DEPENDENCE OF PATH-LENGTH SPREAD ON FFAG LATTICE TYPE

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with the help of
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and
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This work was triggered by Shane's longitudinal studies of the non-scaling FFAG lattices suggested by Carol Johnstone

- lattices with some surprising properties.

- How can such a wide range of momentum be compacted into such a narrow radial band?

- Why does the path length vary parabolically with momentum p?

- How can triplet lattices apparently achieve smaller path length spreads ΔC than the more symmetric FODO?

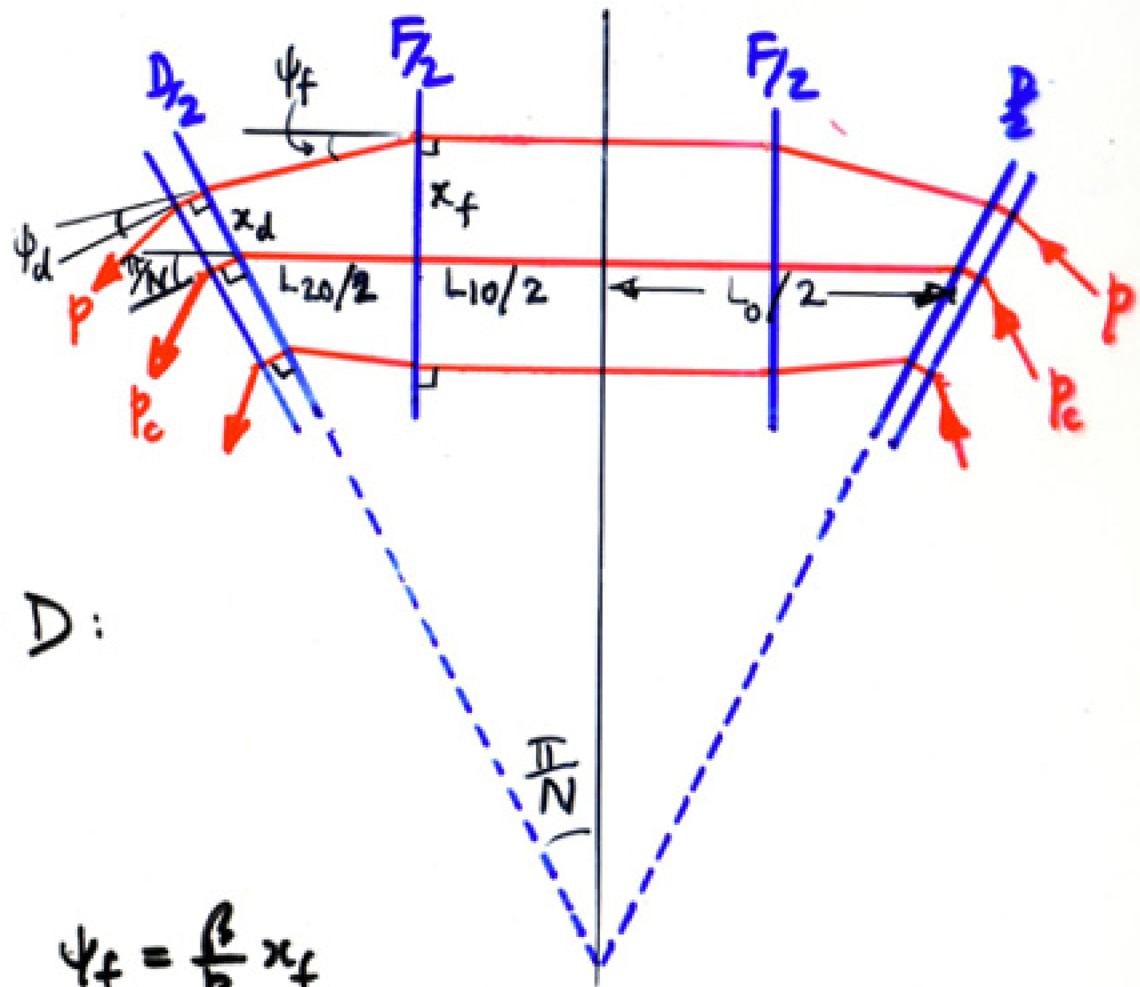
For such basic questions, perhaps a very elementary approach would be sufficient – thin lenses?

- especially if we keep the scope limited to orbit geometry, and ignore β_x, ν_x, B_{max}……!

THIN BEND TRIPLET

Assume FDF cells with thin bends: —

The "central orbit" — N-sided polygon — is on axis in F quad for momentum p_c, so all bending occurs at D:



$$e B_c \frac{l_d}{2} = \frac{\pi}{N} p_c .$$

For other momenta p

$$B_f = B_f' x_f \qquad \psi_f = \frac{\beta}{p} x_f$$

$$B_d = B_c - B_d' x_d \qquad \psi_d = \frac{\pi}{N}\frac{p_c}{p} - \frac{\beta}{p} x_d$$

where $\beta \equiv e B_f' l_f/2 = e B_d' l_d/2$ i.e. equal F and D quad strengths.

