

TeV muon acceleration - update

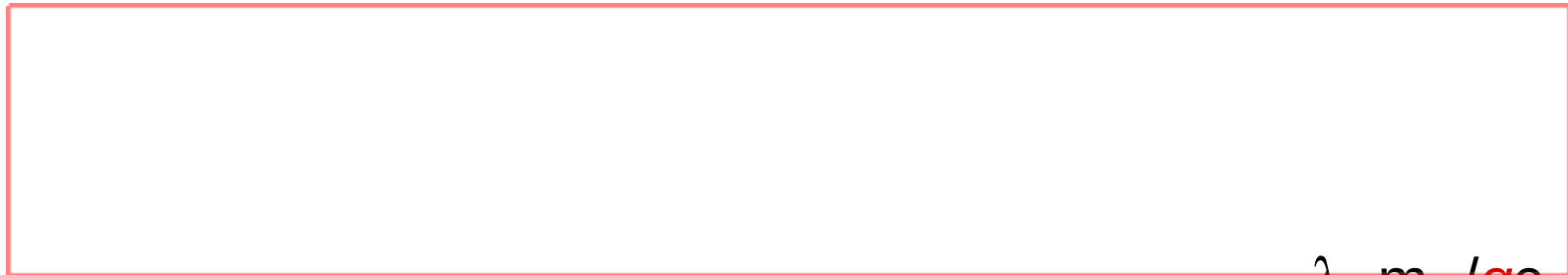
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Getting μ 's from 30 GeV to 2 TeV

before they all decay away...

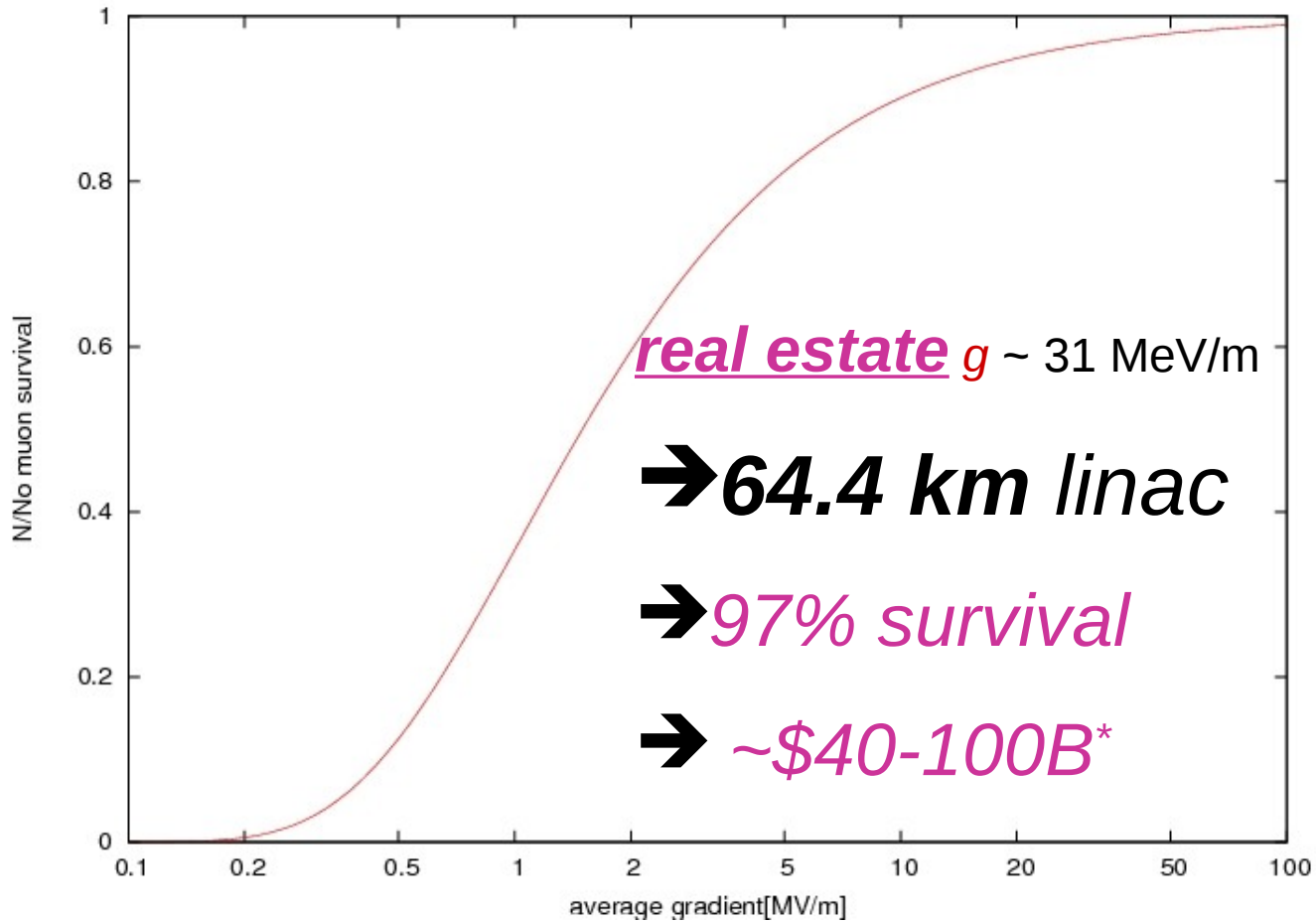
The very big picture.... $dt_{\mu} = dt_L/\gamma$ (time dilation)

