Ionization Cooling with Lithium Lenses

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Objectives

- Lithium lens versus 50 T solenoid
  - Is it competitive?

Outline

- Ionization cooling with lithium lens
- FNAL Lithium lens
- Limitations on lithium lens operation and their mitigations
- Conclusions

Cooling and Diffusion

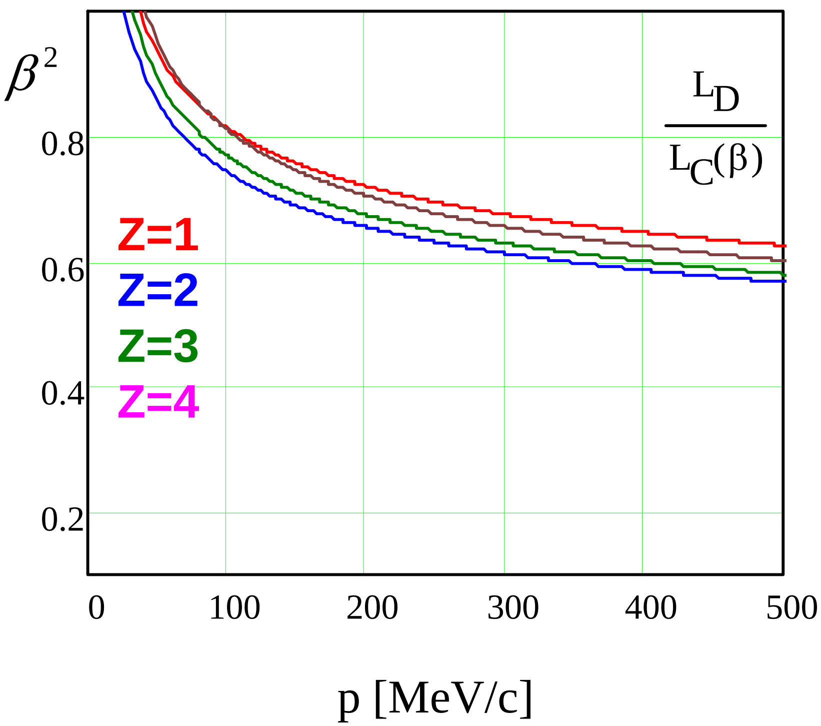
- Energy loss and multiple scattering are closely related

$$\frac{dE}{ds}=\frac{4\pi Z n_a e^4}{mc^2\beta^2}L_C(\beta),\quad L_C(\beta)=\ln\left(\frac{\sqrt{T_{\max}m_e c^2\beta^2\gamma^2}}{I}\right)-\beta^2$$

$$\frac{d\overline{p_x^{\,2}}}{ds}=\frac{d\overline{p_y^{\,2}}}{ds}=\frac{4\pi Z(Z+1)n_a e^4}{c^2\beta^2}L_D,\quad L_D\approx\ln\left(\frac{r_a}{r_n}\right)$$

In PDB: $\theta_0^{\,2}=\left(\frac{13.6\,\mathrm{MeV}}{c\beta p}\right)^2\frac{x}{X_0}\left(1+0.038\ln\left(\frac{x}{X_0}\right)\right)$

where x in $\ln(x/X_0)$ is set by $x\,(dE/dx)\approx E$



- Equilibrium rms angle in thin target approximation ($L \ll \beta_{x,y}$) are

$$\overline{\theta_x^{\,2}}=\overline{\theta_y^{\,2}}=\frac{m_e}{m_\mu}\frac{(Z+1)}{2\gamma}\frac{L_D}{L_C(\beta)}$$

$\Rightarrow$ Normalized emittance

$$\varepsilon_{nx}=\frac{m_e}{m_\mu}\frac{(Z+1)}{2}\frac{L_D}{L_C(\beta)}\beta\langle\beta_x\rangle_{t\arg}$$

	p = 100 MeV/c	
	L_C	L_D
H	10.6	8.7
He	10.3	7.7
Li	9.8	7.6
Be	9.2	7.5

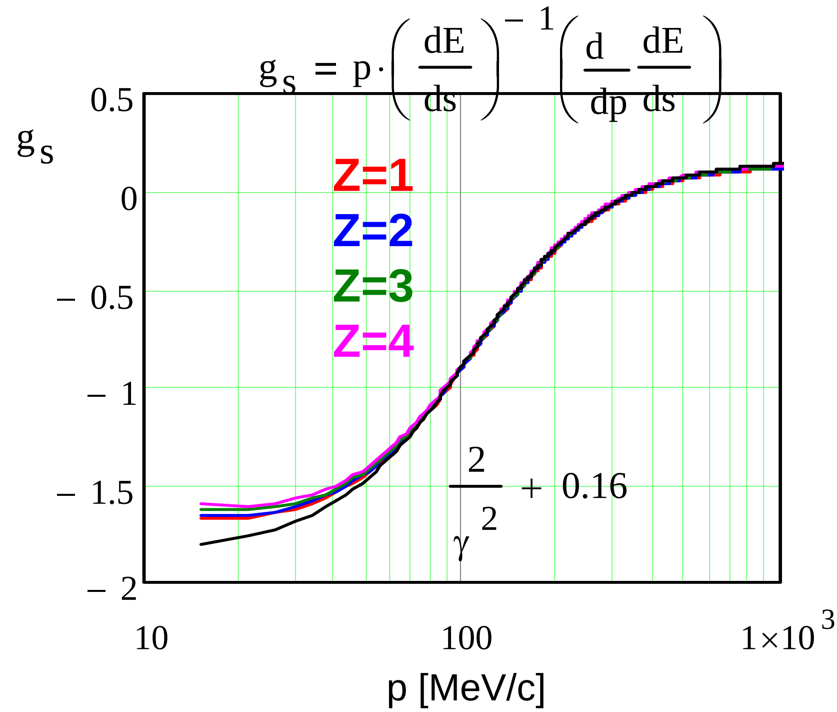
Sum and Redistribution of Cooling Decrements

