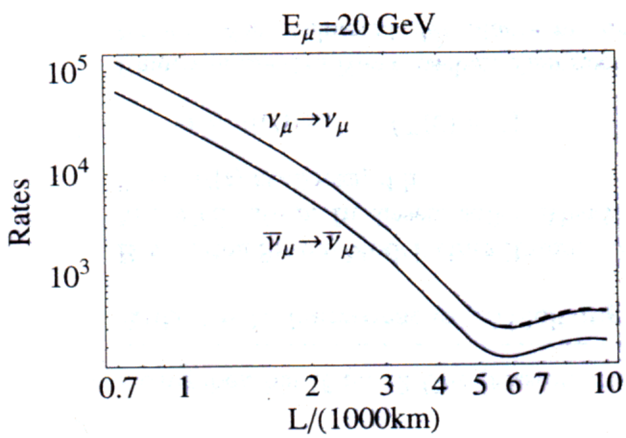
Matter Effects in $\nu_e \to \nu_\mu$ and $\overline{\nu}_e \to \overline{\nu}_\mu$:

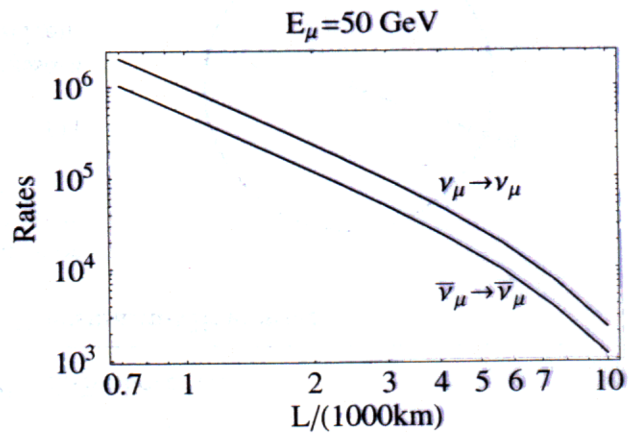




$\Rightarrow$ Sizable asymmetry between neutrinos and anti-neutrinos!

Matter Effects in $\nu_\mu \to \nu_\mu$ and $\overline{\nu}_\mu \to \overline{\nu}_\mu$:

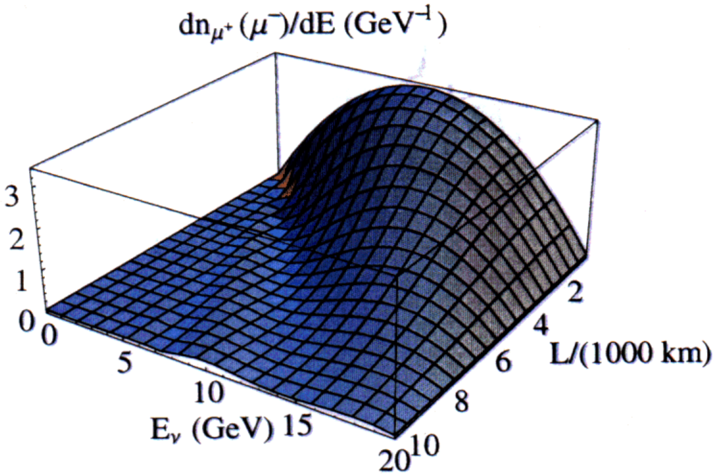


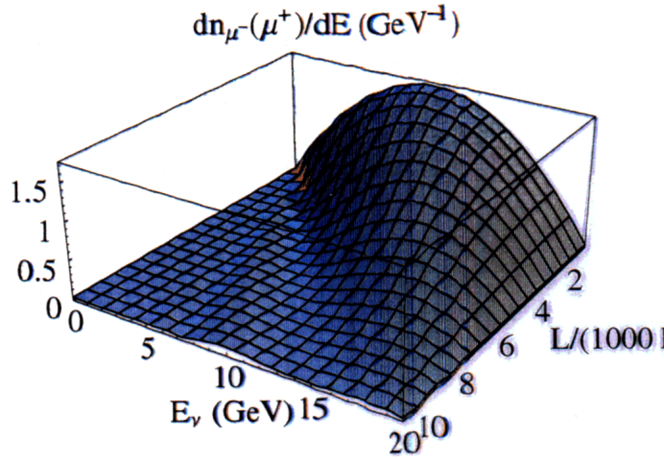


$\Rightarrow$ Negligible!

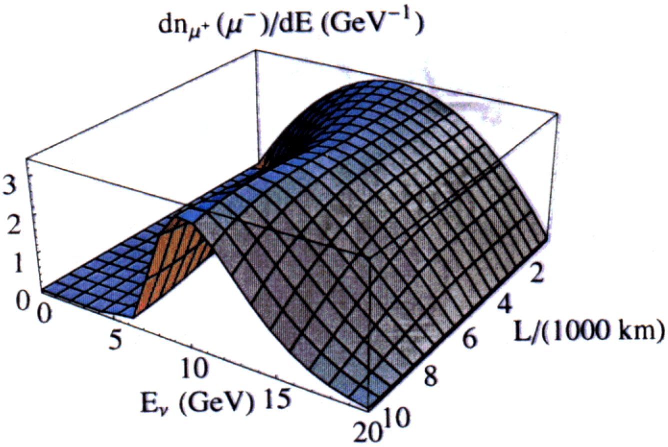
Differential Event Rates:

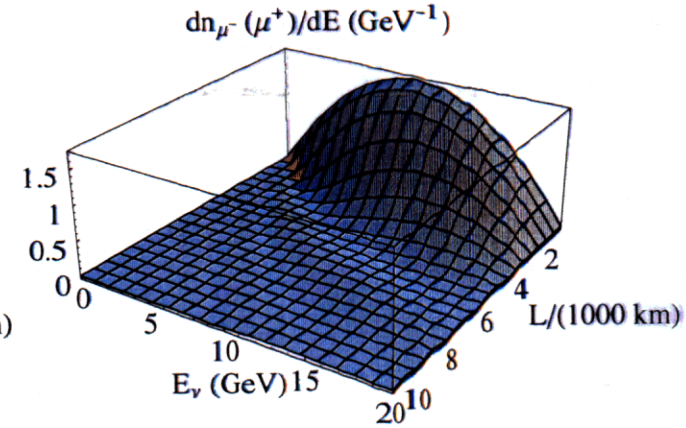
1) $\nu_e \to \nu_\mu$ and $\overline{\nu}_e \to \overline{\nu}_\mu$ in vacuum for $E_\mu = 20$ GeV:



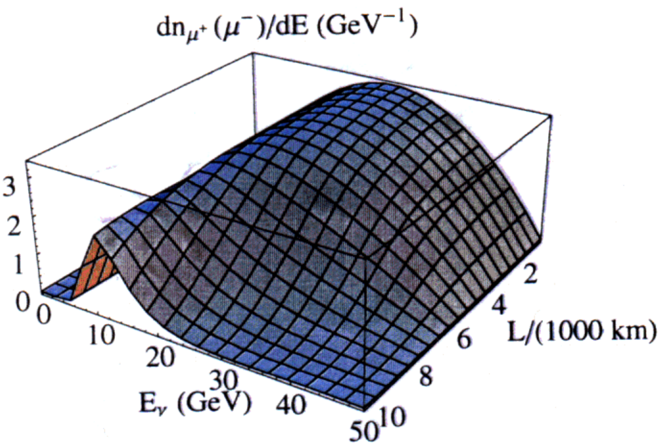


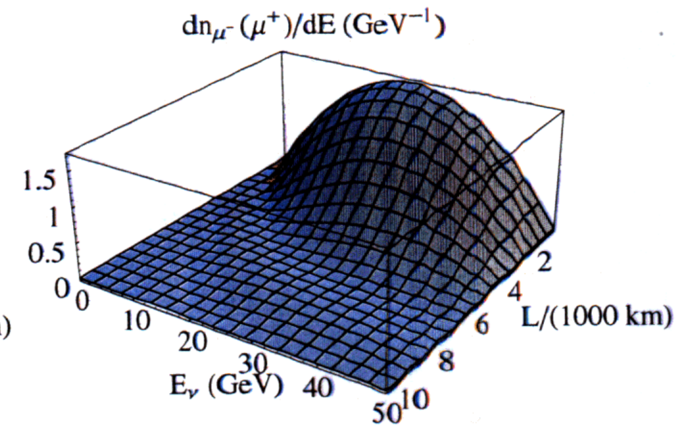
2) $\nu_e \to \nu_\mu$ and $\overline{\nu}_e \to \overline{\nu}_\mu$ in matter for $E_\mu = 20$ GeV:





3) $\nu_e \to \nu_\mu$ and $\overline{\nu}_e \to \overline{\nu}_\mu$ in matter for $E_\mu = 50$ GeV:





Enhancement/supression at MSW resonance energy $\simeq 13$ GeV!

Extraction of Matter Effects:

MSW Resonance of ν's / Suppression of $\overline{\nu}$'s:

$\Rightarrow$ Asymmetry $$A := \frac{\Delta}{\Sigma} = \frac{n_{\mu^+}(\mu^-) - 2\; n_{\mu^-}(\mu^+)}{n_{\mu^+}(\mu^-) + 2\; n_{\mu^-}(\mu^+)}$$

- Very sensitive to effects which go in opposite direction
- Many common systematic effects drop out
- $A = 0$ for vacuum oscillation

Statistical Significance: Two Methods

1) Via Asymmetry:

a) Small Σ^{vac} (i.e. small E_μ): $\boxed{\chi = \Delta / \delta\Delta^{\text{vac}}}$

b) Large Σ^{vac} (i.e. large E_μ): $\boxed{\chi = A / \delta A^{\text{vac}}}$

2) Better: Particle Data Group Confidence Levels:

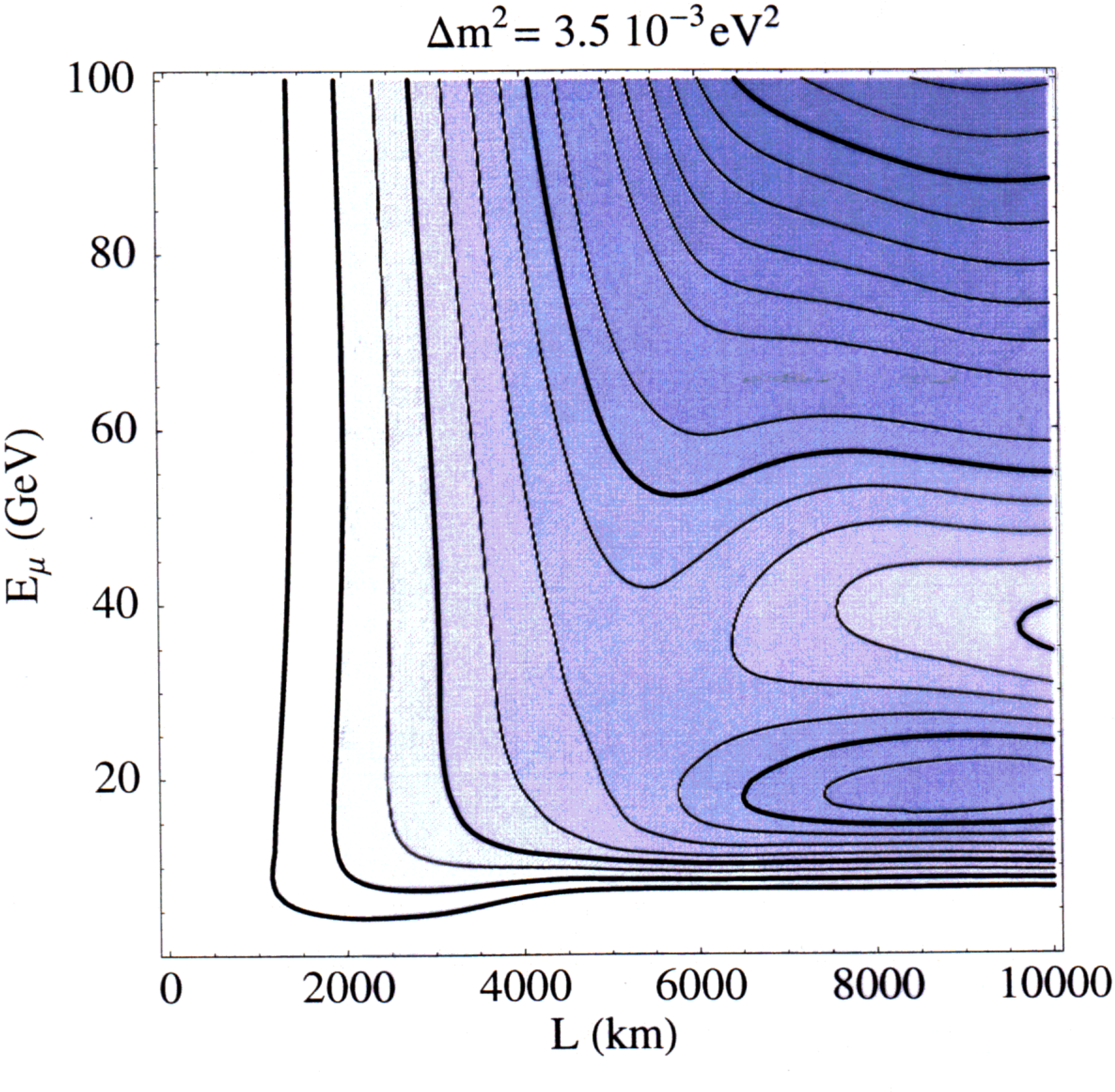
$$\chi^2 = 2\left[\langle n^{\text{vac}}_{\mu^+}(\mu^-)\rangle - n_{\mu^+}(\mu^-)\right] + 2\, n_{\mu^+}(\mu^-) \log\left[\frac{n_{\mu^+}(\mu^-)}{\langle n^{\text{vac}}_{\mu^+}(\mu^-)\rangle}\right]$$
$$+2\left[\langle n^{\text{vac}}_{\mu^-}(\mu^+)\rangle - n_{\mu^-}(\mu^+)\right] + 2\, n_{\mu^-}(\mu^+) \log\left[\frac{n_{\mu^-}(\mu^+)}{\langle n^{\text{vac}}_{\mu^-}(\mu^+)\rangle}\right]$$

