Tapered 6D cooling

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Collaboration Workshop

Old Miss 13/1/10

- Introduction & Efficiency Q
- Emittance exchange without dispersion using ICOOL Matrix command
- Pseudo simulation without taper
- Taper design
- Pseudo simulation with taper
- Other taper designs
- Conclusion

An error of mine concerning Fernow's TRANSPORT command has been fixed in plots of emittances or survival of tapered solution, but not in all others, nor in the table on p 16

Transmission and definition of 'Efficiency' Q

If one multiplies the transmissions of all un-tapered simulations, the result is around 1% and quite unacceptable. But many of the losses come from poor initial matching and lack of tapering. To estimate transmission with good matching and tapering we define a cooling efficiency Q

$$Q_6(z) = \frac{d\epsilon_6/\epsilon_6}{dN/N}$$

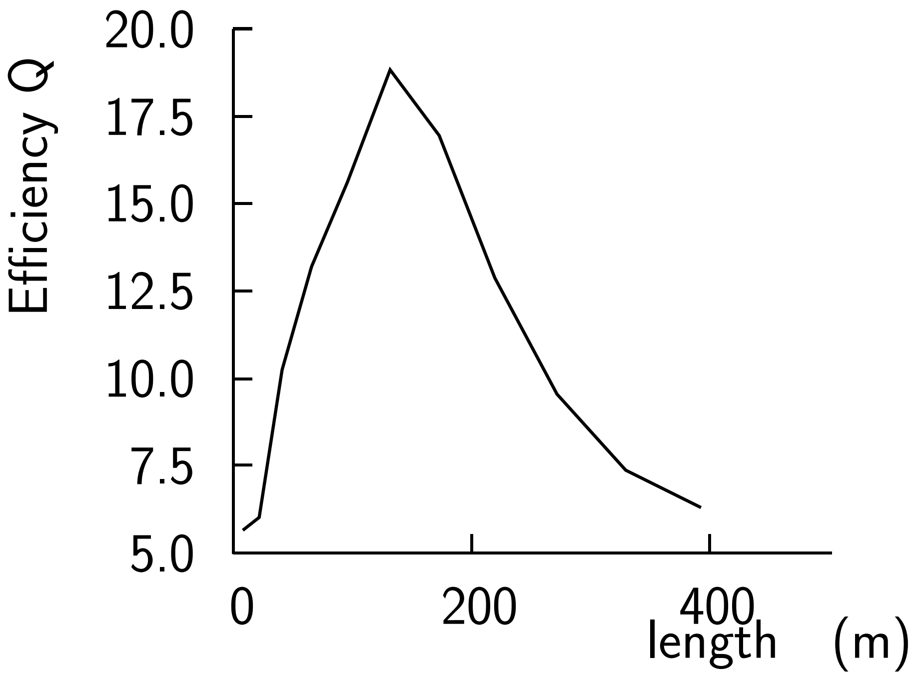
Note, if $Q_6(z)$=constant, then

$$\int_o^n \frac{d\epsilon_6}{\epsilon_6} = Q_6 \int_o^n \frac{dN}{N}$$

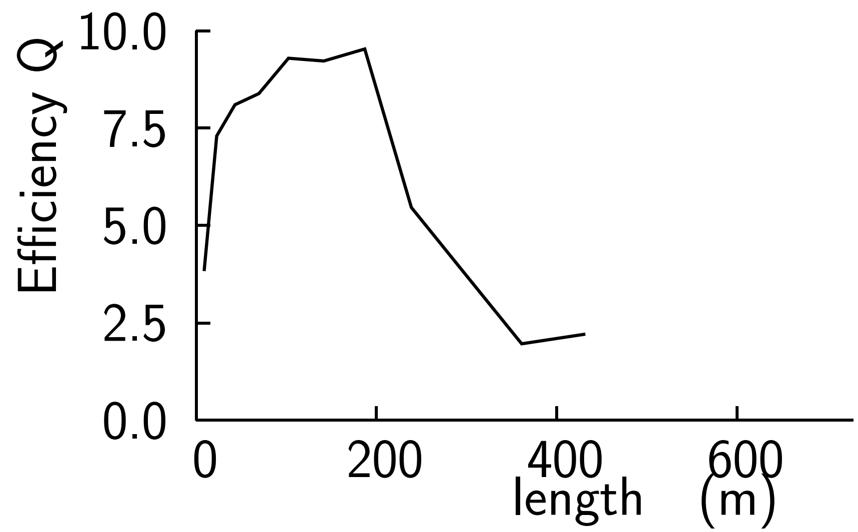
$$\mathrm{Ln}\left(\frac{\epsilon_6(n)}{\epsilon_6(o)}\right) = Q_6 \ \mathrm{Ln}\left(\frac{N(n)}{N(o)}\right)$$

$$\frac{N(n)}{N(o)} = \left(\frac{\epsilon_6(n)}{\epsilon_6(o)}\right)^{1/Q_6}$$

Q vs, length for ICOOL simulations



201 MHz early RFOFO



805 MHz late RFOFO

- Mismatch and Scraping losses at start
- Decay losses as emittances approach equilibrium at end
- Sweet region in between (Q≈ 15 for initial RFOFO, ≈ 8 for late RFOFO)
- If tapered then the entire channel is operated in the sweet region and $< Q >$ greater to or equal to $< Q_{\text{sweet}} >$
- NEED DEMONSTRATION OF THIS HYPOTHESIS

Concept of this study

- Simulate tapered 6D cooling from early through late RFOFO lattices
- Without having to design lattices with bending, dispersion, and wedges
- Design and simulate real, but linear, RFOFO lattices
- Add emittance exchange using matrices in ICOOL's TRANSPORT command

Matrices act on the 6 dimensional vectors: x, x', y, y', σ_z, σ_p/p

Matrix used is

1	1	1	1	1	1
1	$1+\delta$	1	1	1	1
1	1	1	1	1	1
1	1	1	$1+\delta$	1	1
1	1	1	1	1	1
1	1	1	1	1	1 - 2δ