N.B. As $\frac{L_{20}}{2} \rightarrow \frac{L_0}{2}$ so FDF $\longrightarrow$ FODO.

OFFSETS AT THE QUADS

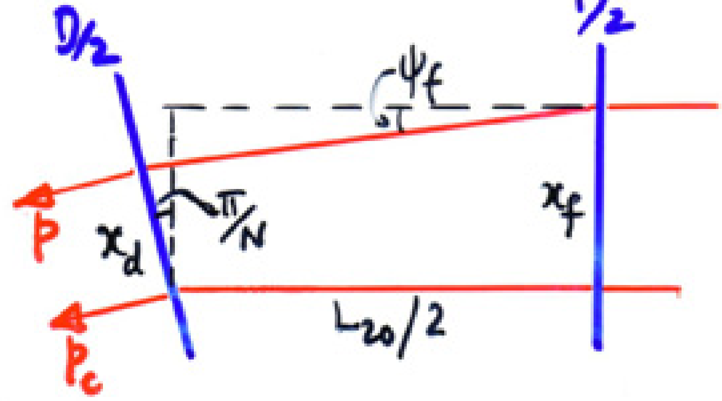
Total bend $\psi_f + \psi_d = \pi/N$

$$\therefore \quad \boxed{x_f - x_d = \frac{\pi}{N\beta}(p - p_c)}$$

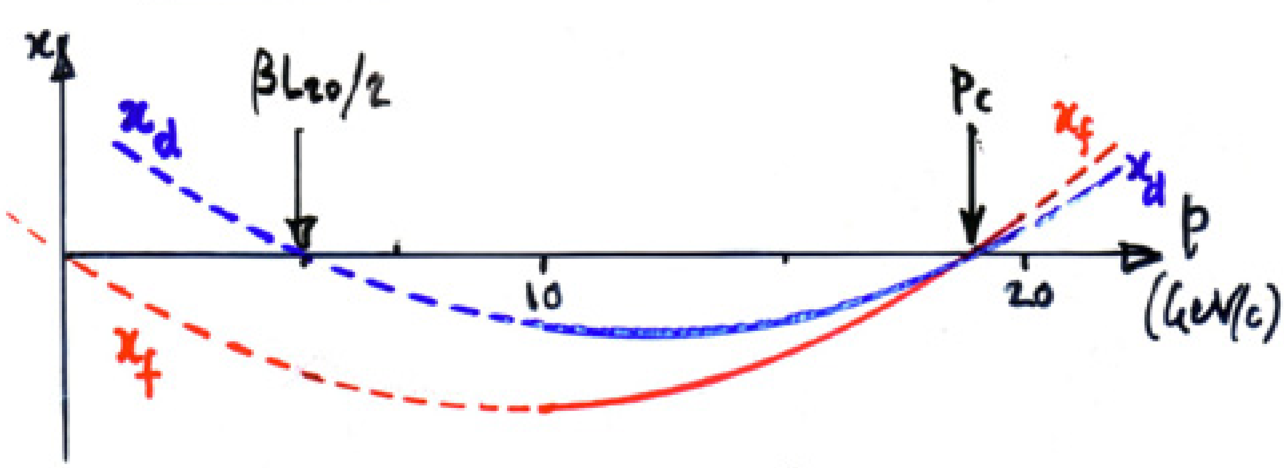
But for $\psi_f \to 0$, $\pi/N \to 0$

$$x_f - x_d \simeq \tfrac{1}{2} L_{20} \psi_f$$

$$= \frac{\beta L_{20}}{2p} x_f$$



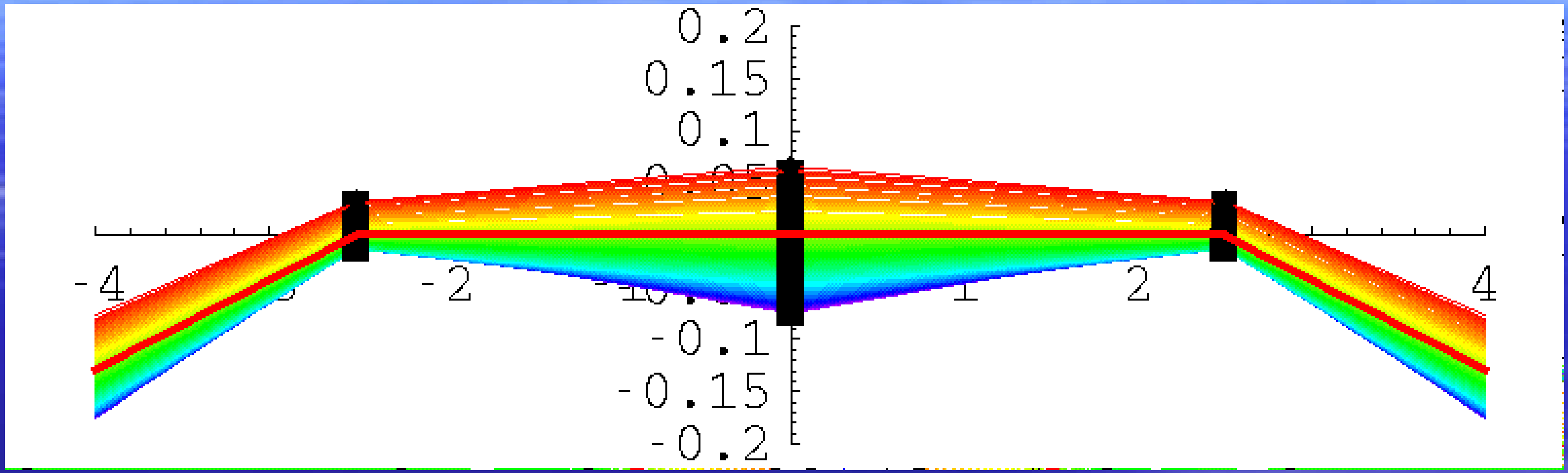
$$\therefore \quad \boxed{\begin{aligned} x_f &= \frac{2\pi}{\beta^2 N L_{20}}\, p(p - p_c) \\ x_d &= \frac{2\pi}{\beta^2 N L_{20}}\, (p - p_c)\left(p - \frac{\beta L_{20}}{2}\right) \end{aligned}}$$