$\Rightarrow$ "Standard deviations": $\boxed{n_\sigma \equiv \sqrt{\chi^2}}$

MSW Effect as Deviation from Vacuum:

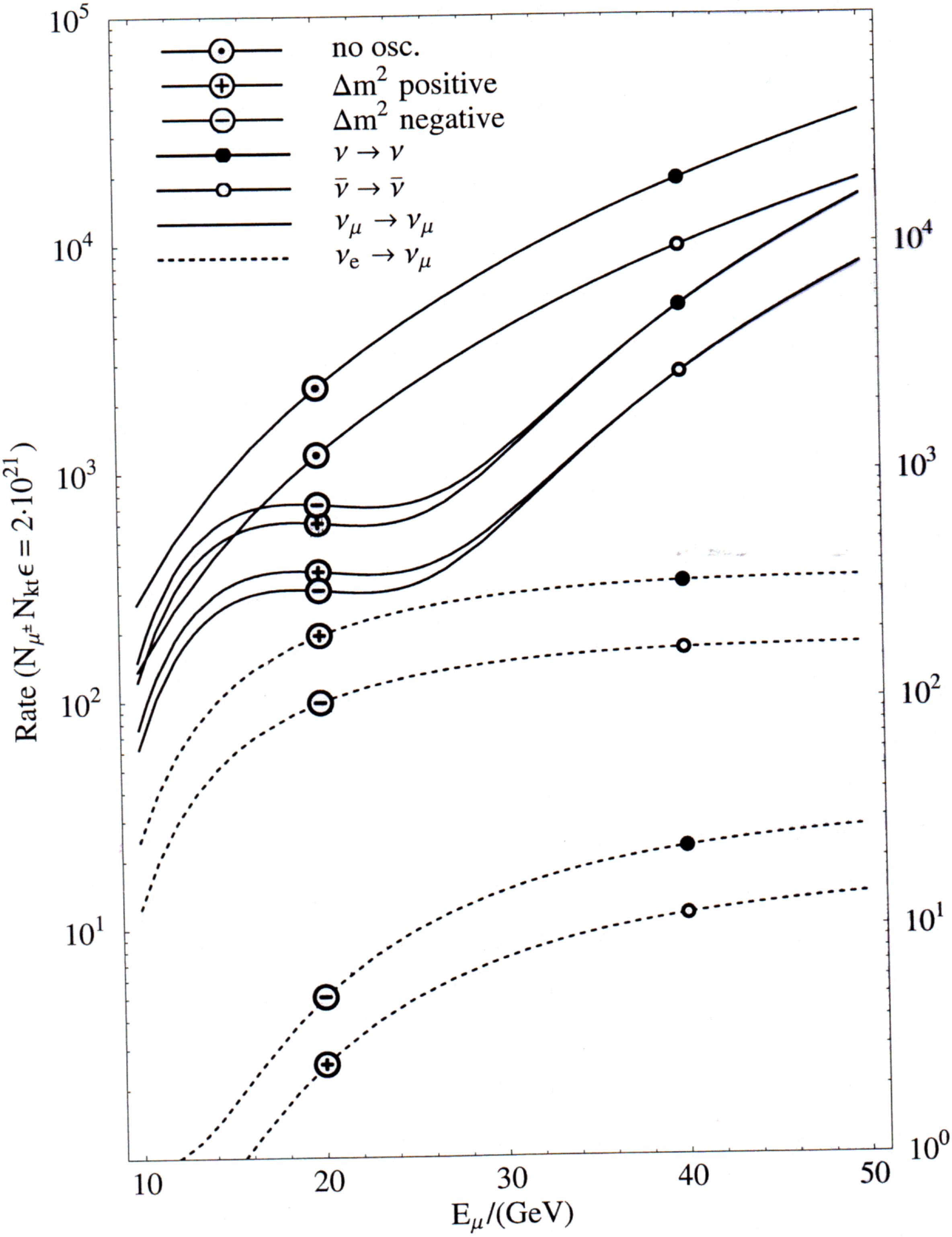
Contour Lines of n_σ Deviations from Vacuum:

Thick solid lines $\Leftrightarrow$ $n_\sigma = 100 \sin^2 2\theta_{13} \cdot \{1, 2, 4, 8, 16\}$
I.e. for $\sin^2 2\theta_{13} = 0.01 \Rightarrow$ 1,2,4,8,16 standard deviations



Used parameters:
Central $\Delta m^2_{31} = 3.5 \cdot 10^3\,\mathrm{eV}^2$
θ_{13} arbitrary $\Leftrightarrow$ scaling
Neutrino factory and detector $N_\mu N_{\mathrm{kT}} \epsilon = 10^{21}$

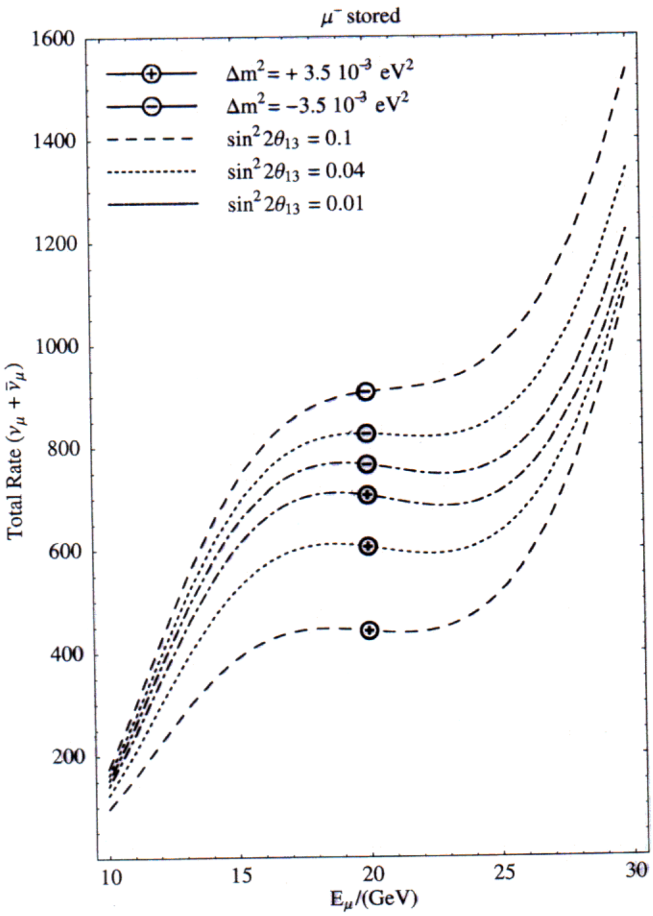
Total Event Rates

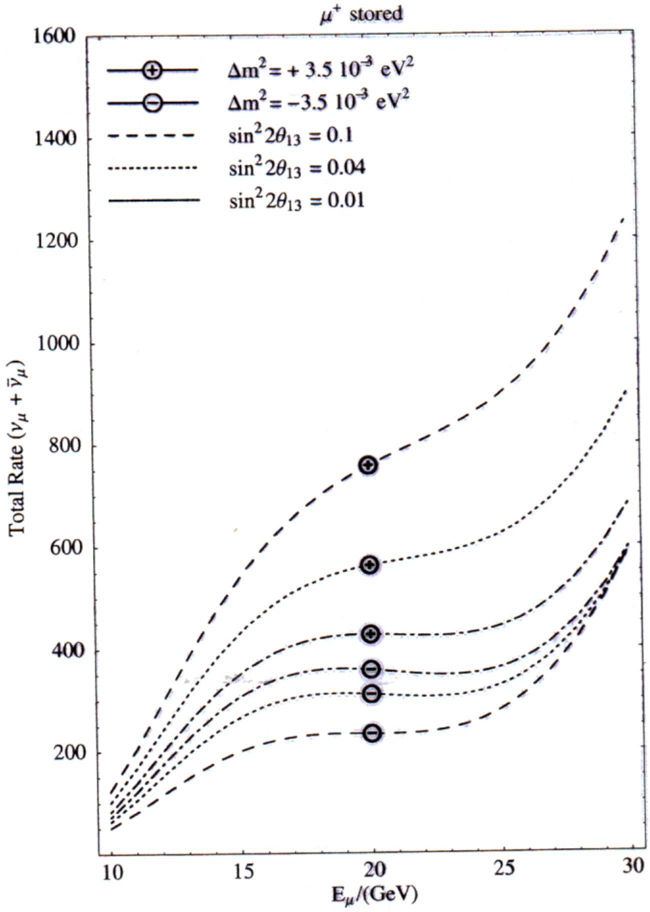


Barger, Geer, Raja and Whisnant: hep-ph/9911524

Freund, Huber, Lindner: hep-ph/0004085

Combined Appearance and Disappearance Rates





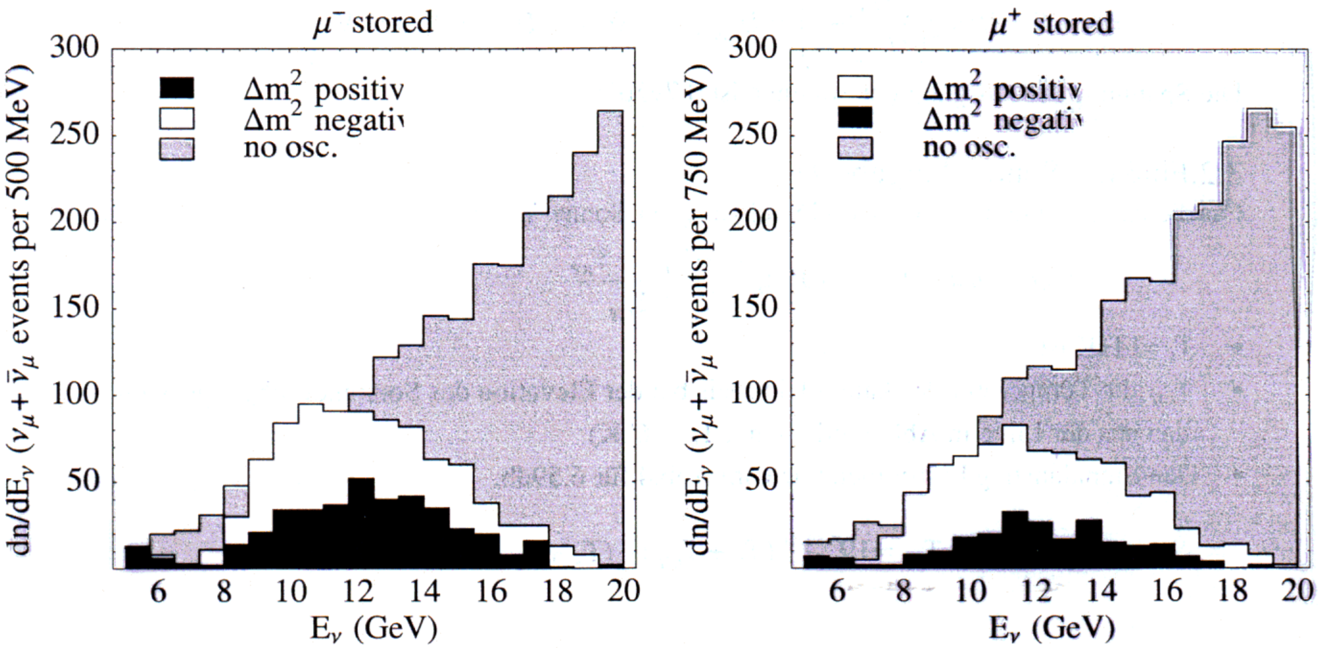
$\Rightarrow$ amplification of matter effects!