- Sum of the decrements does not depend on details of cooling scheme

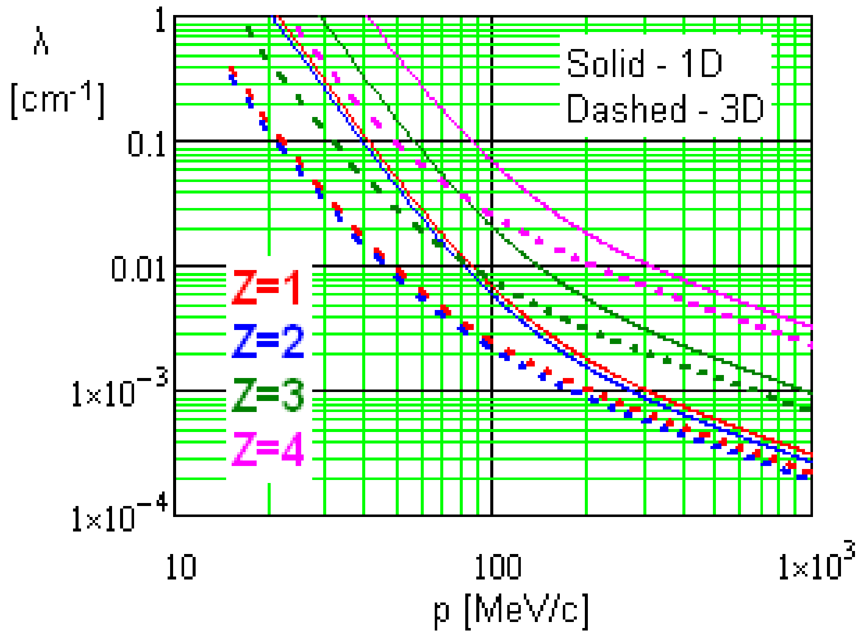
$$\lambda_{x0} \equiv \frac{1}{p_x}\frac{dp_x}{ds} = \frac{1}{\beta cp}\frac{dE}{ds}, \qquad \lambda_{y0} = \lambda_{x0},$$

$$\lambda_{s0} = \frac{1}{\beta c}\frac{d}{dp}\left(\frac{dE}{ds}\right)$$

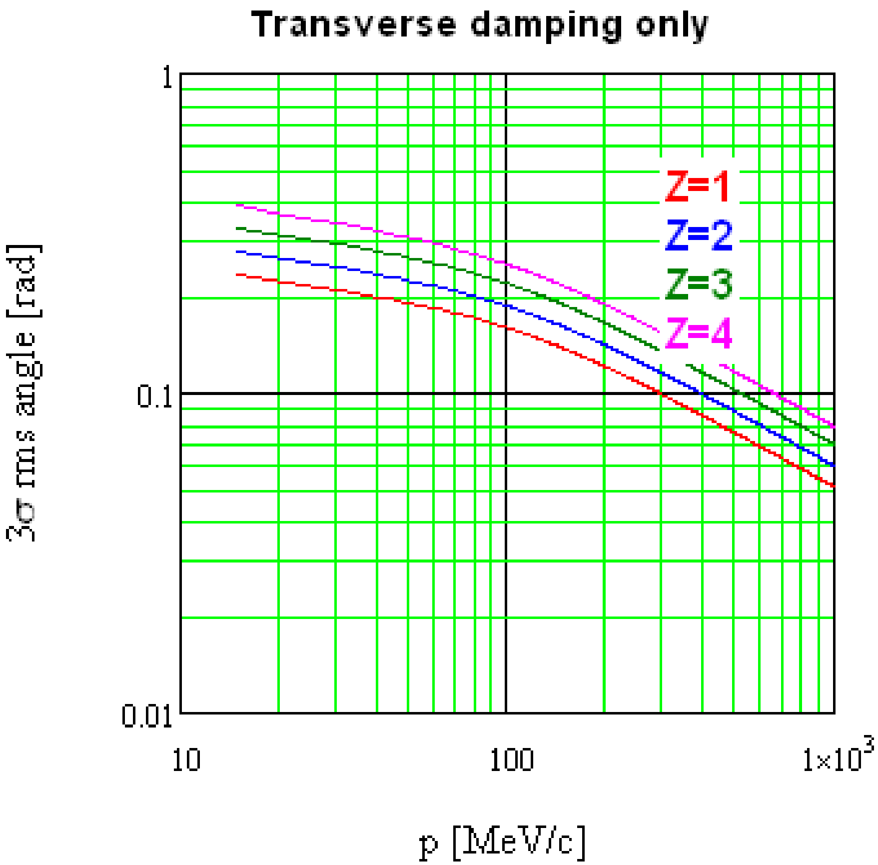
$$\Rightarrow \quad \sum_k \lambda_k = \frac{1}{\beta cp}\left(2\frac{dE}{ds} + p\frac{d}{dp}\left(\frac{dE}{ds}\right)\right)$$
$$\approx 2\lambda_{x0}\left(\beta^2 + 0.08\right), \quad 0.5 \le \beta\gamma \le 5$$