Tapering Scheme

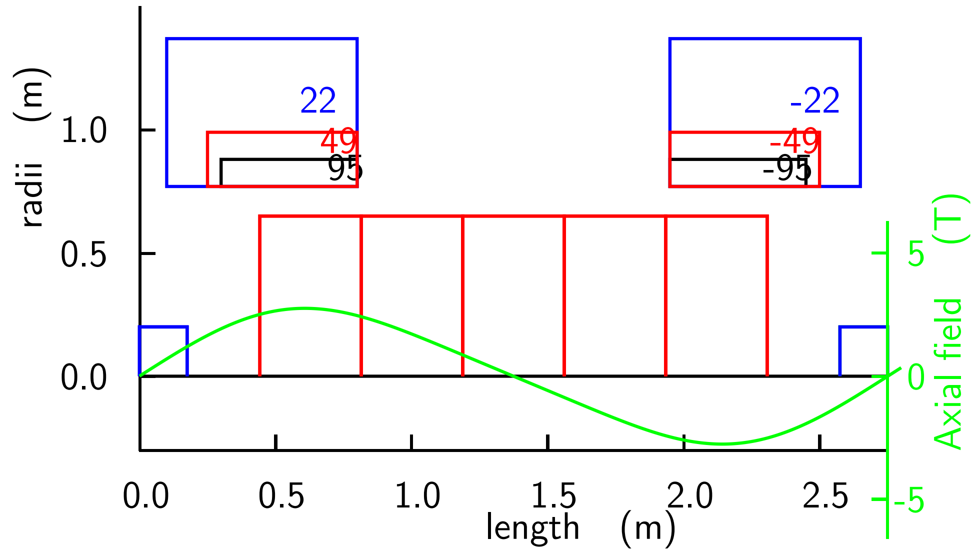
- Use 15 different lattices with betas differing by factors of $4^{(1/9)} \approx 0.857$
- The first 10 stages use old 201 MHz RFOFO lattices with coils outside the rf
- Cell lengths, hydrogen lengths & radii, aluminum window radii & thicknesses, emittance exchange δ, and rf wavelengths $\propto \beta_\perp$
- The relative beta dependences on momentum are kept the same, but coil dimensions are modified to keep current densities reasonable (for same proportions j$\propto 1/L^2$)
  - 4 segments with the original lattice dimensions scaled for the require betas
  - 3 segments with larger coils to reduce the current densities
  - 3 segments with yet larger coils

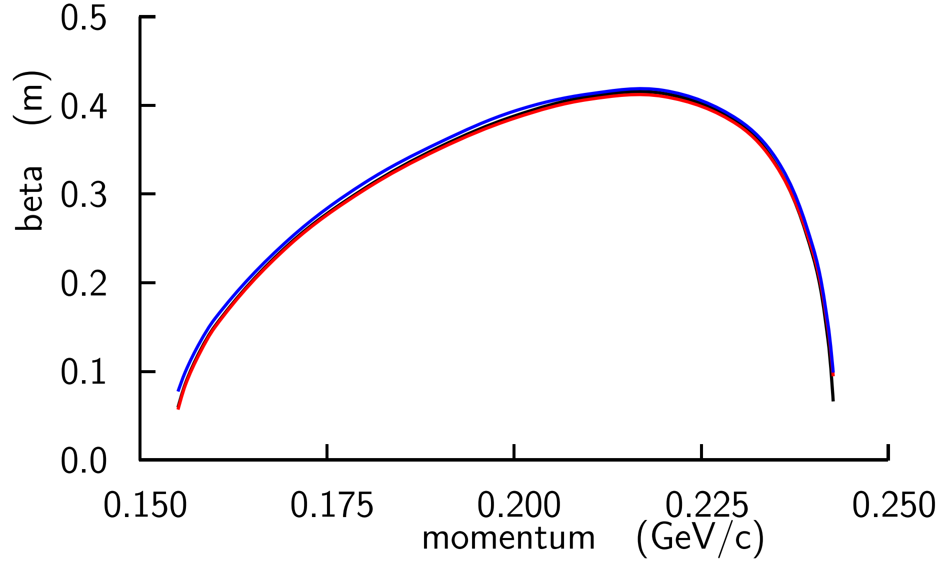
- The 5 following segments have the same cell length (68.75), hydrogen length (10.6 cm), and frequencies of 805 MHz
- With coils designed to give progressively lower betas and momentum acceptances that are also reduced

- For each stage, the number of cells is adjusted to keep the stage lengths approximately equal

Lattice designs for Scaled Cells

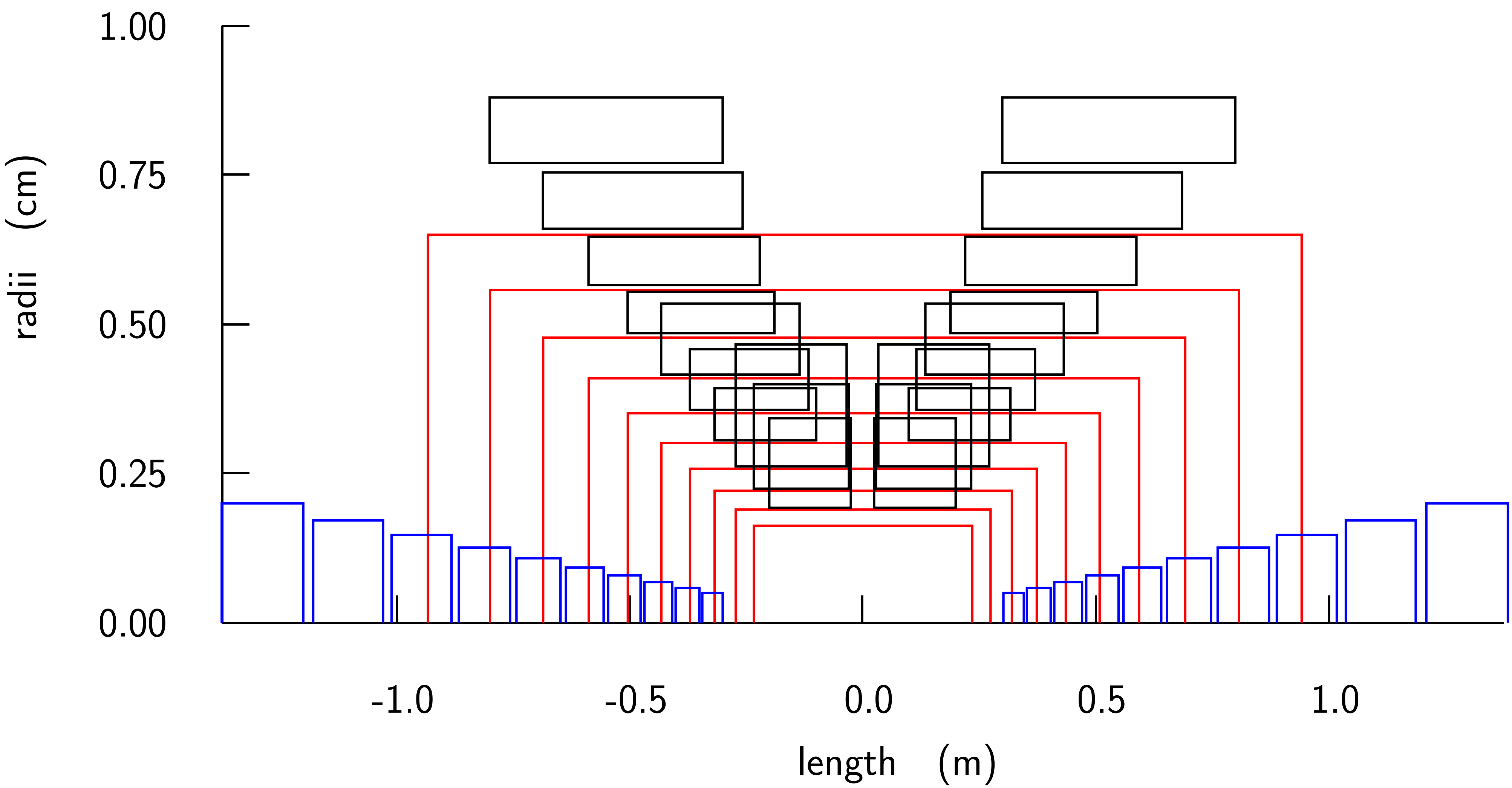
Cell length	2.75	m
Hydrogen length	42.6	cm
Al window thickness	500	μm
Al window radius	18	cm
rf length	1.88	m
rf fraction of len	68.4	%
emittance exchange δ	2.5	%
For first 4 segment		
Coil z	30-80	cm
Coil r	77-88	cm
Current density	95	A/mm^2
For next 3 segment		
Coil z	25-80	cm
Coil r	77-99	cm
Current density	49	A/mm^2
For first 4 segment		
Coil z	10-80	cm
Coil r	77-137	cm
Current density	22	A/mm^2