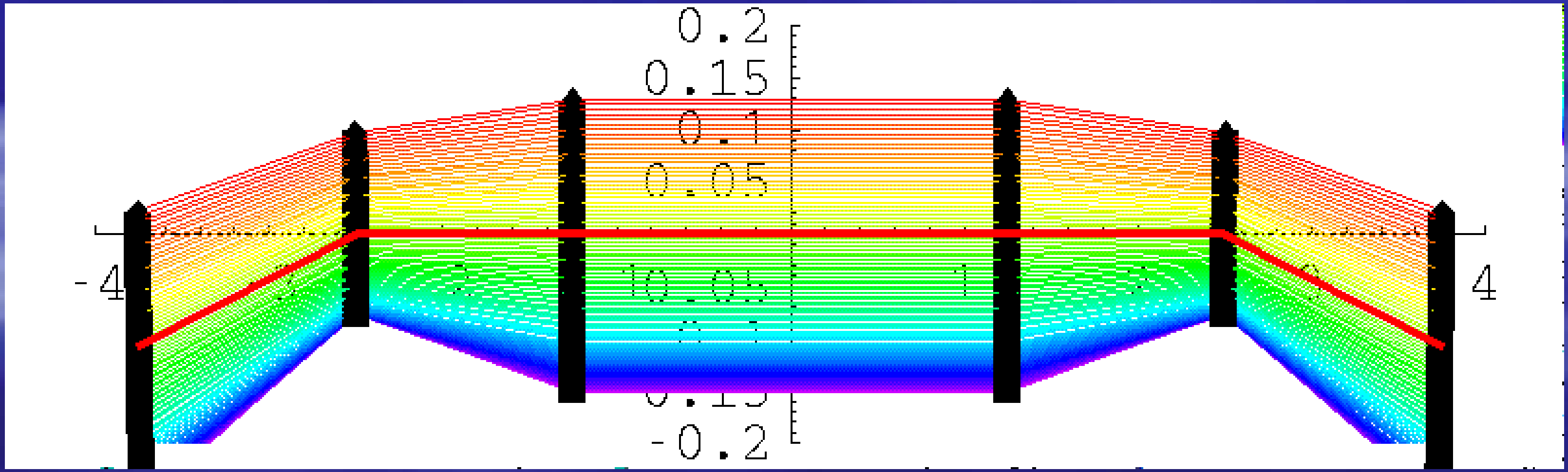


N.B. x amplitude factor $\propto 1/L_{20}$

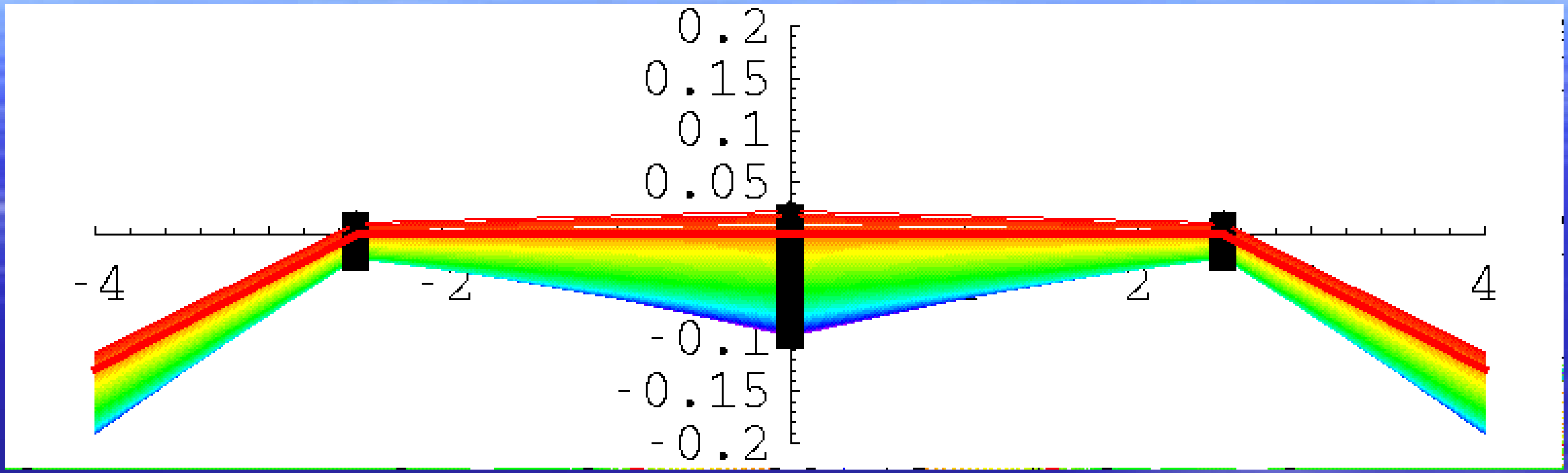
– so for the same β and N $\dfrac{x\,(\text{FDF})}{x\,(\text{FODO})} = \dfrac{L_{20}\,(\text{FODO})}{L_{20}\,(\text{FDF})} \simeq 3$