$$\Delta t_{\mu} = \int 1/\gamma dt_L \approx \int m_0/(E_i + g x) dx/c = m_0/gc \ln(E_f/E_i)$$



Simple straight linac...

μ



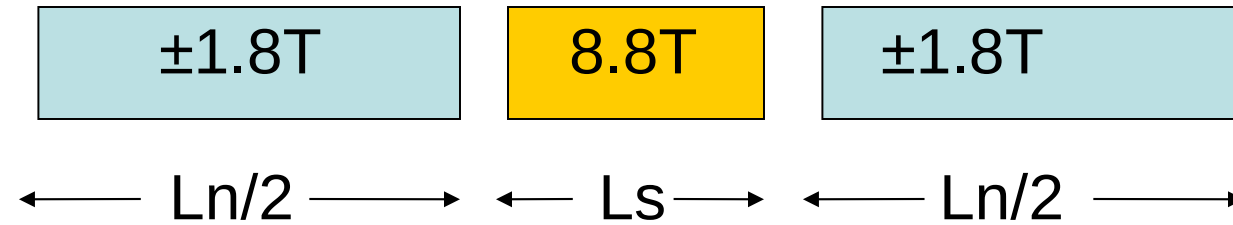
g



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Hybrid magnets...

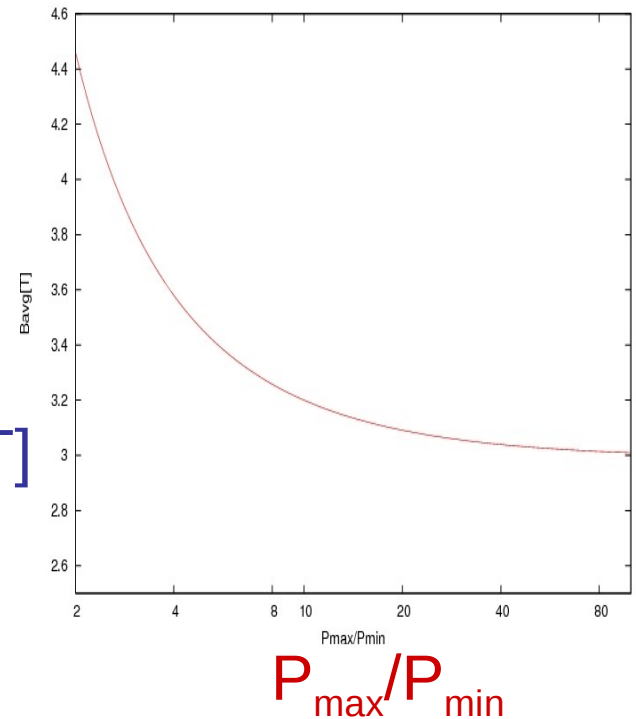


$$P_{\max}/P_{\min} = B_{\max}/B_{\min} = \frac{(B_s \cdot L_s + B_n \cdot L_n)}{(B_s \cdot L_s - B_n \cdot L_n)}$$

$$x \equiv (P_{\max}/P_{\min} - 1)/(P_{\max}/P_{\min} + 1)$$

$$B_{\text{avg}} = f(x+1)/(x/B_n + 1/B_s)$$

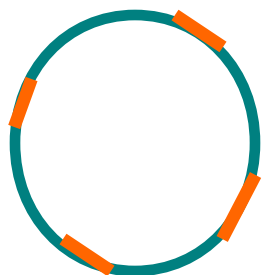
$B_{\text{avg}}[\text{T}]$



$$P_{\max}/P_{\min} \rightarrow \infty, \quad \underline{B_{\text{avg}} \rightarrow 3.0\text{T}}$$