Numbers of cells in each segment given below (p 11)

Scaled Cells



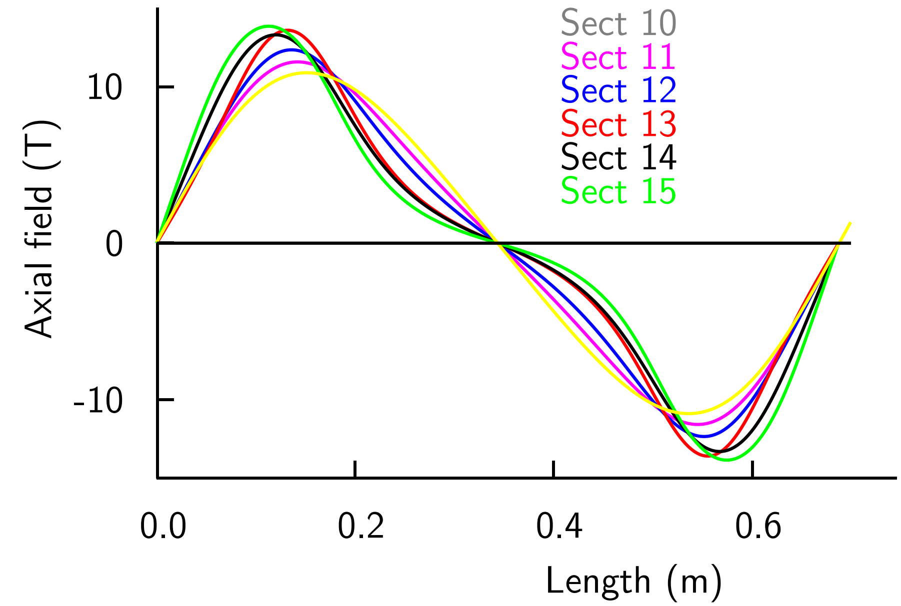
Interfaces between segments are at absorber center where Bz=0 and are not strictly Maxwellian

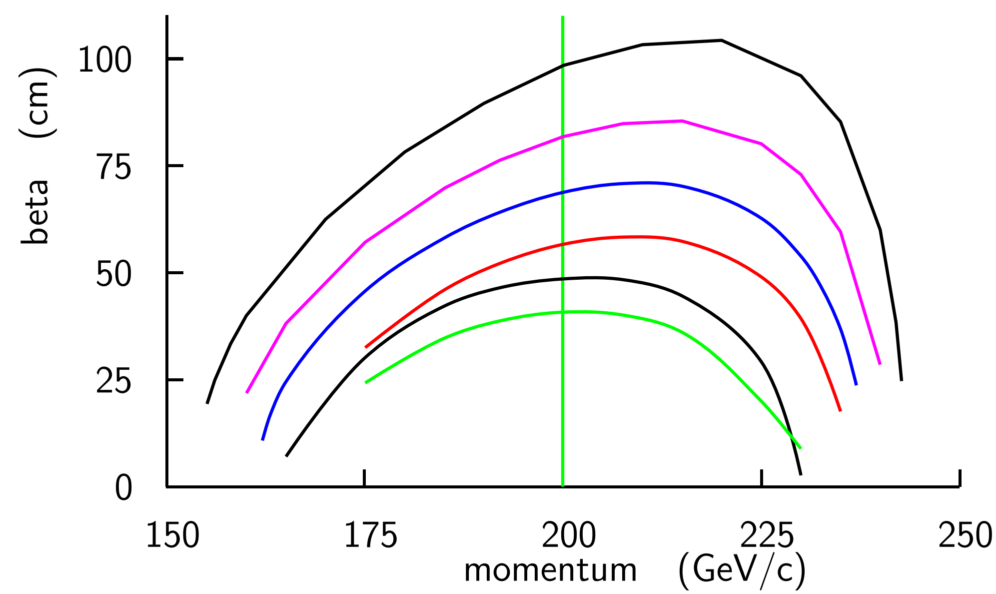
Non-Scaled Cells

- Coils modified to lower betas,
- by moving coils nearer the ends
- But momentum acceptance reduced