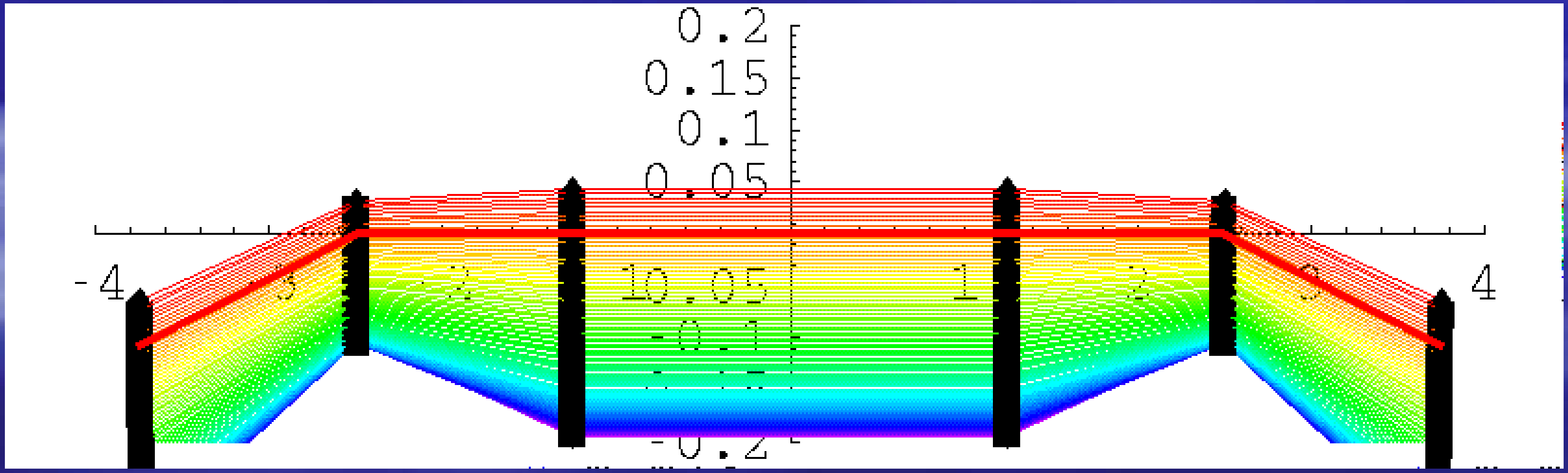
Doublet & triplet lattices *a la* Carol





Doublet & triplet lattices *a la* Dejan





FODO v. Triplet

The equations show a difference
– but what is the physical reason?