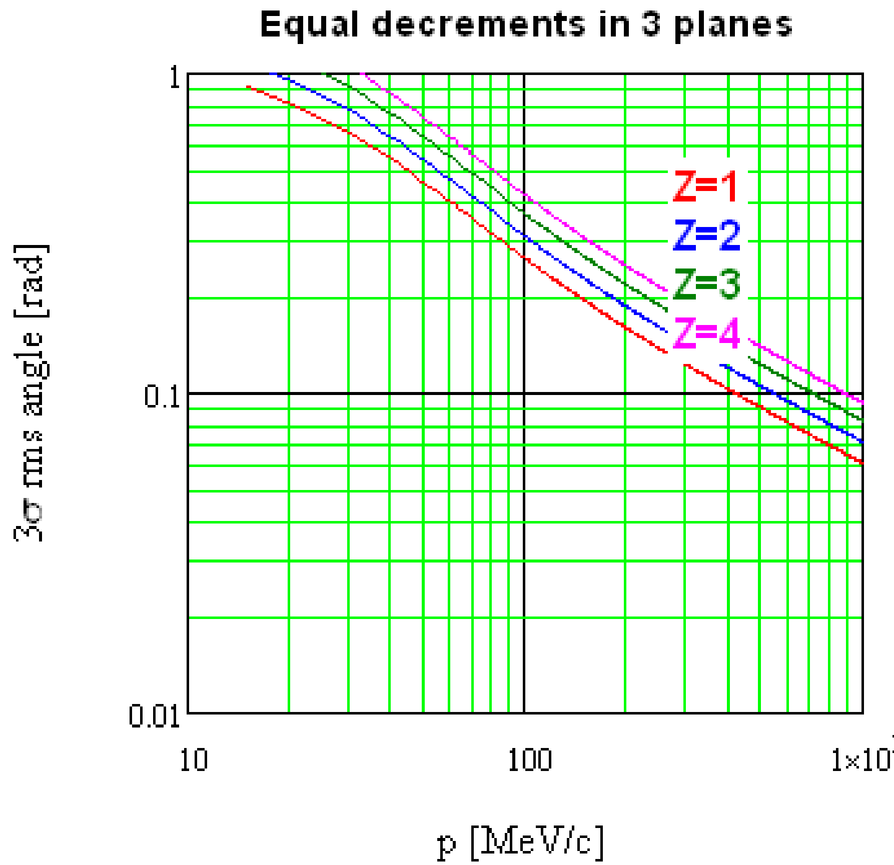


- Long. motion is unstable for p≤300 MeV in absence of decrement redistribution
- Redistribution of decrements allows one to have good cooling for all 3 degrees of freedom for smaller energy



Equilibrium Angular Spread



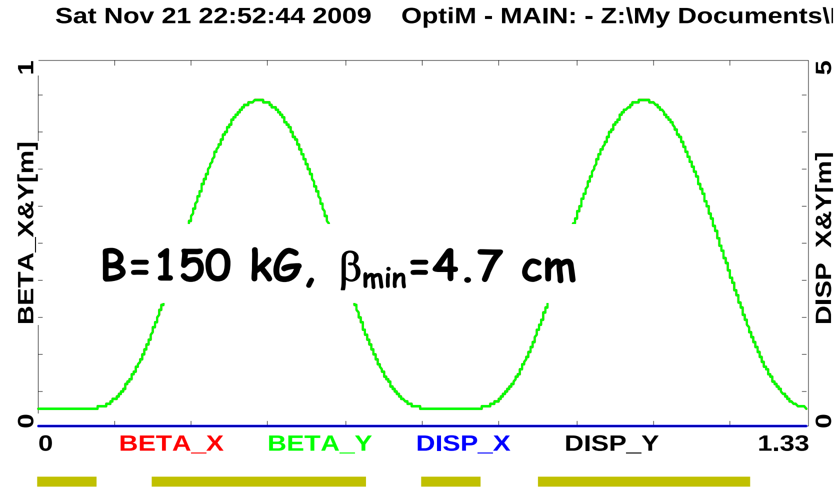


- Decrease of cooling energy results in an increase of equilibrium angular spread and, consequently, an increase of non-linear effects
- Equilibrium rms angle in lithium is ~2 times larger than in liquid hydrogen

Beam focusing with Lithium Lenses & Solenoids

- For solenoidal focusing (Edwards -Teng β-functions)

$$\boxed{\beta_{\perp sol} = \frac{2pc}{eB_0}}$$

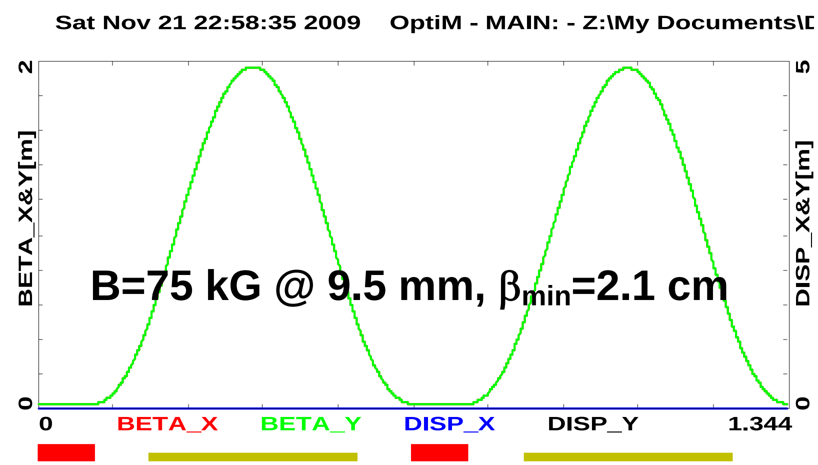


- Lithium lens gradient is limited by magnetic field at its aperture
  - At the final stage of cooling the equilibrium β-function is

$$\beta_{\perp LiL} = \sqrt{\frac{pc}{eG}}, \quad G = \frac{B_0}{A_\sigma \sqrt[4]{\beta_{\perp LiL}{}^2 \, \overline{\theta_x{}^2}}}$$

$$\Rightarrow \quad \boxed{\beta_{\perp LiL} = \frac{pc}{eB_0}\left(A_\sigma \sqrt{\overline{\theta_x{}^2}}\right)}$$

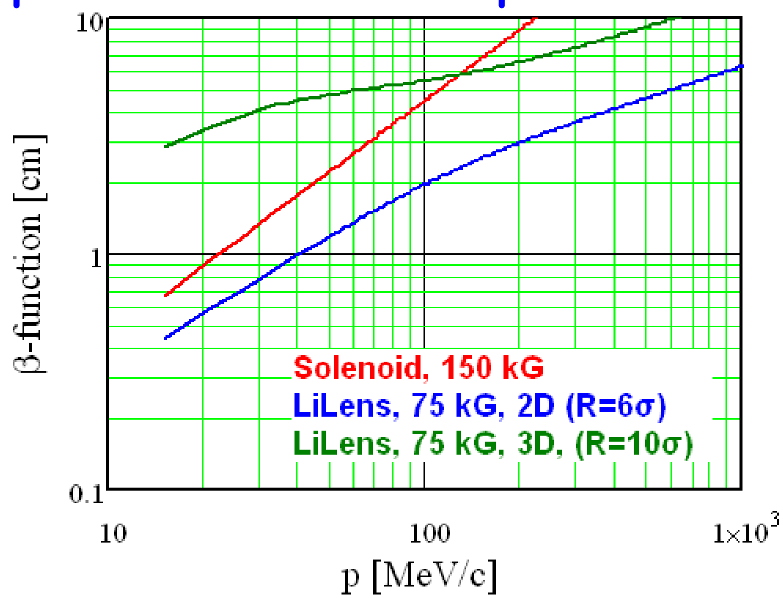
where A_σ is the lens aperture over rms beam size in equilibrium