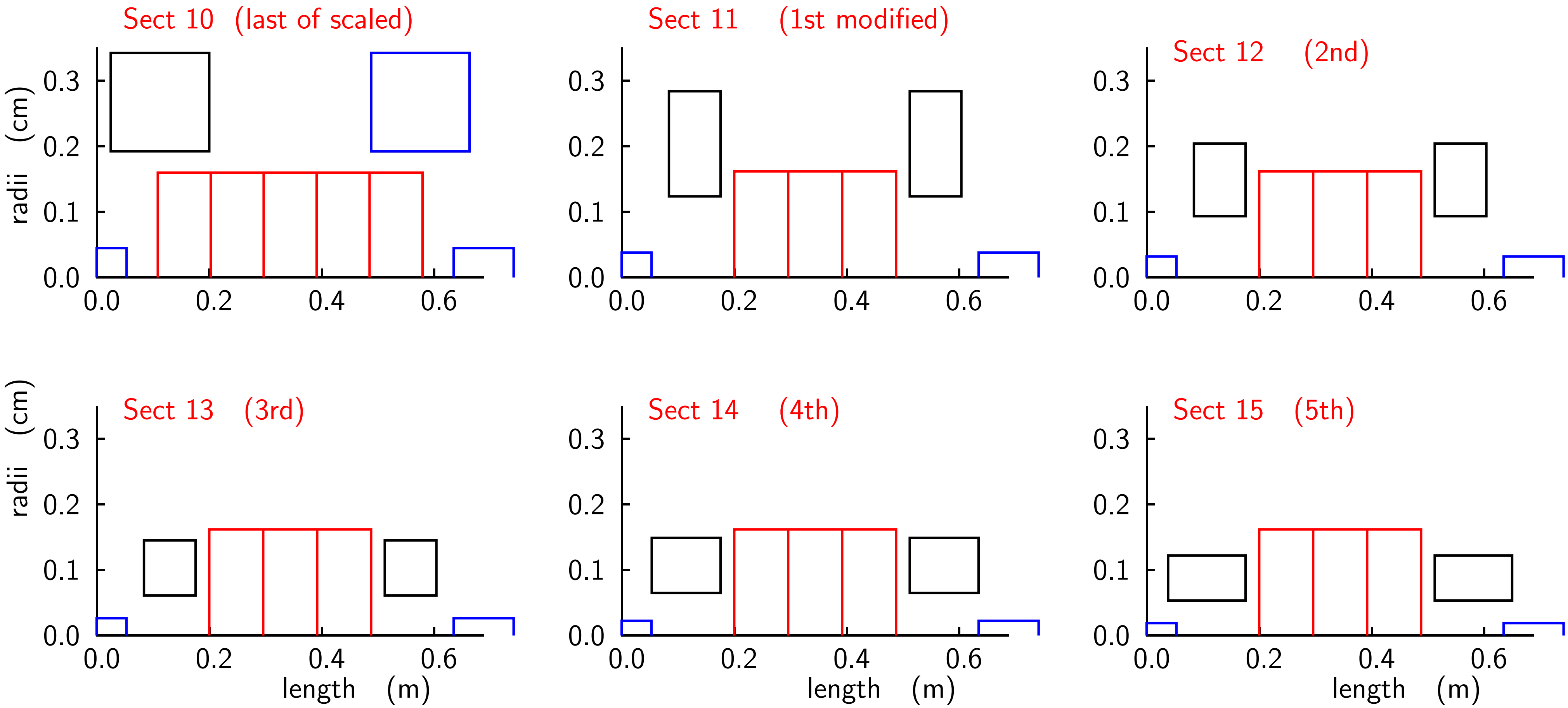
Stage	cells	z1	z2	r1	r2	j
		cm	cm	cm	cm	a/mm^2
11	28	11	23	16.2	37.2	338
12	28	11	23	12.25	26.75	339
13	28	11	23	8	19	327
14	40	7	23	8.5	19.5	274
15	80	5	23	7	16	279

Hydrogen length	5.32	cm
H2 and window rad	4.5	cm
Al window thickness	125	μm
ϵ exchange δ	0.625	%





Non-Scaled Lattices

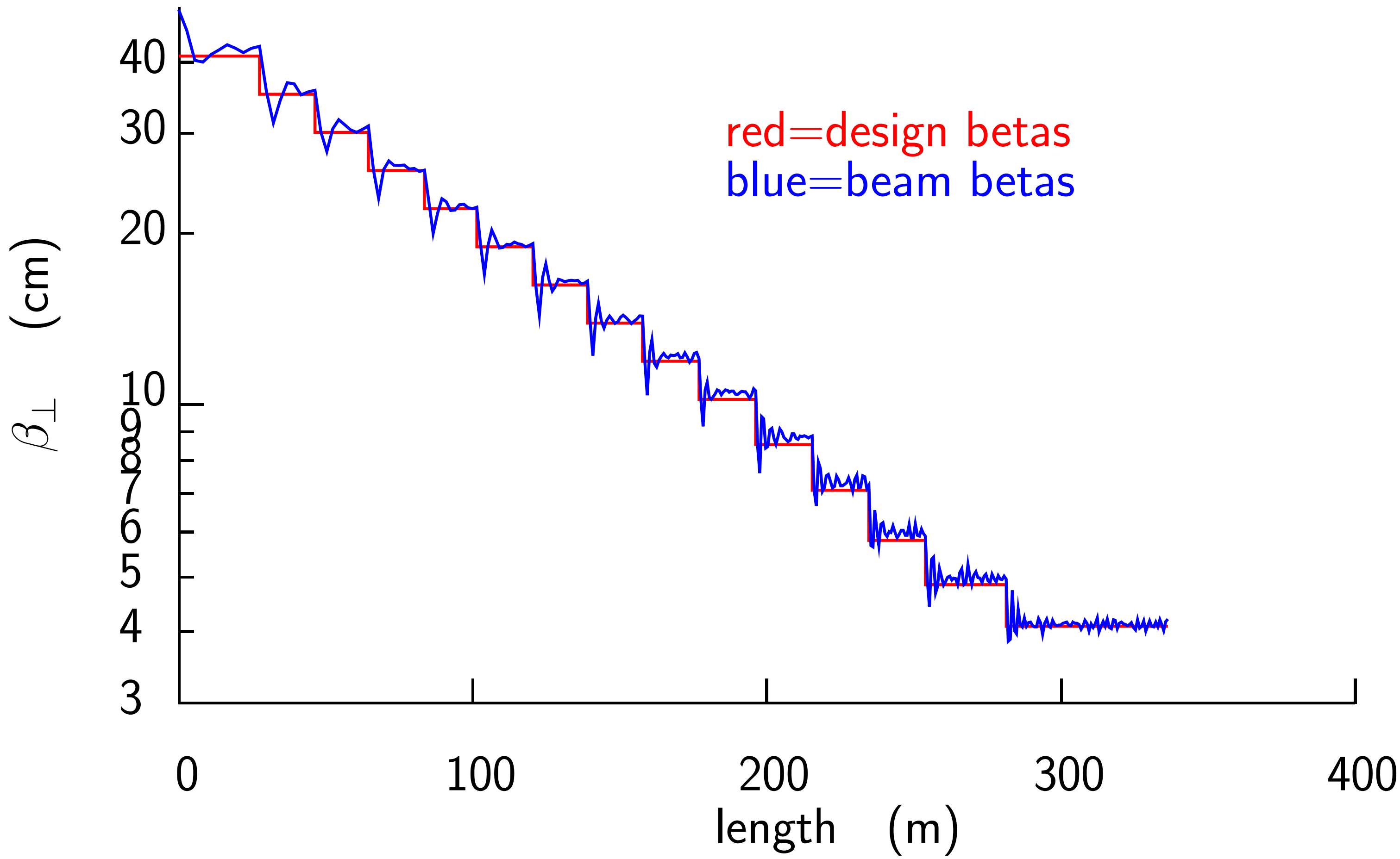


- Lengths of rf are 61% of those in scaled lattices
- Gradient increased from 17.75 to 25.9 MV/m
- Gradients assume that use of Be removes breakdown in field problem