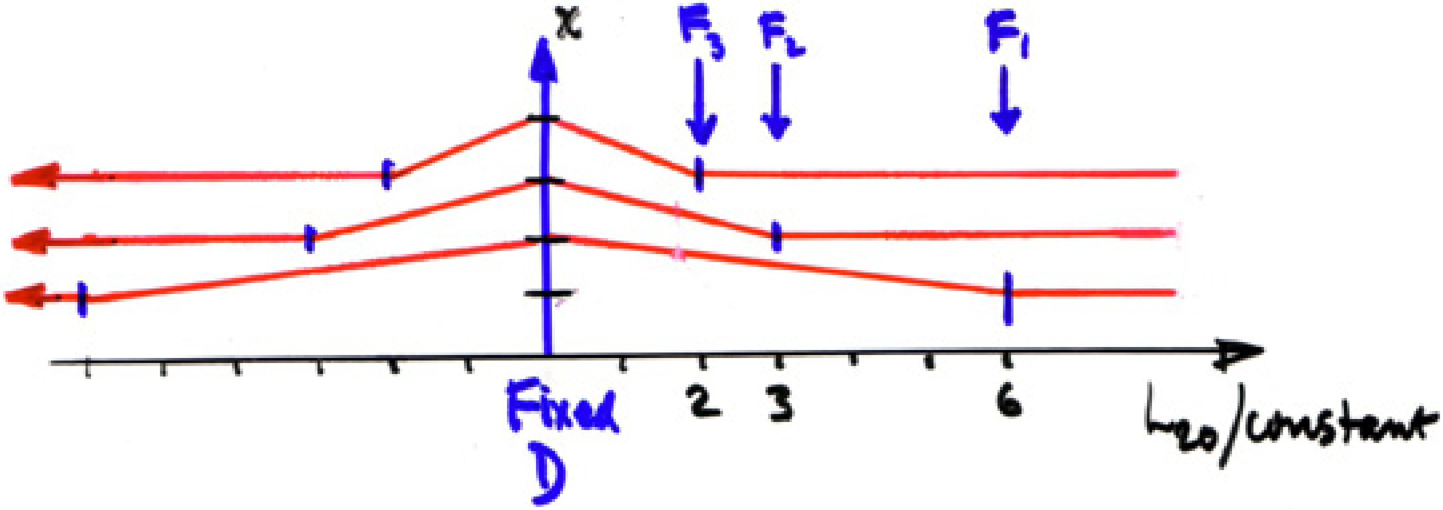
Recall $x_f - x_d = \frac{\pi}{N\beta}(p - p_c)$

$= \text{constant for given} \begin{cases} N \\ \beta \\ p \end{cases}$

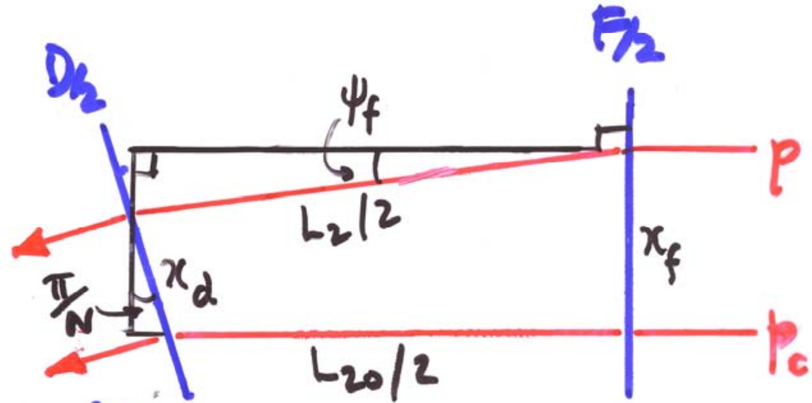
+ independent of F-D-F spacing L_{20}

But also $x_f - x_d = \frac{1}{2} L_{20} \psi_f$

– so a short lever arm L_{20} requires larger deflexion ψ_f
– i.e. larger amplitude x_f



PATH LENGTH



$$\frac{L_2}{2}\cos\psi_f = \frac{L_{20}}{2} + x_d \sin\frac{\pi}{N}$$

$$\therefore\ L_2 \simeq \left(L_{20} + \frac{2\pi}{N}x_d\right)\left(1+\frac{1}{2}\beta^2\frac{x_f^2}{p^2}\right)$$

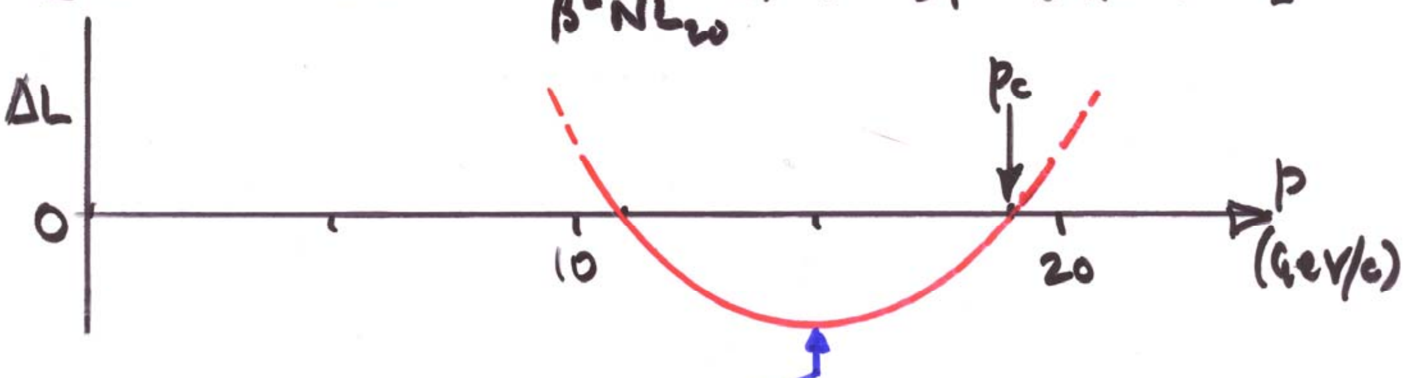
$$\& \quad \Delta L \equiv L_2 - L_{20}$$

$$\simeq \frac{2\pi}{N}x_d + \frac{1}{2}\beta^2 L_{20}\left(\frac{x_f}{p}\right)^2 \qquad \text{for } x_d \ll NL_{20},\ \psi_f \to 0$$

i.e.
$$\boxed{\Delta L = \frac{6\pi^2}{\beta^2 N^2 L_{20}}(p-p_c)\left[p - \frac{1}{3}(p_c + \beta L_{20})\right]}$$