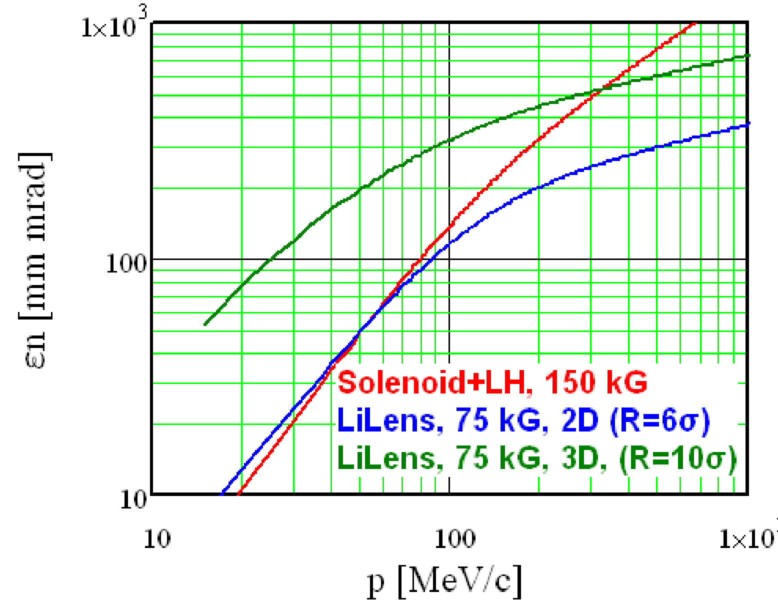


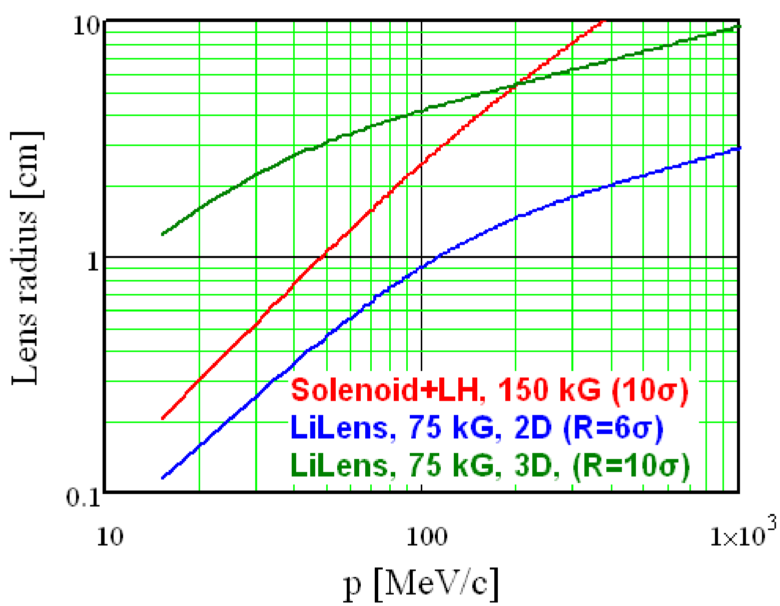
- Both β-functions linearly depend on B
  - B=150 kG for solenoid & B=75 kG for Li lens are based on present technology
- Accelerating cavities are located inside low field solenoids

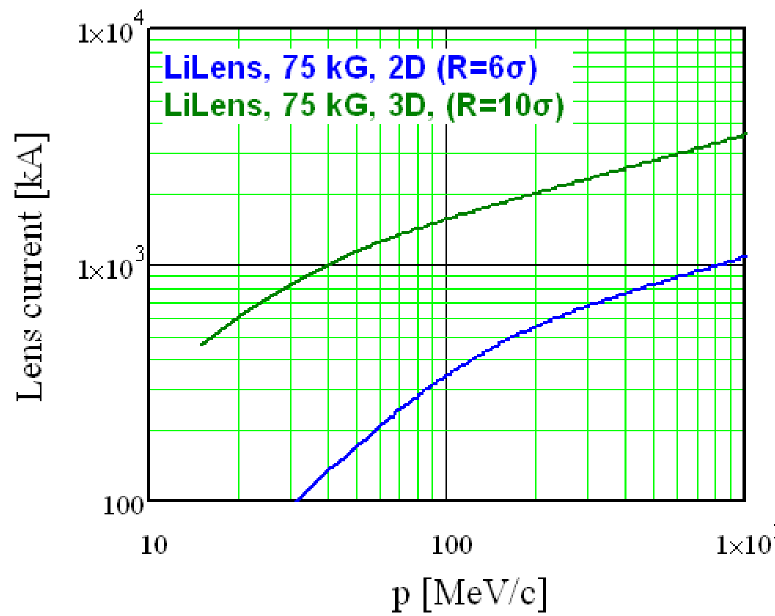
Lithium Lens versus Solenoid

- Decr. redistribution requires aperture increase ⇒larger $\beta_\perp$ & ε
  - Aperture is not a problem for solenoid