Freund, Huber and Lindner: hep-ph/0004085

Optimal beam energy:

- $E_\mu > 20$ GeV
- Higher energies add only events away from resonance region
  $\Rightarrow$ increased total rates, constant matter effects
- Note however: Increasing Δm^2 from $3.5 \cdot 10^{-3}\mathrm{eV}^2$ to say $7.0 \cdot 10^{-3}\mathrm{eV}^2$ required factor 2 in E_μ

Differential Event Rates

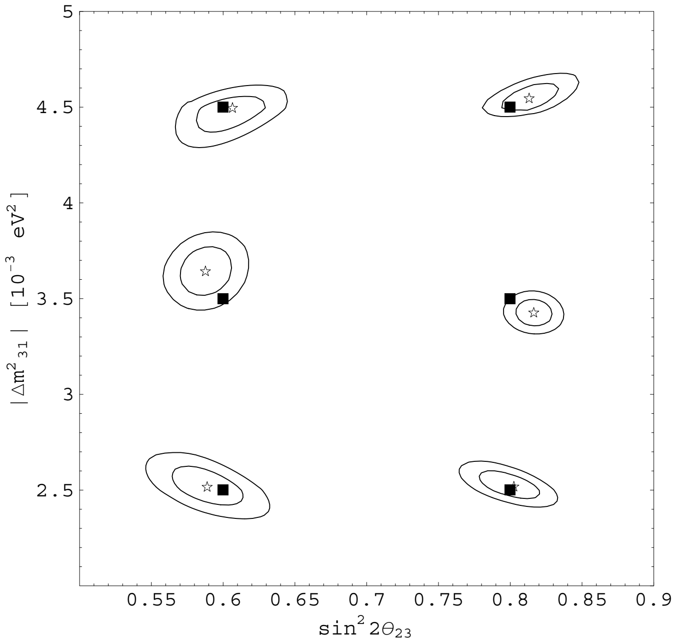


Two Step Analysis:

1) Extract $|\Delta m^2_{31}|$ and $\sin^2 2\theta_{23}$ from combined rates where matter effects cancel

2) Fit Δm^2_{31} and $\sin^2 2\theta_{13}$ to individual channels

Step 1: Fitting $|\Delta m^2_{31}|$ and $\sin^2 2\theta_{23}$



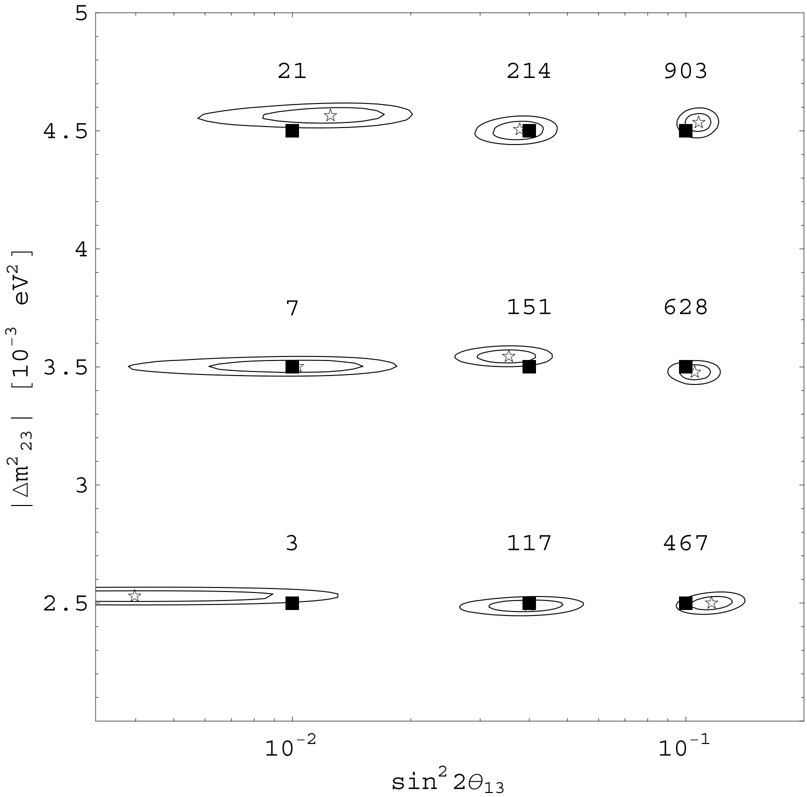
$\Rightarrow$ Our results agree for 2800 km with Barger et. al

Our analyis: Significance at 7332 km similar to 2800 km

Starting point for step 2:

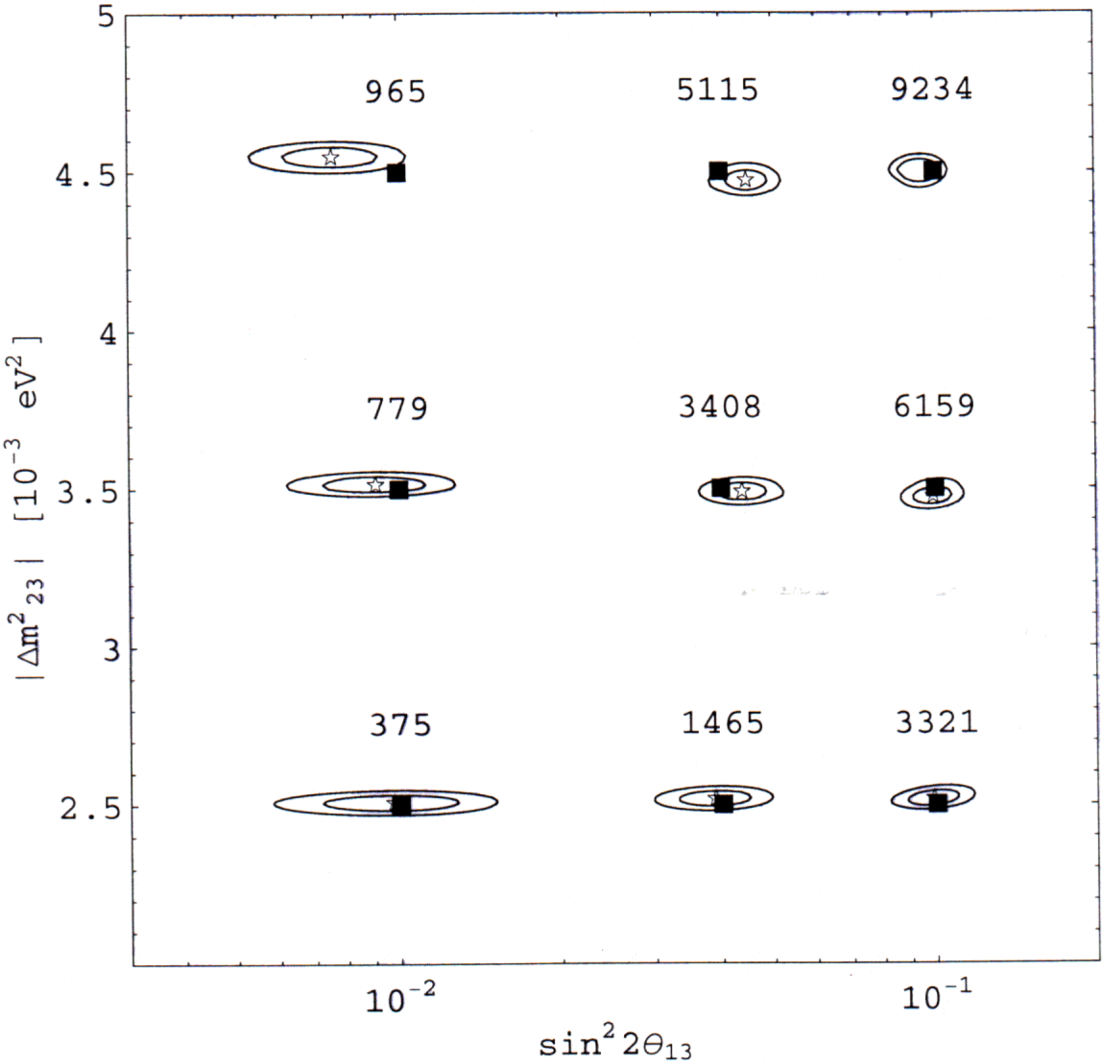
Our results of the sign of Δm^2_{31} and θ_{13}

Fit to total muon spectrum w/o CID



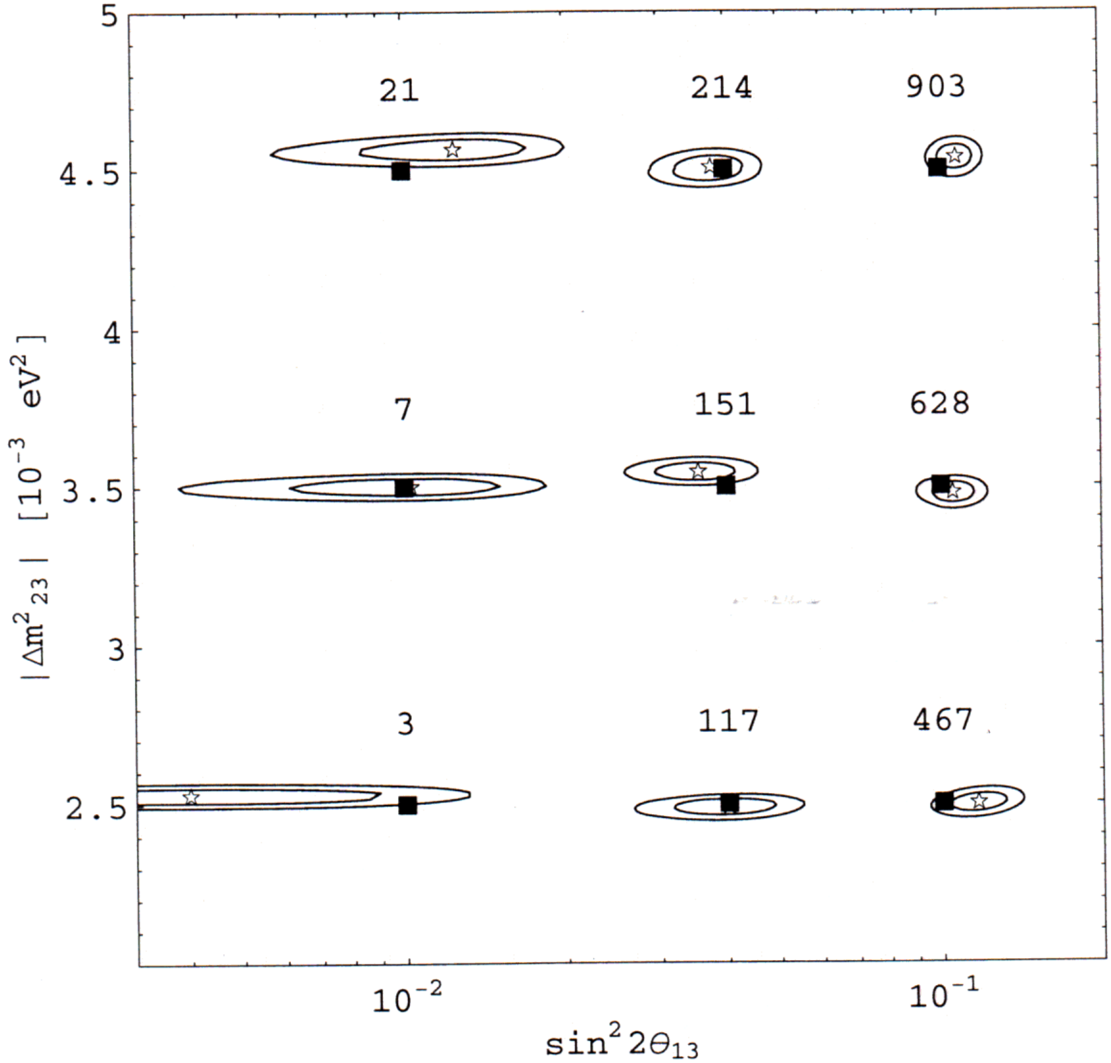
$$\sin^2 2\theta_{23} = 1 \quad L = 7332\ \mathrm{km} \quad \Delta m^2_{13} > 0$$