Over the whole circumference

$$\Delta C = N\Delta L = \frac{6\pi^2}{\beta^2 N L_{20}}(p-p_c)\left[p-\frac{1}{3}(p_c+\beta L_{20})\right]$$



Minimum L occurs for $\quad p_{L^\vee} = \frac{1}{3}\left(2p_c + \frac{\beta L_{20}}{2}\right)$

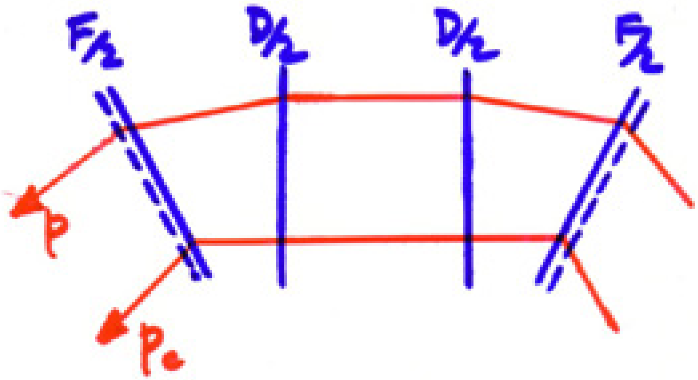
– or inversely $\quad p_c = \frac{1}{2}\left(p_{L^\vee} - \frac{\beta L_{20}}{2}\right)$

– so for $p_{L^\vee} = 15$ GeV/c, $\frac{\beta L_0}{2} = 7$ GeV/c $\rightarrow p_c = 19$ GeV/c.

N.B. $\Delta C \propto 1/\beta^2 N L_{20}$ — just like x_f, x_d.

FDF or DFD?

DFD can be analysed in the same way as **FDF**.



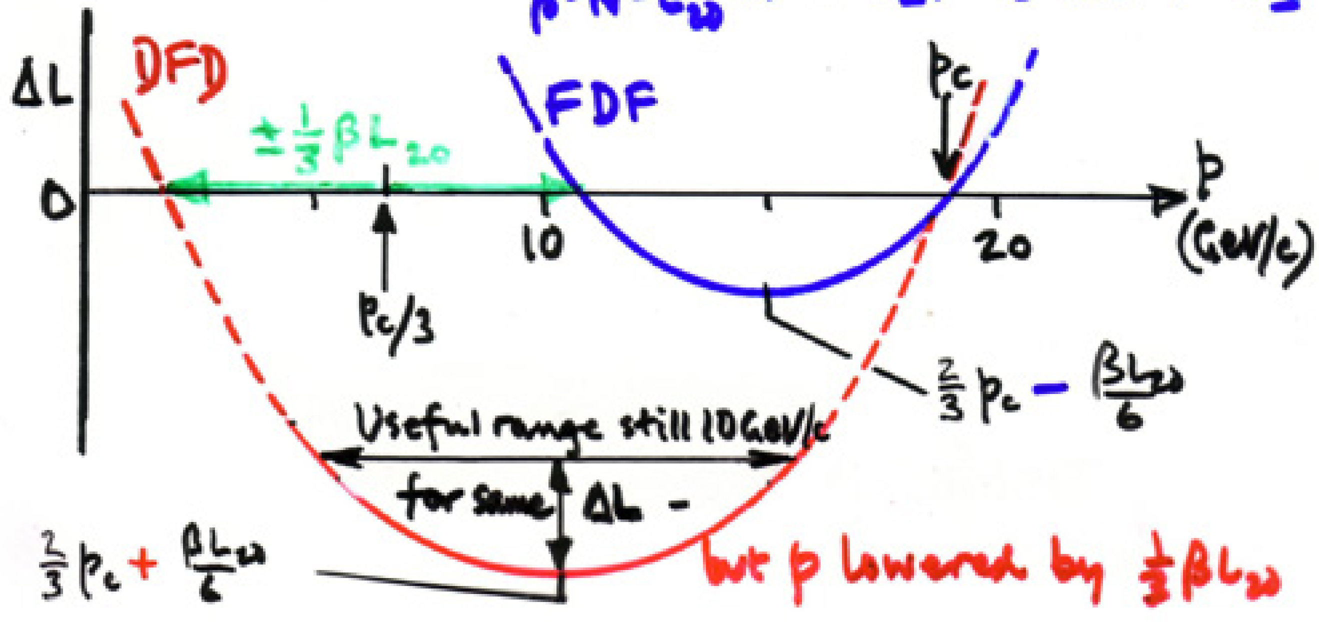
For the same geometry and magnet strengths, the results can be simply obtained by – exchanging subscripts $f \longleftrightarrow d$

– reversing the sign of β.