Lithium Lens and Limitations on its Gradient

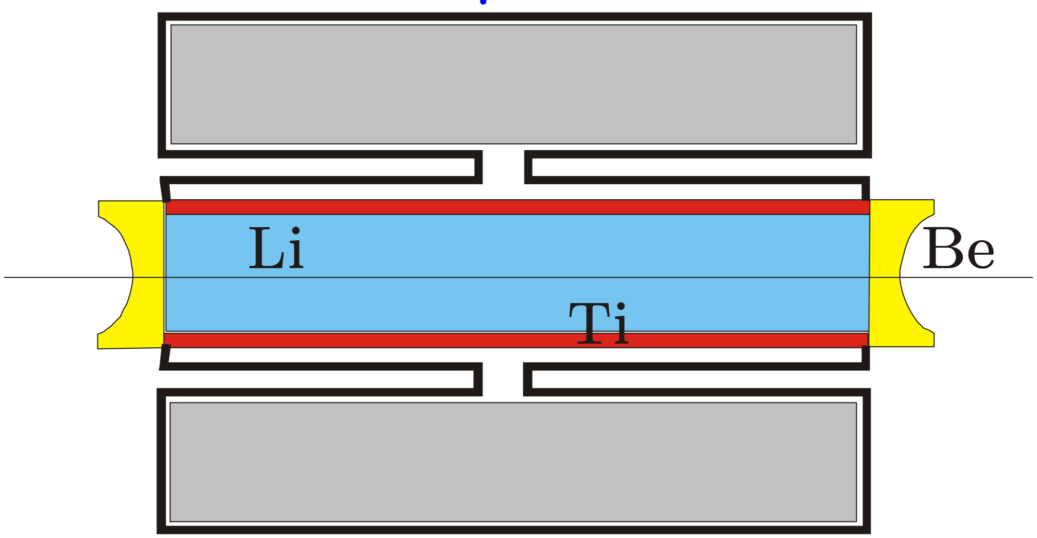
- Li lens is competitive to solenoid for the last stages of cooling when only transverse cooling is used
  - It additionally suggests a correction for spherical aberrations
- Li lens is not good for initial stage of cooling when the beam size is large

  $\Rightarrow$ Large radius

  $\Rightarrow$ Large lens current
- Major limitations of Li lens focusing
  - Surface magnetic field
    - Pressure and material fatigue
  - Lens overheating
    - Skin-effect requires long pulses

Fermilab Lithium Lens

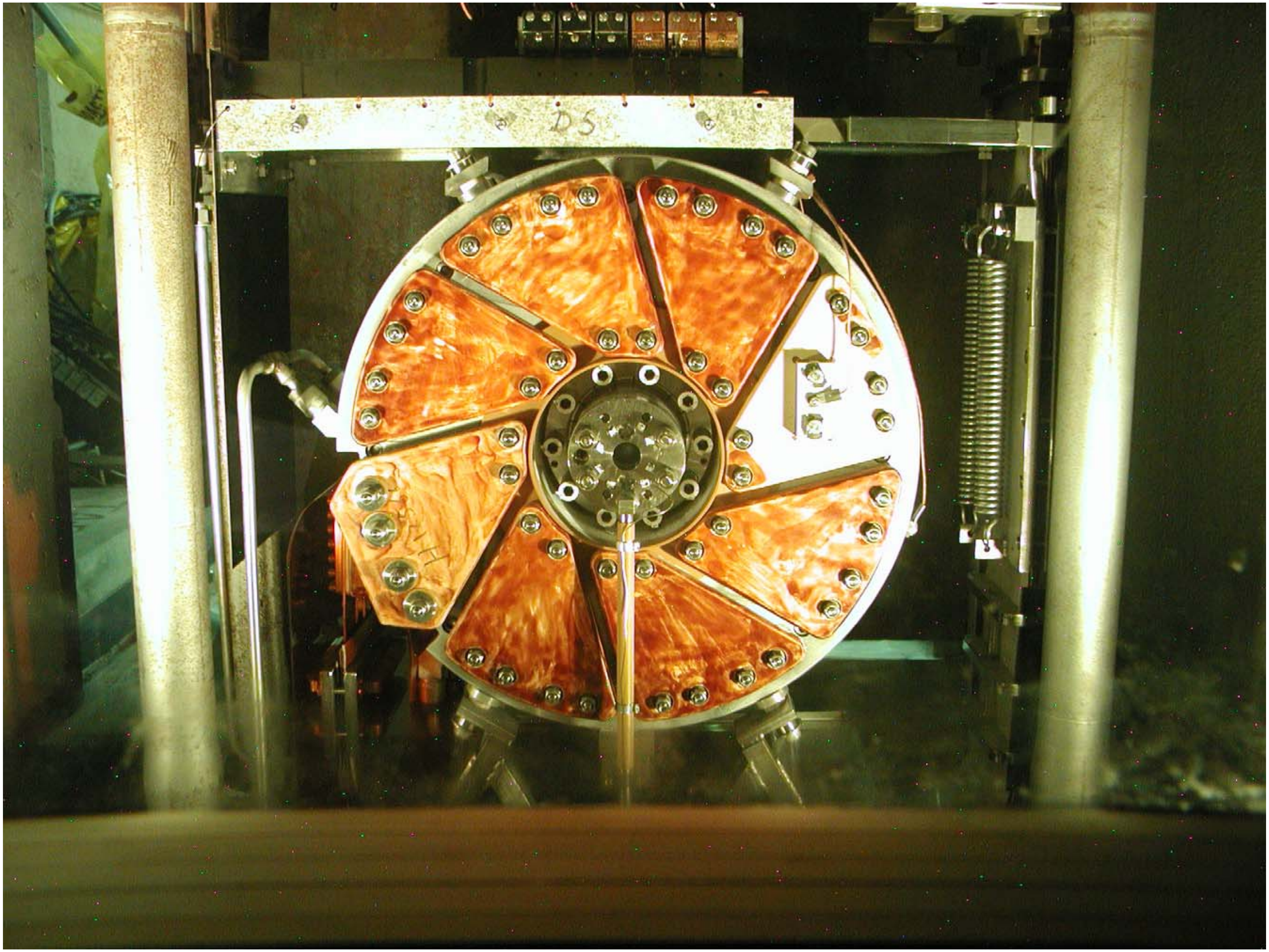
- Long evolution of the design
  - Diffusion bonded welding
  - 3-6 month lifetime
  - ~$(5\text{-}10)\cdot 10^6$ pulses
- Material fatigue is the major problem limiting the lifetime
  - Pressure preloaded Li
- 1-to-8 current transformer
  - ~4 kV capacitor bank