Some parameters for all cells

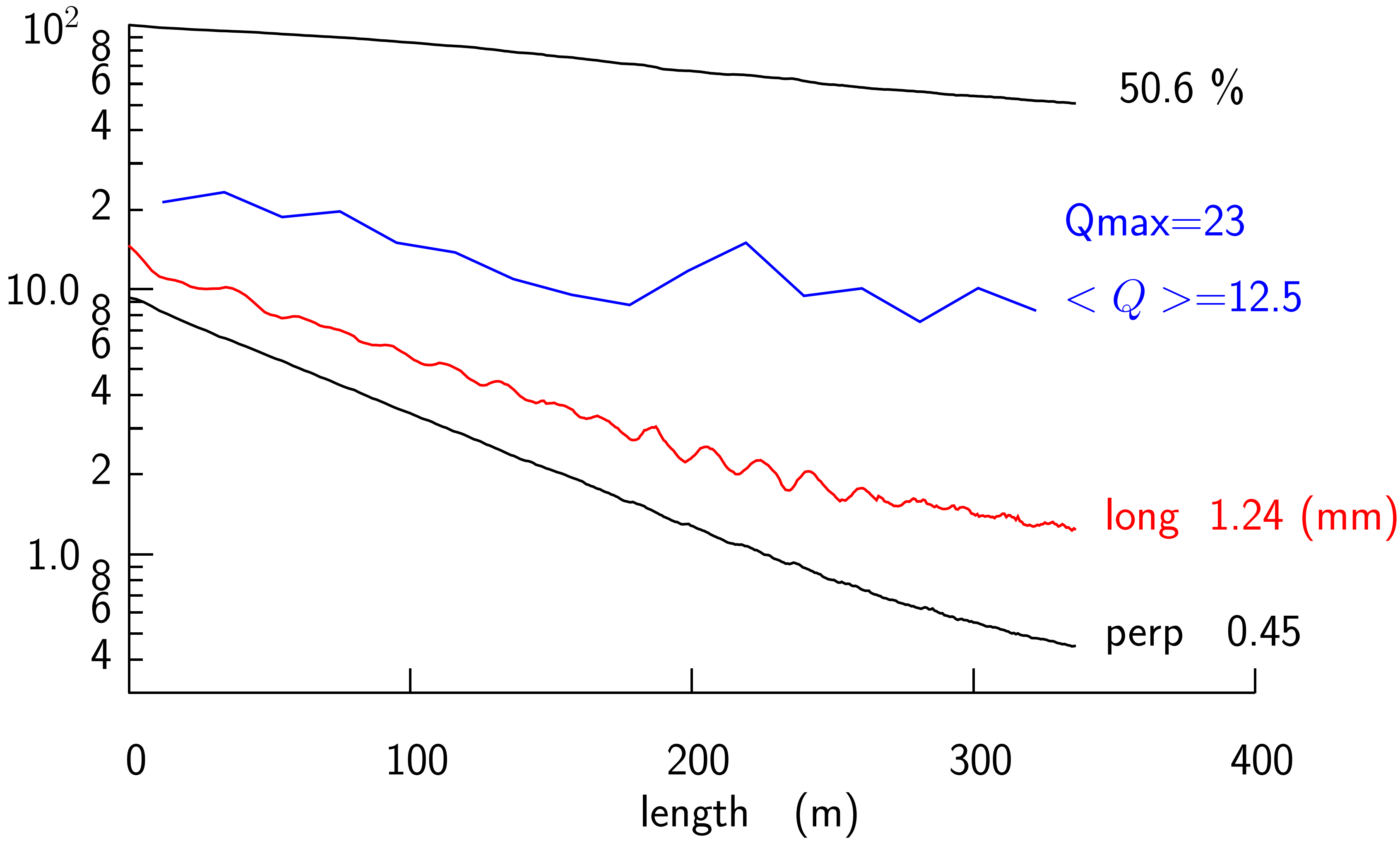
segment	n_{cells}	cell Len m	z_{end} m	$\beta_\perp$ cm	Bmax T	r_{H2} cm	t_{Al} μm	f MHz	grad MV/m	j_{sc} a/mm^2
1	10	2.75	27.5	41.0	2.8	18.0	500	201	15.8	95
2	8	2.36	46.4	35.1	3.2	15.4	429	235	15.8	130
3	9	2.02	64.5	30.1	3.7	13.2	367	274	15.8	177
4	11	1.73	83.6	25.8	4.4	11.3	315	319	15.8	241
5	12	1.49	101.4	22.1	5.1	9.7	270	373	15.8	170
6	15	1.27	120.5	19.0	6.0	8.3	231	435	15.8	231
7	17	1.09	139.1	16.3	6.9	7.1	198	507	15.8	314
8	20	0.94	157.8	13.9	8.1	6.1	170	592	15.8	191
9	24	0.80	177.0	12.0	9.4	5.2	146	690	15.8	261
10	28	0.69	196.3	10.2	11.0	4.5	125	805	15.8	355
11	28	0.69	215.5	8.5	11.6	4.5	125	805	25.9	338
12	28	0.69	234.8	7.1	12.3	4.5	125	805	25.9	339
13	28	0.69	254.0	5.8	13.6	4.5	125	805	25.9	327
14	40	0.69	281.5	4.9	13.3	4.5	125	805	25.9	274
15	80	0.69	336.5	4.1	13.8	4.5	125	805	25.9	279