$$x_{bend} = x_f = \frac{2\pi}{\beta^2 N L_{20}} (p - p_c)(p + \beta L_{20})$$

$$x_d = \frac{2\pi}{\beta^2 N L_{20}} p (p - p_c)$$

$$\Delta L = \frac{6\pi}{\beta^2 N^2 L_{20}} (p - p_c)\left[p - \frac{1}{3}(p_c - \beta L_{20})\right]$$



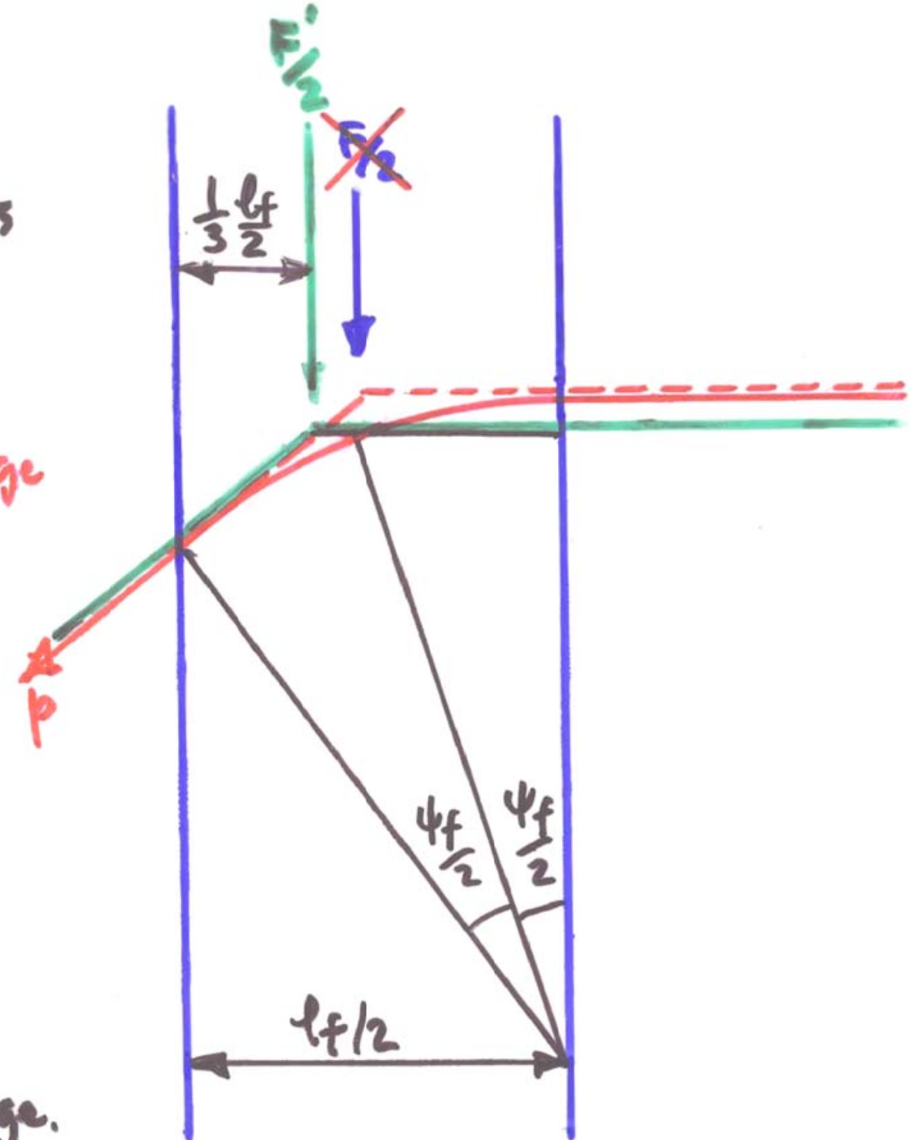
THICK BENDS

Orbit curvature within the magnets makes

$$\hat{x}_{thick} \neq \hat{x}_{thin}$$

– so placing thin bend at hi-x edge of a thick ½-bend → B error.

∴ shift the thin bend so that $B(F'/2) = \langle B(arc) \rangle$.



For ||-ended magnets place $\frac{F'}{2}$ $\frac{1}{3}\frac{\ell_f}{2}$ from D-facing edge.

This reduces the D-F spacing L_{20} to

$$L'_{20} = \ell_{fd} + \frac{1}{3}\left(\frac{\ell_f}{2} + \frac{\ell_d}{2}\right)$$ – a significant correction to $L_{20} = \ell_{fd} + \frac{\ell_f}{4} + \frac{\ell_d}{2}$ when $\ell_{fd} \simeq 20$ cm

For sector magnets, $L'_{20} \simeq \ell_{fd} + \frac{1}{\cancel{2}\,3}\left(\frac{\ell_f}{2} + \frac{\ell_d}{2}\right)$