FNAL Lens Parameters

Length	15 cm
Radius, r_L	1 cm
Repetition rate	0.455 Hz
Pulse type	half of sine wave
Pulse duration, T_p	360 μs
Skin-depth@ f=1/(2 T_p)	0.46 cm
Total lens resistance	53 μΩ
Lens current	430 kA
Lens gradient	75 kG/cm
dP/ds at surface	9 W/cm^2
ΔT across lens	5 C^o
Magnetic field pressure, $B^2/8\pi$	230 kg/cm^2
Long. stress in 1.3 mm thick Ti shell	2*9 kg/mm^2

Recent FNAL Lens Picture (Apr. 2009)



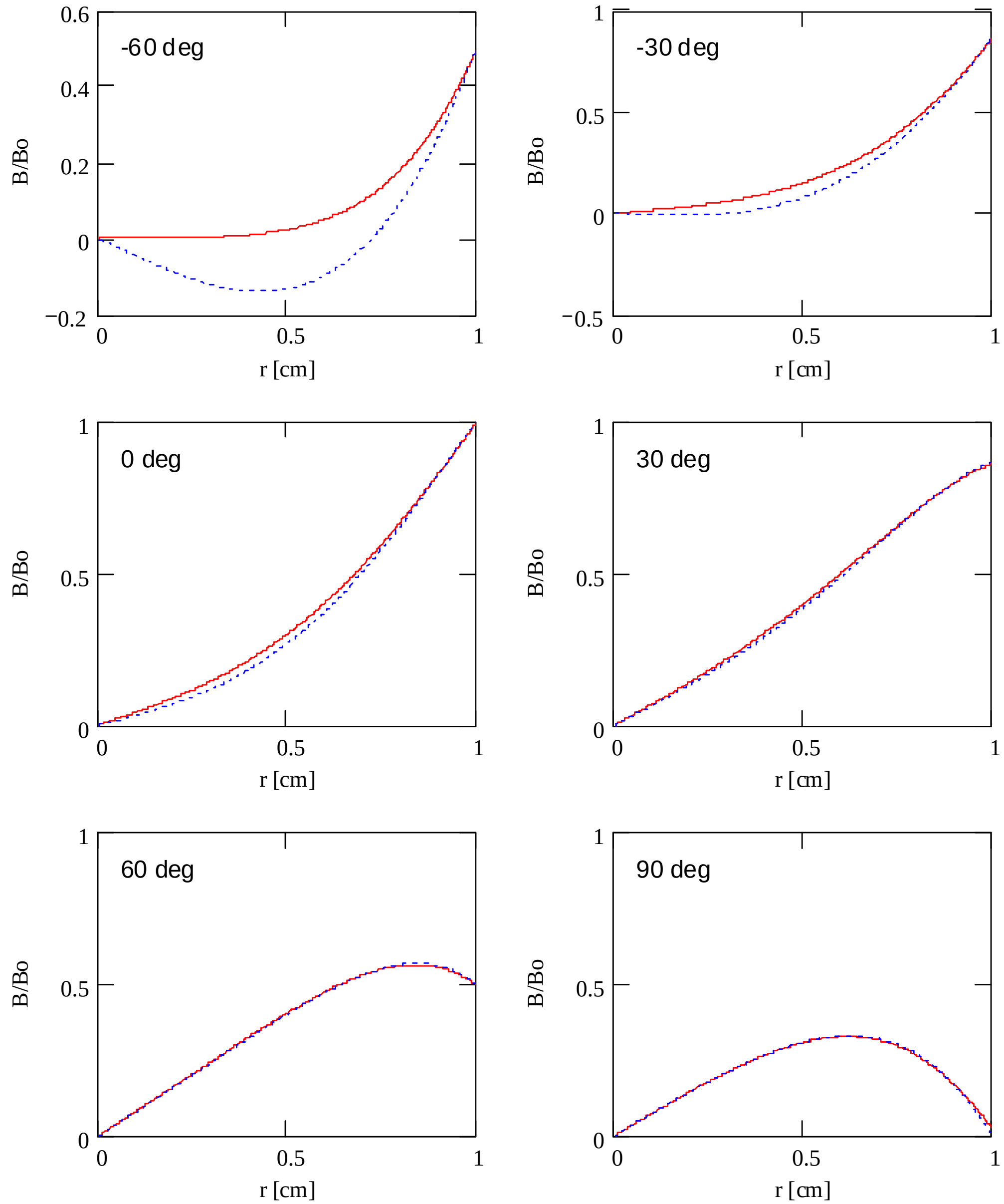
Skin-Effect

- Pulse length should be long enough for field penetration into lithium
- For FNAL lens: $\delta / r_L = 0.46$

$$\delta = \frac{c}{\sqrt{2\pi\sigma\omega}}$$

If current pulse is close to a half sinusoid (FNAL) the field penetration can be approximated well by result obtained for harmonic lens current
(ϕ > 0 for practical cases)

$$B(r,t) = \frac{2I_0}{cr_0}\mathrm{Re}\left(\frac{\mathrm{ber}_1\left(\sqrt{2}\, r/\delta\right) + i\,\mathrm{bei}_1\left(\sqrt{2}\, r/\delta\right)}{\mathrm{ber}_1\left(\sqrt{2}\, r_L/\delta\right) + i\,\mathrm{bei}_1\left(\sqrt{2}\, r_L/\delta\right)} e^{i\omega t}\right)$$



B(r) for different times during half period sinusoidal pulse of 350 μs. Time is expressed through the RF phase so that the pulse end and beginning correspond to ±90 deg. Dotted line represents solution for continuous sinusoidal wave.