Simulated beta vs length



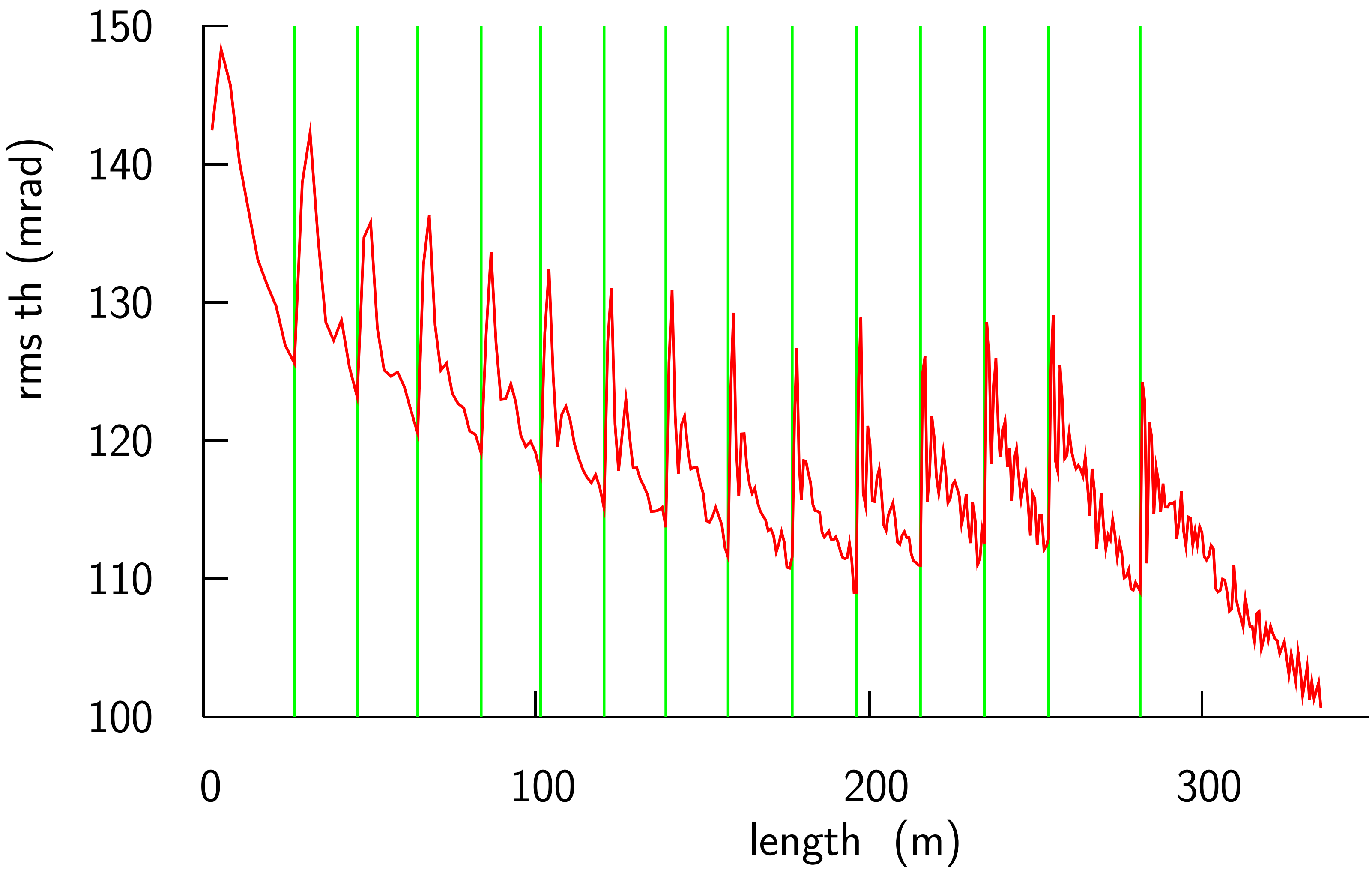
- FS2 simulated input distributions used avoids initial miss-match
- Some beta beat is seen after each segment
- Designed matching could reduce this

Emittances vs length



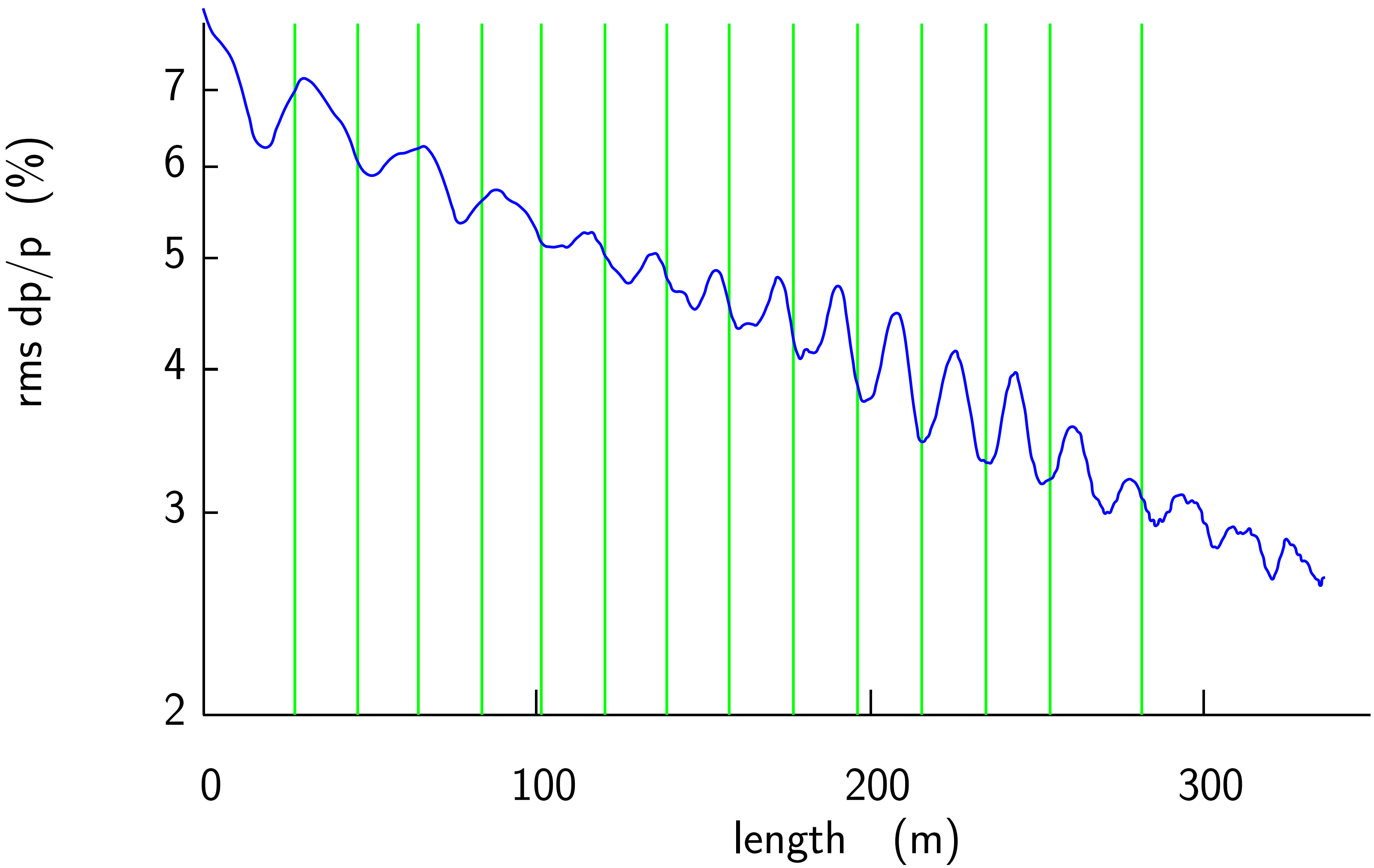
- No transverse emittance growth at matches is observed
- But significant longitudinal effects seen
- Initial Q is better than in un-tapered lattice (23 vs. 15)
- Final Q is better than in un-tapered lattice (12 vs. 8)