Fit to muon spectrum with CID



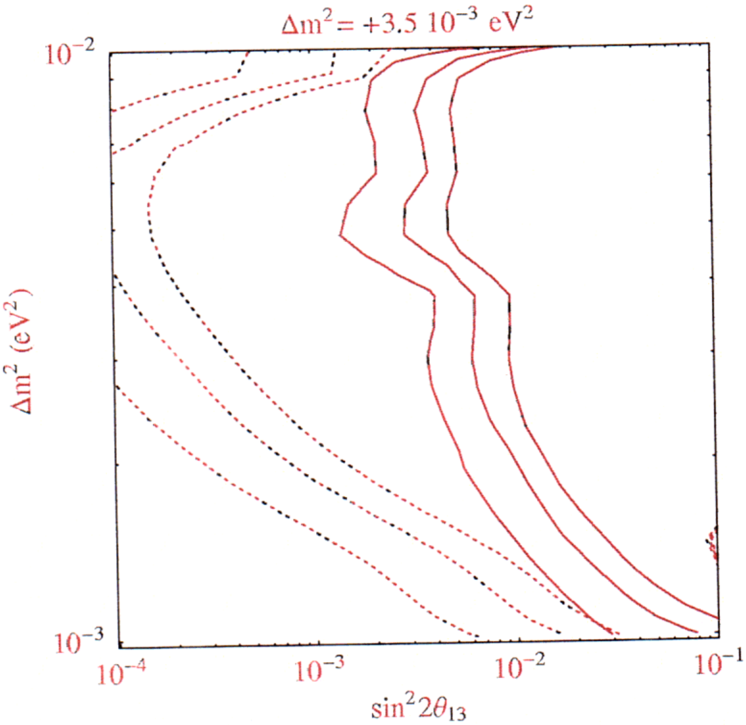
$$sin^2 2\theta_{23} = 1 \quad L = 7332km \quad \Delta m^2_{13} > 0$$

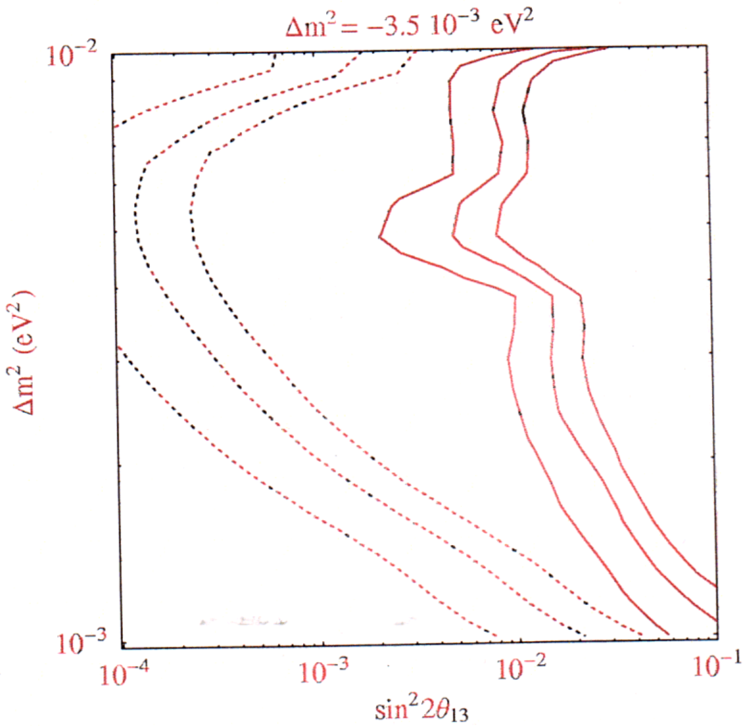
Fit to muon spectrum w/o CID



$$sin^2 2\theta_{23} = 1 \quad L = 7332km \quad \Delta m^2_{13} > 0$$

Sensitivity





Solid lines: $1, 2, 3\sigma$ limits without CID

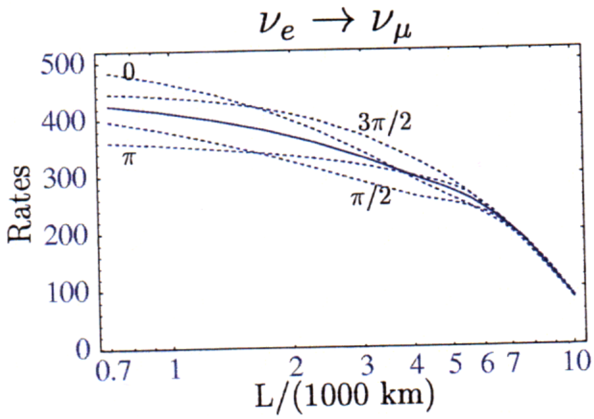
Dashed lines: $1, 2, 3\sigma$ limits with CID

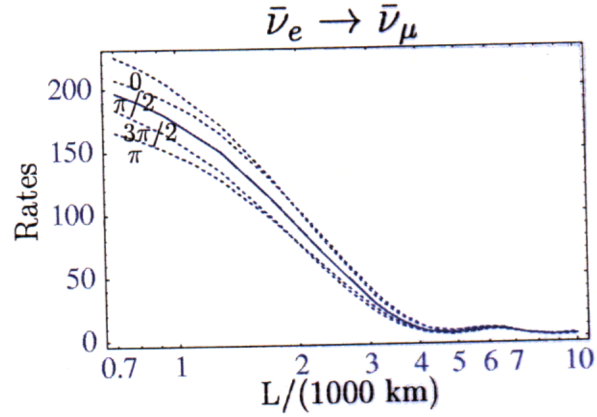
Will CID work: Efficiency, backgrounds, ... ?