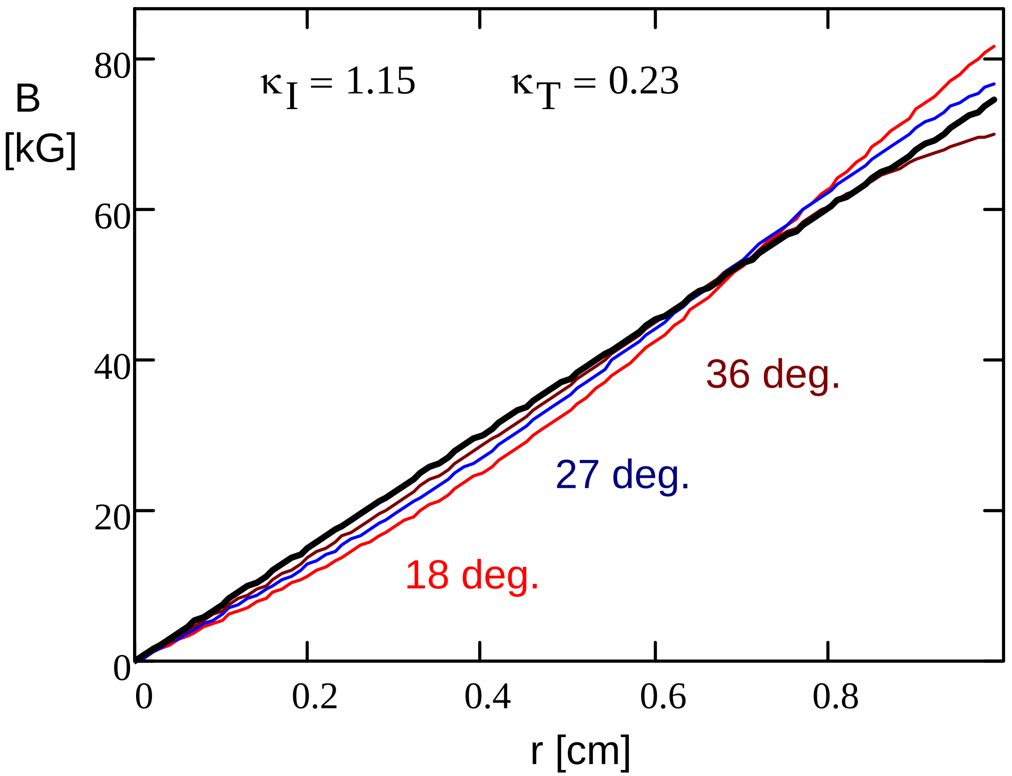
Lens Heating

- Pulse length should be long enough for field penetration
- Combining

$$T_{pulse} = \kappa_T \frac{\pi}{\omega_{cr}} , \qquad r_L = \delta = \frac{c}{\sqrt{2\pi\sigma\omega_{cr}}} ,$$

$$B_0 = \kappa_I \frac{2I_0}{cr_L} , \qquad R = \frac{1}{\sigma} \frac{L}{\pi r_L^2} ,$$

$$P = \frac{RI^2}{2} f_{rep} T_{pulse}$$

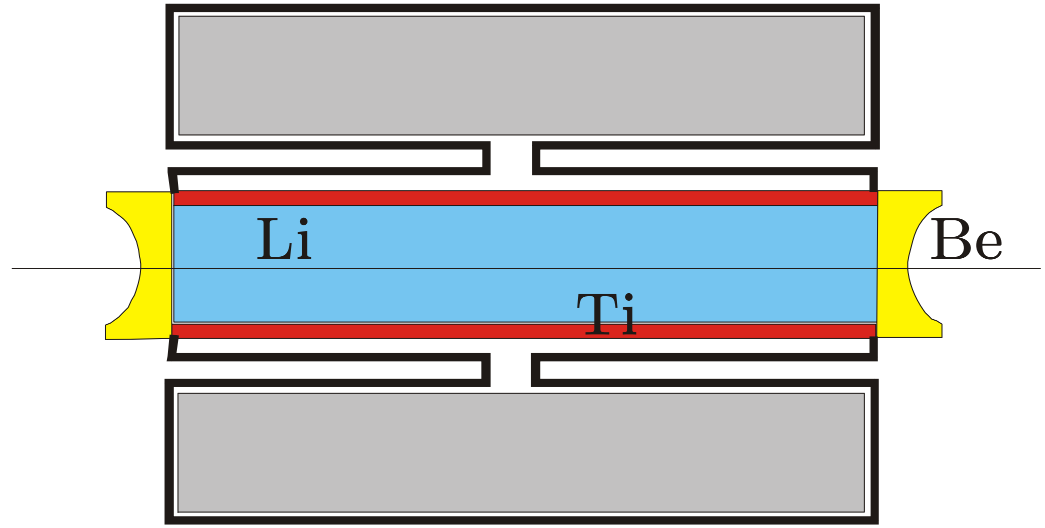


One obtains the power density on the lens surface

$$\frac{dP}{ds} = \frac{\kappa_T \kappa_I^2}{8} B_0^2 f_{rep} r_L$$

- The power density does not depend on material conductivity!!!
  - 15 Hz, 1 cm, 75 kG ⇒ ~300 W/cm^2

Mechanical Stress



- Relationship between magnetic field and mechanical pressure

$$\nabla\left(P+\frac{B^2}{8\pi}\right)=0 \quad \Rightarrow \quad P+\frac{B^2}{8\pi}=P_{preload}+\frac{B_0{}^2}{8\pi}$$

- That results in a radial compressing and an axial force on Be windows

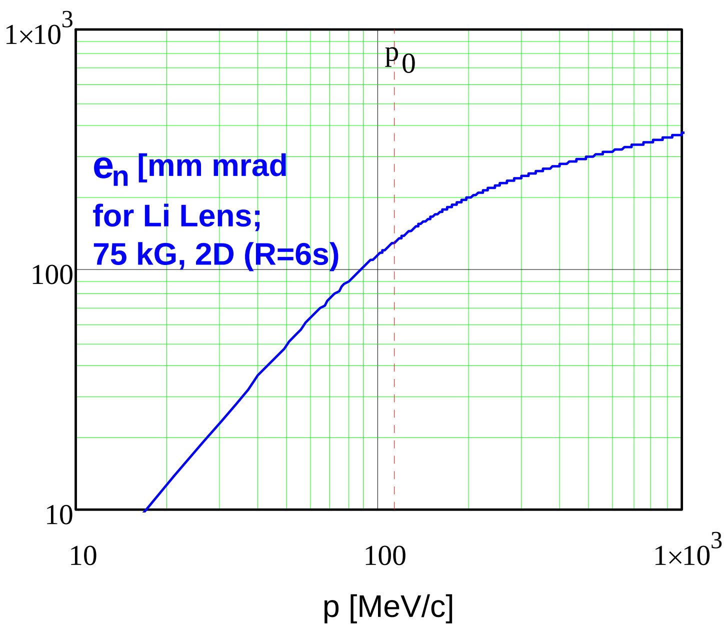
B=0 at window $\Rightarrow$ $$F=\pi r_L{}^2\left(P_{preload}+\frac{B_0{}^2}{8\pi}\right)$$

