• Introduction & Efficiency Q
• Emittance exchange without dispersion using ICOOL Matrix command
• Pseudo simulation without taper
• Taper design
• Pseudo simulation with taper
• Other taper designs
• Conclusion

An error of mine concerning Fernow's TRANSPORT command has been fixed in plots of emittances or survival of tapered solution, but not in all others, nor in the table on p 16
Transmission and definition of 'Efficiency' $Q$

If one multiplies the transmissions of all un-tapered simulations, the result is around 1% and quite unacceptable. But many of the losses come from poor initial matching and lack of tapering. To estimate transmission with good matching and tapering we define a cooling efficiency $Q$

$$Q_6(z) = \frac{d\epsilon_6/\epsilon_6}{dN/N}$$

Note, if $Q_6(z)=$constant, then

$$\int_o^n \frac{d\epsilon_6}{\epsilon_6} = Q_6 \int_o^n \frac{dN}{N}$$

$$\ln\left(\frac{\epsilon_6(n)}{\epsilon_6(o)}\right) = Q_6 \ln\left(\frac{N(n)}{N(o)}\right)$$

$$\frac{N(n)}{N(o)} = \left(\frac{\epsilon_6(n)}{\epsilon_6(o)}\right)^{1/Q_6}$$
Q vs, length for ICOOL simulations

201 MHz early RFOFO

- Mismatch and Scraping losses at start
- Decay losses as emittances approach equilibrium at end
- Sweet region in between ($Q \approx 15$ for initial RFOFO, $\approx 8$ for late RFOFO)
- If tapered then the entire channel is operated in the sweet region
  and $<Q>$ greater to or equal to $<Q_{\text{sweet}}>$

805 MHz late RFOFO

- NEED DEMONSTRATION OF THIS HYPOTHESIS
**Concept of this study**

- Simulate tapered 6D cooling from early through late RFOFO lattices
- Without having to design lattices with bending, dispersion, and wedges
- Design and simulate real, but linear, RFOFO lattices
- Add emittance exchange using matrices in ICOOL’s TRANSPORT command

Matrices act on the 6 dimensional vectors: $x, x', y, y', \sigma_z, \sigma_p/p$

Matrix used is

```
1 1 1 1 1 1 1
1 1 + \delta 1 1 1 1 1
1 1 1 1 1 1 1
1 1 1 1 + \delta 1 1 1
1 1 1 1 1 1 1
1 1 1 1 1 1 1 - 2\delta
```
Tapering Scheme

• Use 15 different lattices with betas differing by factors of $4^{(1/9)} \approx 0.857$

• The first 10 stages use old 201 MHz RFOFO lattices with coils outside the rf

• Cell lengths, hydrogen lengths & radii, aluminum window radii & thicknesses, emittance exchange $\delta$, and rf wavelengths $\propto \beta_{\perp}$

• The relative beta dependences on momentum are kept the same, but coil dimensions are modified to keep current densities reasonable (for same proportions $j \propto 1/L^2$)
  – 4 segments with the original lattice dimensions scaled for the require betas
  – 3 segments with larger coils to reduce the current densities
  – 3 segments with yet larger coils

• The 5 following segments have the same cell length (68.75), hydrogen length (10.6 cm), and frequencies of 805 MHz

• With coils designed to give progressively lower betas and momentum acceptances that are also reduced

• For each stage, the number of cells is adjusted to keep the stage lengths approximately equal
Cell length 2.75 m
Hydrogen length 42.6 cm
Al window thickness 500 µm
Al window radius 18 cm
rf length 1.88 m
rf fraction of len 68.4 %
emittance exchange δ 2.5 %

For first 4 segment
Coil z 30-80 cm
Coil r 77-88 cm
Current density 95 A/mm²

For next 3 segment
Coil z 25-80 cm
Coil r 77-99 cm
Current density 49 A/mm²

For first 4 segment
Coil z 10-80 cm
Coil r 77-137 cm
Current density 22 A/mm²

Numbers of cells in each segment given below (p 11)
Interfaces between segments are at absorber center where $B_z=0$ and are not strictly Maxwellian.
Non-Scaled Cells

- Coils modified to lower betas,
- by moving coils nearer the ends
- But momentum acceptance reduced

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<tr>
<th>Stage cells</th>
<th>$z_1$</th>
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<th>$r_1$</th>
<th>$r_2$</th>
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<table>
<thead>
<tr>
<th>Hydrogen length</th>
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<tr>
<td>H2 and window rad</td>
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<tr>
<td>Al window thickness</td>
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<tr>
<td>$\epsilon$ exchange $\delta$</td>
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</table>
• Lengths of rf are 61% of those in scaled lattices
• Gradient increased from 17.75 to 25.9 MV/m
• Gradients assume that use of Be removes breakdown in field problem
### Some parameters for all cells