rms angles vs length



- Ideal tapering would imply a constant σ_θ
- Achieved in later stages, but reduces performance if forced on earlier stages

dp/p vs length



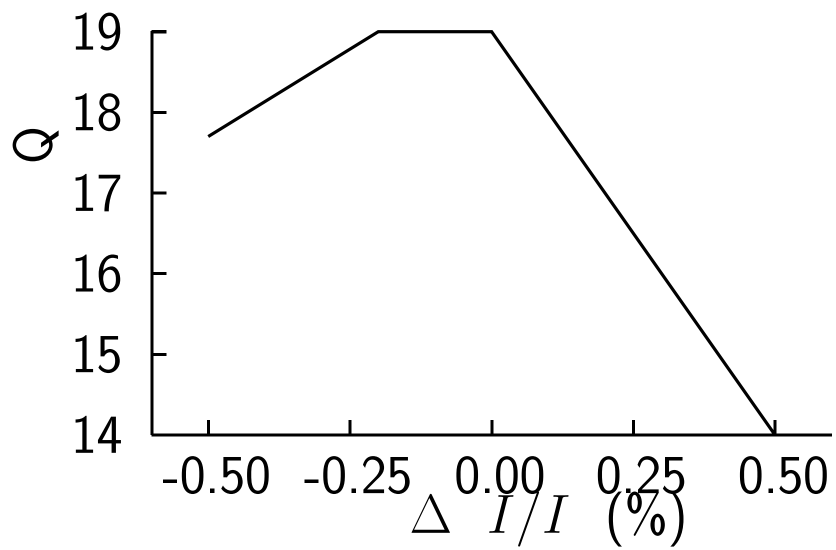
- Miss-matching clearly visible
- Worse than transverse because longitudinal beta similar to segment lengths

Other designs simulated

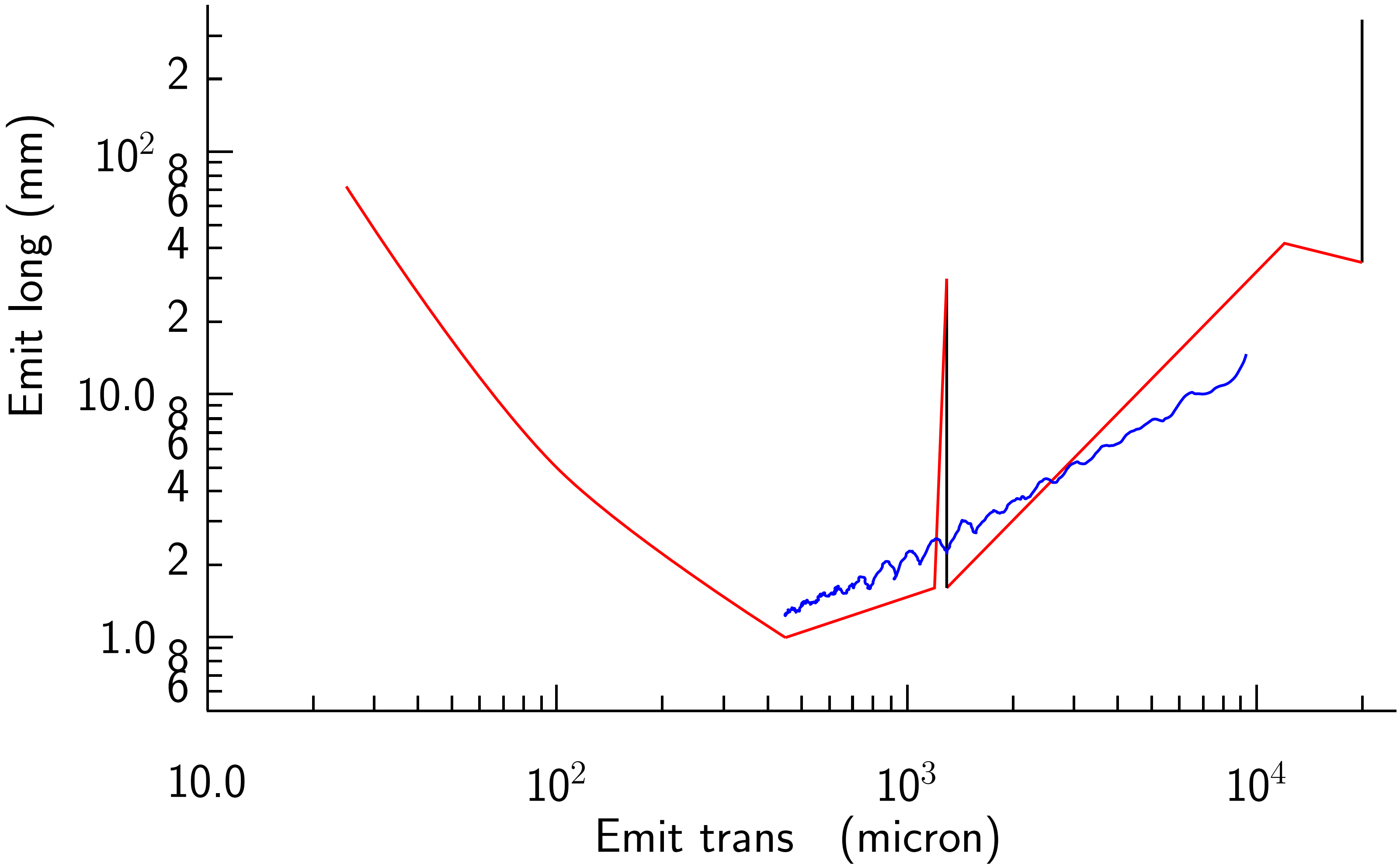
	file	$\epsilon_\perp$ μm	$\epsilon_\parallel$ mm	survival %	$< Q >$
Baseline	zfo4s	.39	.79	61.6	19
$I = 1.05\ I_o$	zfo4p5	.41	.93	52.2	14
$I = 0.95\ I_o$	zfo4m5	.38	.81	59.2	17.7
$I = 0.98\ I_o$	zfo4m2	.40	.81	61.8	19
$\mathcal{E}(805)$=15.8 L_{H2}=6.5 cm	zfo4ss	.46	.72	63	14
No Al windows	zfo4nw	.37	.75	65.9	22.5
Only 3 frequencies* (201 402 805)	zfo4nf	.39	.81	52.8	14