$\Rightarrow$ Increase of radius results in a thickness increase of windows and Ti shell

- ♦ Li has to be loaded under pressure to avoid a pinch instability
  - It approximately doubles the effect of magnetic pressure and stress on windows

Tentative Beam and Lens Parameters

- Beam energy choice is a compromise between $\varepsilon_{n\perp equilibrium}$ and the $d\varepsilon_L / ds$
- Further energy increase would make the lens easier but introduces too large long. heating



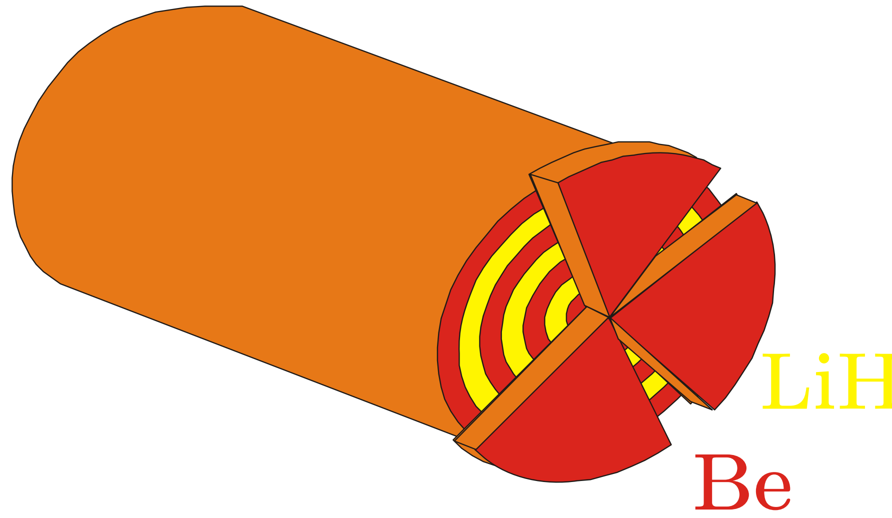
Beam energy	49 MeV
Beam momentum	113 MeV
Longitudinal cooling factor, g_s	-.0.775 !!!
Surface field	75 kG
Equilibrium emittance	130 mm mrad
Energy loss	1.25 MeV/cm
Length	8 cm
Radius, r_L	1 cm
Repetition rate	15 Hz
Pulse type	half of sine wave
Pulse duration, T_p	400 μs
Lens current	430 kA
dP/ds @ on surface	320 W/cm^2 !!!
ΔT across lens	200 C^o !!!
Magnetic field pressure	230 kg/cm^2
Long. stress in 1.3 mm thick Ti shell	2*9 kg/mm^2

Lithium Lens Heating Mitigation

- Solid lithium lens has two problems
  - Temperature gradient across the lens
  - Too large power density at lens boundary to be removed by water cooling
- Both problems can be addressed by liquid lithium lens (Silvestrov, BINP)
  - Required velocity of the lithium ~ 20 cm/s (3.6 l/min)
- Fermilab had a program for the liquid lens development but it was not finished
  - More difficult than expected
    - It can be easier for 75 kG/cm than for 100 kG/cm
  - Liquid lens gradient of 100 kg/cm is not really necessary for Run II
    - Further development of solid lens satisfied our needs
  - Safety issues
- Lens reliability is one of the main problems
  - Present lens lifetime, ~0.5 year, is hardly sufficient when ~10 lenses or more are operating in the cooling channel
  - 30 time increase of the repetition rate does not make it easier

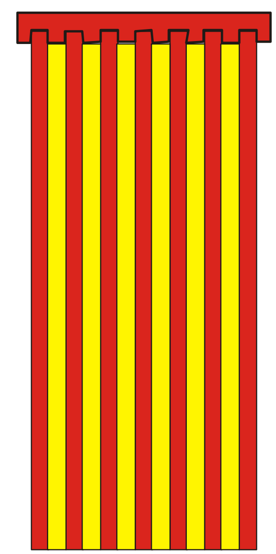
Hybrid Lens

- Splitting lens conductor into separate smaller thickness current layers could be used to shorten pulse length



$$T_{pulse} \propto \sigma d^2$$

- LiH prevents beryllium cylinders from collapse
  - Much smaller force on caps (windows) in comparison with Li lens
- Lens filling: Liquid LiH is filled to take the rest of the space after beryllium construction is assembled



- Major limitations are
  - Pulser making 500 kA in ~50 μs time looks feasible but not easy (~30 kV)
  - Ability to withstand pulsed mechanical stress
    - Requires more insight and actual tests
  - LiH is not good thermo-conductor and should take smaller fraction of the volume

Conclusions

- Cooling with lithium lenses looks feasible
- However there are problems which limit its possible use in the paradigm of present lens design
  - Surface magnetic field is limited to ~75 kG because of mechanical stresses
  - Ohmic lens heating limits the repetition rate to <5 Hz
  - Surface field decrease reduces stresses $\propto B^2$ but power density as $\propto B$
- Liquid lithium lens can address the problem of heat load and can be competitive to schemes with solenoidal focusing
- To apprehend possibilities created by hybrid lens we need better understanding of its mechanical properties and powering scheme
- It is rather improbable that 3D cooling can be created with lithium lenses
  - Required aperture increase results in a reduction of lens gradient and, consequently, increases the equilibrium emittance