<table>
<thead>
<tr>
<th>segment</th>
<th>n&lt;sub&gt;cells&lt;/sub&gt;</th>
<th>cell Len</th>
<th>z&lt;sub&gt;end&lt;/sub&gt;</th>
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<th>t&lt;sub&gt;Al&lt;/sub&gt;</th>
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<td>4.5</td>
<td>125</td>
<td>805</td>
<td>25.9</td>
<td>279</td>
</tr>
</tbody>
</table>
Simulated beta vs length

- FS2 simulated input distributions used avoids initial miss-match
- Some beta beat is seen after each segment
- Designed matching could reduce this
- No transverse emittance growth at matches is observed
- But significant longitudinal effects seen
- Initial $Q$ is better than in un-tapered lattice (23 vs. 15)
- Final $Q$ is better than in un-tapered lattice (12 vs. 8)
rms angles vs length

- Ideal tapering would imply a constant $\sigma_\theta$
- Achieved in later stages, but reduces performance if forced on earlier stages
• Miss-matching clearly visible
• Worse than transverse because longitudinal beta similar to segment lengths
Other designs simulated

<table>
<thead>
<tr>
<th>Baseline</th>
<th>file</th>
<th>$\epsilon_\perp$</th>
<th>$\epsilon_\parallel$</th>
<th>survival</th>
<th>$&lt; Q &gt;$</th>
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<td>.79</td>
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<td>.81</td>
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<tr>
<td>$\mathcal{E}(805)=15.8$</td>
<td>zfo4m2</td>
<td>.40</td>
<td>.81</td>
<td>61.8</td>
<td>19</td>
</tr>
</tbody>
</table>

| $L_{H2}=6.5$ cm   | zfo4ss    | .46               | .72                    | 63       | 14      |

| No Al windows     | zfo4nw    | .37               | .75                    | 65.9     | 22.5    |

| Only 3 frequencies* (201 402 805) | zfo4nf   | .39               | .81                    | 52.8     | 14      |

* In this simulation the coils and rf interfere in some cases New coil design would be needed

- Currents approximately optimum
- 25% Q loss with lower rf gradients
- 18% Q gain without Al windows
- 26% Q loss with 3 frequencies
- All cases meet min requirements

![Graph showing Q vs. \( \Delta I/I \)](image)
Long vs. trans emittances

- Final trans emittance ok
- Final long emittance close
- It is assumed that a FOFO snake would be used rather than initial 4D cooling
Slope of loss vs 6D emittance ($Q=12.5$) is a little better than previous estimates ($Q=11$ & 9)
Conclusion

• Initial ave Q is better than max Q in un-tapered lattice (23 vs. 15)
• Final ave Q is better than max Q in un-tapered lattice (12 vs. 8)
• Hypothesis seems confirmed
• 17% abrupt beta changes give little emittance dilution

• Use of only 3 frequencies, vs. 9, without matching, reduced Q by only 26% Matching will help
• No evidence of transmission loss from the reduced momentum acceptance of the final lower beta lattices, suggesting that even lower emittances using lower beta lattices, may be possible
Further work

- Design lattices with lower betas (and momentum acceptance) to see when these lower momentum acceptances significantly hurt transmission
- Longitudinally match between 3 frequencies and design coils that do not interfere with rf
- Check magnetic fields on conductors
- Optimize a) strengths of emittance exchange, b) length of absorbers
- Study performance vs. rf gradients

- Include bunch merging in the simulation
- Simulate with real dispersion and wedges using the Fourier representations of fields (this method can underestimate performance, but would give a useful lower performance limit)
- Simulate with real dispersion, wedges, and 6D magnetic fields - a major effort - but only after the above optimizations
Appendix: Super-conductor performance

![Graph showing superconductor performance](image)

- **YBCO B⊥ Tape Plane**
- **YBCO B∥ Tape Plane**
- **Nb-Ti**
- **RRP Nb₃Sn**
- **MgB₂**
- **Bronze Nb₃Sn**

**Legend:**
- YBCO Insert Tape (B∥ Tape Plane)
- YBCO Insert Tape (B⊥ Tape Plane)
- MgB₂ 19Fil 24% Fill (HyperTech)
- 2212 OI-ST 28% Ceramic Filaments
- NbTi LHC Production 38%SC (4.2 K)
- Nb₃Sn RRP Internal Sn (O1-ST)
- Nb₃Sn High Sn Bronze Cu:Non-Cu 0.3

Maximal $J_E$ for entire LHC Nb-Ti strand production

Compiled from A2C'02 and ICIncUS papers (C. Parcell O1-ST)

427 filament strand with Ag alloy outer sheath tested at NHMFL

10+1 MgB₂/Nb/Cu/MoNi

10+1 MgB₂/Nb/Cu/MoNi (courtesy M. Tomsic, 2007)

4347 filament High Sn Bronze-16wt. %Sn-0.3wt.%Ti (M. Tomsic - MT18-IEEE'04)