* In this simulation the coils and rf interfere in some cases New coil design would be needed

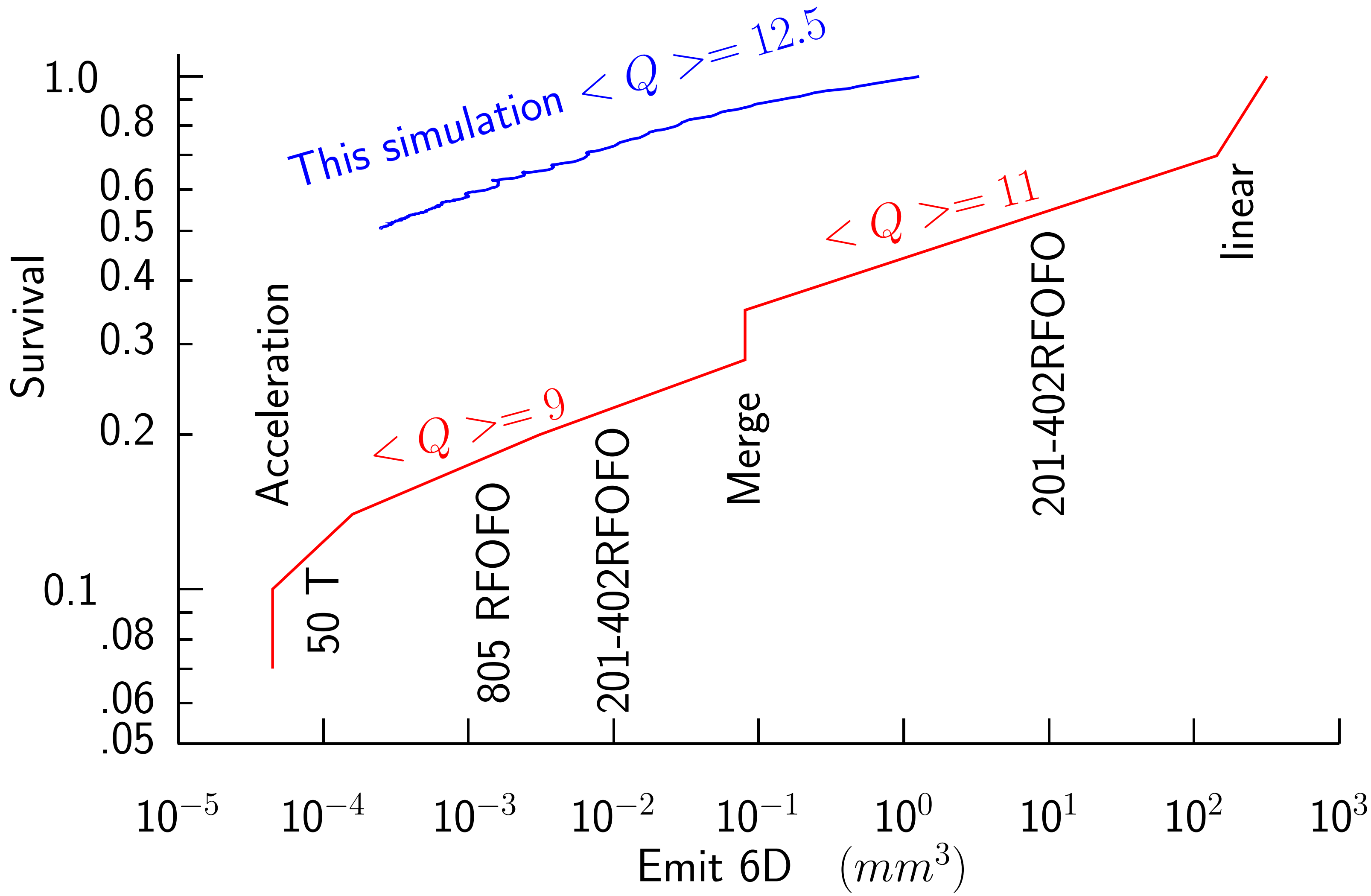
- Currents approximately optimum
- 25% Q loss with lower rf gradients
- 18% Q gain without Al windows
- 26% Q loss with 3 frequencies
- All cases meet min requirements



Long vs. trans emittances



- Final trans emittance ok   Final long emittance close
- It is assumed that a FOFO snake would be used rather than initial 4D cooling



- Slope of loss vs 6D emittance (Q=12.5) is a little better than previous estimates (Q=11 & 9)

Conclusion

- Initial ave Q is better than max Q in un-tapered lattice (23 vs. 15)
- Final ave Q is better than max Q in un-tapered lattice (12 vs. 8)
- Hypothesis seems confirmed
- 17% abrupt beta changes give little emittance dilution
- Use of only 3 frequencies, vs. 9, without matching, reduced Q by only 26% Matching will help
- No evidence of transmission loss from the reduced momentum acceptance of the final lower beta lattices, suggesting that even lower emittances using lower beta lattices, may be possible

Further work

- Design lattices with lower betas (and momentum acceptance) to see when these lower momentum acceptances significantly hurt transmission
- Longitudinally match between 3 frequencies and design coils that do not interfere with rf
- Check magnetic fields on conductors
- Optimize a) strengths of emittance exchange, b) length of absorbers
- Study performance vs. rf gradients

- Include bunch merging in the simulation
- Simulate with real dispersion and wedges using the Fourier representations of fields (this method can underestimate performance, but would give a useful lower performance limit)
- Simulate with real dispersion, wedges, and 6D magnetic fields - a major effort - but only after the above optimizations

Appendix: Super-conductor performance