CONTENT:

Update with our lattice design:

- A distributed RF looks very promising.
- Tools: SYNCH, COSY, MAD
- Lattice properties – a ring picture.
- Longitudinal simulation of the acceleration with the latest lattice solutions (Mike Blaskiewicz).
- Conclusions
• The minimum emittance lattice requires reduction of the function $H$:
  - The normalized dispersion amplitude corresponds to the $<H>^{1/2}$
What are the basic parameters?

- **Required Range of Energies (or \( \frac{dp}{p} \))**
  - the “central” energy or momentum \( p_0 \) is in two examples presented later set to 10 GEV. The acceleration would be possible from 10 GeV up to 20 GeV.
  - Aperture limitation is defined by the maximum value of the DISPERSION function:
    - \( \Delta x < +/- 30 \text{ mm} \)
    - if the \( 0.5 < \frac{dp}{p} < 1.5 \) then:
      - \( D_x < 60 \text{ mm} \)

- **Why is the Minimum Emittance Lattice for the electronic Storage Rings Relevant?**
  - The normalized dispersion amplitude Corresponds to the \( <H>^{1/2} \) !!!
What was our promise given at the last meeting (BNL editors meeting):

• Construct a lattice where the dispersion will oscillate between positive and negative values but not exceeding 6 cm without opposite bending magnets.

\[ \Delta x < D \frac{dp}{p} = 0.06 \times (+/-0.5) = +/-0.03 \, m \]

• Make a change in the circumference smaller to reduce the RF phase change.

• Try to combine the linac with a single arc.

• Or make enough room for the cavities within the ring.

• Longitudinal simulation of the multiple turns (10 – 20 turns)
The major result: reduced change of the circumference the ‘SYNCH’ result (with Ernie’s combined function dipole subroutine correction)
TEST data for different tools: SYNCHROTRON
The major result: reduced change of the circumference
the ‘MAD’ file result

Change in circumference on $\Delta p/p$

The FFAG lattice without opposite bands (f9.mad file)
The FFAG lattice without opposite bends

The RF cavities are distributed within every third cell
Betatron functions within the two cells

FFAG lattice without opposite bends
RF is distributed within the ring

Beta [m], Dispersion [cm]

- Beta X
- Beta Y
- Dispersion

Path length [m]

Cavities 87 92 107 120 133 147 160
Cavities 97
A part of the ring

The FFAG lattice without opposite bends

The RF cavities are distributed within every third cell

\[ \theta = \frac{1.6 * 0.25}{Bp} \]

- 12 mrad @ 10 GeV
- 6.235 mrad @ 20 GeV

\[ \theta = 0.16998 \text{ rad @ 10 GeV} \ (9.7^\circ) \]
\[ \theta = 0.11331 \text{ rad @ 15 GeV} \ (6.5^\circ) \]
\[ \theta = 0.087 \text{ rad @ 20 GeV} \ (4.87^\circ) \]
Two CELLS:

The FFAG lattice without opposite bends

Two Cells
Betatron tunes during acceleration

FFAG lattice without opposite bends - distributed RF

Betatron tunes versus momenta

- Horizontal tune
- Vertical tune
Chromaticities during acceleration – Corrected to zero at the central muon energy of 15 GeV

FFAG lattice without opposite bends – distributed RF
Chromaticities versus momenta (corrected to zero @ 15 GeV)

- Horizontal Chromaticity
- Vertical chromaticity

Chromaticities $\xi_x, \xi_y$ vs. $\Delta \rho/\rho$
The slipping factor $\eta$ during acceleration
All previous results have a ~1m dipole divided into 100 pieces and quadrupoles divided into 26 and 46 pieces, as well they include the first attempt to include the end of the quadrupole field.
At negative $\Delta p/p$ lattice is ‘imaginary $\gamma$’

Momentum Compaction Dependence on $\Delta p/p$

The FFAG lattice without opposite bends (f9.mod file)
Maximae of the betatron functions during acceleration
Maximum of the dispersion function during acceleration

FFAG lattice without opposite bends – distributed RF

Maximum of $D_x$ versus momenta

Maximum of Dispersion function $D_x$
A picture tells a story: particle path in the basic cell during acceleration

Transverse offsets along the basic cell for all $\Delta p$

The FFAG without opposite bands

Distance along the basic cell (m)

Transverse orbit offset (mm)
Particle path in one of the recent examples:

Muons paths along the basic cell for different momenta

Circumference 163.20 meters long is made of 32 cells
660 ns lattice from D. Trbojevic and 900 ns lattice from E. Courant. Assume negligible energy input to the RF system during acceleration[1]

1-D update equations are

\[ \tau_{n+1} = \tau_n + T(E_n) \]  
(1)

\[ \left( \frac{R}{Q} \right) I(t) = \frac{1}{\omega_{rf}} \frac{dV(t)}{dt} + \omega_{rf} \int_0^t dt_1 V(t_1) \]  
(2)

\[ E_{n+1} = E_n + qV(\tau_{n+1}) \]  
(3)

\( I(t) \) smoothed by 0.5 ps. \( V(t) \) updated with \( \Delta t = 0.15 \) ps.
Energetics of the RF system
For $6.25 \times 10^{12}$ muons the total charge is $1\mu$C. Assuming a factor of 2 voltage drop the initial stored energy in the RF cavities is

$$U = 10GV \times 1\mu C \times \frac{4}{3} = 13kJ$$

The stored energy is related to the voltage and impedance by

$$U = \frac{V^2}{2\omega_{rf} \left( \frac{R}{Q} \right)}$$

Taking a total voltage of 500 MV and $\omega_{rf} = 2\pi \times 200MHz$ one obtains $(R/Q) = 7.6$ k$\Omega$. The simulations used this impedance and $V = 600$ MV so the voltage dropped to 400 MV at the end of the cycle. Taking 10 MV per cavity the requisite $R/Q$ per cavity is 126$\Omega$. The stored energy per cavity is 300 J. For $E = 10$ MV/m the volume is 0.7$m^3$. With 60 cavities some extra straight sections may be required but, since $10 GeV \gg 106$ MeV = $m_\mu c^2$, the straights will have a negligible effect on $dT/dE$.

References

Conclusions:

- The latest results in the FFAG lattice without opposite bends with distributed RF are very encouraging.

- Present codes MAD and SYNCH should be checked by either other codes or by an analytical calculation.

- If it is shown that the presented idea is really possible the whole muon acceleration should be redone.