$\not{C}\not{P}$– Effects for the LMA-MSW Solution

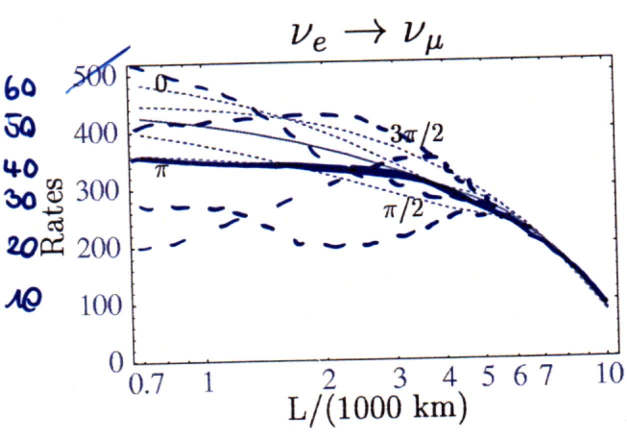
$\Rightarrow$ Δm^2_{21} and δ can no longer be ignored

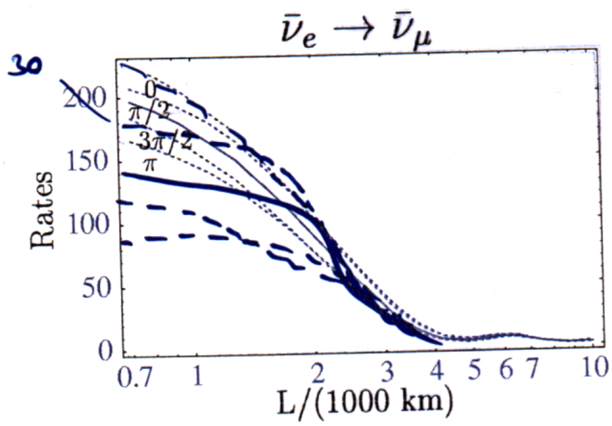
Effects for $\sin^2 2\theta_{13} = 0.1$ for different δ





Effects for $\sin^2 2\theta_{13} = 0.01$ for different δ





- smaller CP-violating effects for $L = 7332$ km

$\Rightarrow$ LMA-MSW: 2 baselines to separate matter and CP effects

- Would become less relevant if Δm^2_{atm} is larger (K2K)

Approximation $0 < \Delta m^2_{12} \ll \Delta m^2_{23}$

$$P^{3\nu}_E(\nu_e \to \nu_\mu) \cong$$

$$\begin{aligned}
& s^2_{23}\,[1 + \cos\delta \cot\theta_{23} \sin 2\theta'_{23}]\, P^{2\nu}_E(\,\overline{\Delta m}^2_{31}, \theta_{13}) \\
& + \ \cos\theta'_{12}\ \sin 2\theta_{23}\ \sin 2\overline{\theta}^m_{13}\ \sin\frac{\Delta\overline{E}_m L}{2} \\
& \times \ \left[\cos\delta \left(\sin(\overline{\kappa} + \tfrac{\Delta\overline{E}_m L}{2}) - \cos 2\overline{\theta}^m_{13} \sin\tfrac{\Delta\overline{E}_m L}{2}\right)\right. \\
& \left. - 2\sin\delta\ \sin\tfrac{\overline{\kappa}}{2}\ \sin(\tfrac{\overline{\kappa}}{2} + \tfrac{\Delta\overline{E}_m L}{2})\right]
\end{aligned}$$

where

$$\overline{\Delta m}^2_{31} \equiv \Delta m^2_{31} - s^2_{12}\Delta m^2_{21}$$

$$\overline{\kappa} = \frac{L}{2}\left[\frac{\overline{\Delta m}^2_{31}}{2E} + V - \Delta\overline{E}_m\right] - \frac{L\Delta m^2_{21}}{2E}\cos 2\theta_{12}$$

and

$$\cos\theta'_{12} = \frac{\Delta m^2_{21} c_{12} s_{12} c_{13}}{2EV + s^2_{13}\ \overline{\Delta m}^2_{31} - \Delta m^2_{21}\cos 2\theta_{12}} \cong \frac{\Delta m^2_{21} c_{12} s_{12} c_{13}}{2EV + s^2_{13}\ \Delta m^2_{31}}$$

$$\sin 2\theta'_{23} = -\frac{\Delta m^2_{21} s_{13}\sin 2\theta_{12}}{\overline{\Delta m}^2_{31} c^2_{13} - \Delta m^2_{21}\cos 2\theta_{12}} \cong -\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\, s_{13}\sin 2\theta_{12}$$

Conclusions:

With Charge Identification

- **appearance rates (or appearance rates + disappearance rates)**
- **extraction of θ_{13} and the sign of Δm^2_{31} downto $\sin^2\theta_{13} \approx 10^{-4}$**

Without Charge Identification

- **θ_{13} - effects add constructively in the combined μ^+ and μ^- Rates**
- **extraction of θ_{13} and the sign of Δm^2_{31} downto $\sin^2\theta_{13} \approx 10^{-2}$**

Baseline

- **matter effects better at larger $L \simeq 7332$ km**
- **CP violation better at smaller baselines $L \simeq 1000$ km**

$\Rightarrow$ **ideally both baselines!**

Copy of transparencies:
`http://www.ph.tum.de/~mfreund/NUFACT2000.ps`