		Dejan			Scott					Carol
		FDF Triplets			FDF	Doublet	F0D0-1	F0D0-2	F0D0PAC	F0D0
Circumference (m)	**C**	**323**	**328**	**348**	**481.74**	**436.53**	**612.18**	**470.00**	**1200**	**2041**
Number of cells	**N**	66	72	80	93	101	113	82	200	314
Cell length (m)	**$C/N = L_0$**	4.89	4.56	4.35	5.18	4.32	5.42	5.73	6.00	6.5
D-bend length (m)	**$l_d = BL$**	**1.50**	**1.00**	**0.80**	**1.28**	**1.01**	**0.81**	**0.99**	**1.00**	**0.35**
F-quad length (m)	**$l_f/2$ =QLF**	0.50	0.58	0.58	0.45	0.41	0.30	0.37	0.50	0.075
F-D spacing (m)	**D3P = dl3**	0.17	0.20	0.20	0.50	0.50	2.00	2.00	2.00	3.00
Effective F-D spacing / L_0	**L'_{20} / L_0**	0.24	0.25	0.24	0.33	0.26	0.83	0.80	0.78	0.95
F gradient (T/m)	**B'_f = GF1**	67.42	62.43	66.74	50.1	43.09	39.42	30.64	20.38	75.90
D gradient (T/m)	**B'_d =-GDD2**	35.57	56.65	75.30	30.4	35.68	29.97	23.70	20.42	32.45
Quad strength factor (mean)	β	9.05	9.67	10.32	6.30	5.32	3.61	3.46	3.06	1.70
Time-of-flight range/cavity (ps)	**DeltaC/(c*n_{cav})**	**11.9**	**9.0**	**6.9**	**8.3**	**7.9**	**6.6**	**6.7**	**2.8**	**5.5**
Path length variation (cm)	**DeltaC**	**23.5**	**19.5**	**16.5**	**23.3**	**24.0**	**22.5**	**32.9**	**17.0**	**51.6**
" " (tracked)		**14.2**	**11.7**	**9.9**	**17.1**	**21.4**	**24.0**	**35.0**	**18**	**51**
D-bend field (T) @ 15 GeV/c	**B_d = by0**	5.08	6.69	7.49	4.53	4.89	4.85	5.61	2.34	
F-quad field (T) @ 15 GeV/c	**B_f = byq**	-2.85	-2.00	-1.78	-2.68	-2.23	-1.93	-2.30	-0.77	
Momentum (GeV/c) for B_f = 0	**p_c**		19.1	19.0	19.8	20.4	18.5	18.5	18.9	16.6
Momentum @ $x_d = x_c$	**Beta*L'_{20}/2**	5.3	5.5	5.4	5.4	4.3	8.1	7.9	7.1	5.3
Momentum @ minimum path	**p(C_{min})**		**14.5**	**14.5**						**12.8**
" " (tracked)			**14.5**	**14.5**	**15**	**15**	**15**	**15**	**15**	**12.7**
										6 GeV/c
x offset (cm) in F @ 10 GeV/c	**x_f_{10}-x_f_{15}**		**-2.1**	**-1.8**	**-2.2**	**-2.6**	**-3.0**	**-4.4**	**-2.1**	**-7.0**
" " (tracked)	**XCOf**	**-2.9**	**-2.5**	**-2.1**	**-2.5**	**-2.8**	**-3.3**	**-4.8**	**-2.5**	**-7.0**
x offset (cm) in F @ 20 GeV/c	**x_f_{20}-x_f_{15}**		**6.4**	**5.4**	**7.2**	**7.7**	**7.8**	**11.3**	**5.7**	**7.8**
" " (tracked)	**XCOf**	**4.6**	**3.9**	**3.3**	**5.3**	**7.4**	**8.4**	**12.3**	**6.1**	**7.7**

N.B. Data computed with thick bend correction factor 1/3 (in place of 1/2).

THEORY v. EXPT

$p(L')$ – good

ΔL, $x_{f,d}$ – excellent for Carol (thin lens, small angles)
– within ~~20%~~ for "thick bends"
– very sensitive to L'_{20}
– need to refine correction for thick sectors.
↳ < 7% for FODO
< 70% for triplet!

FODO v. TRIPLET

Time of flight spread during acceleration:

$$\Delta t_{cav} N_{cav} N_{turns} = \Delta t_{cav} \frac{\Delta E}{\Delta V} \leq \text{constant} \times T_{rf}$$

∴ crucial factor for phase slip is:

$$\boxed{\Delta L_{cav} = c\Delta t_{cav} = \frac{\Delta L_{cell}}{n_{cav}} = \frac{(\Delta p/2)^2}{\beta^2 N^2 L'_{20}\, n_{cav}}}$$

So for the same energy range $\Delta E = c\Delta p$ and quad strength β

$$\frac{N_{FDF}}{N_{FODO}} = \sqrt{\frac{(L'_{20}\, n_{cav})_{FODO}}{(L'_{20}\, n_{cav})_{FDF}}} \simeq \sqrt{\frac{2.36}{0.76} \times 2} = 2.5\,!$$

– a very different prediction from Scott's findings.

Apparently other factors (B_{max}?, tune, ...)
play a key role in Scott's optimization.

My project this week is to find out why
the potential of FODO is not being realised!