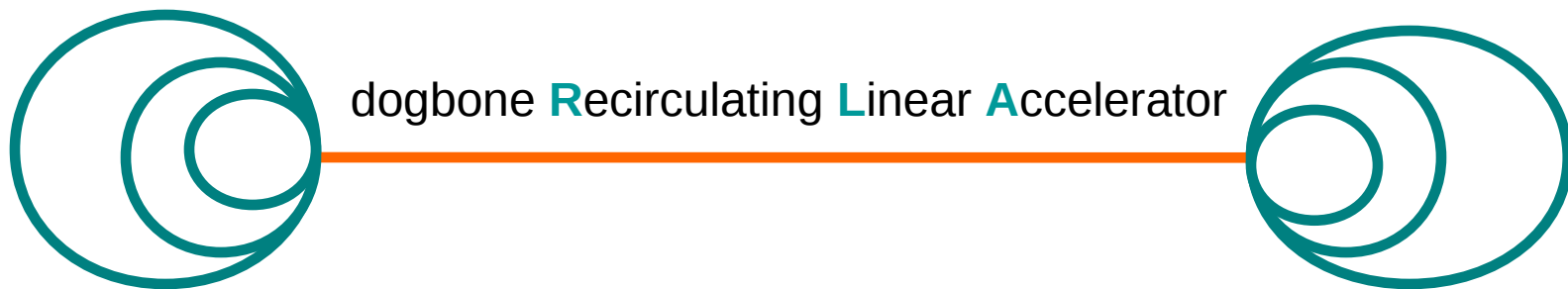
Reuse RF ... lots of schemes...



Rapid **C**ycling **S**ynchrotron



racetrack recirculating linac



dogbone **R**ecirculating **L**inear **A**ccelerator



How do we decide?

- N_{μ}/N_{μ_0} survival -
- Cost --
- Size -- fit at FNAL?
- Complexity -- switchyards
- ...

RLAs as polygons...

N - # of sides

L_T - total length of linacs

L - length of linac on one side $L = L_T / N$

n - # of momentum ranges

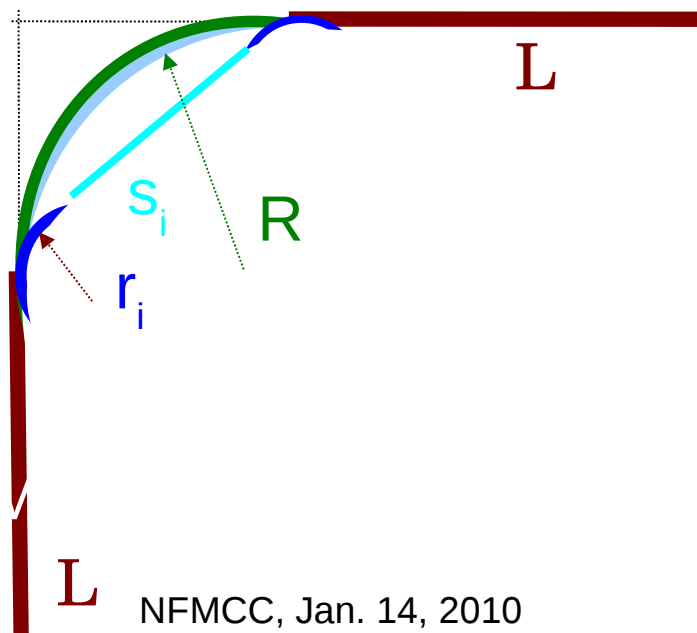
R - largest arc radius

r_i - arc radius for range #i $r_i = R (P_f/P_o)^{i-n}$

s_i - drift for one range $s_i = 2 (R - r_i) \sin(\pi/N)$

P_f = final momentum

P_o = initial momentum



RLAs as polygons...

$$C_i = \text{path for range\#i} \quad C_i = L_{\text{tot}} + 2\pi r_i + N s_i$$

g_L = real estate gradient in linac

$g_{\text{avg.i}}$ = average real estate gradient over path#i

$$g_{\text{avg.i}} = g_L \frac{L_T}{(L_T + 2\pi r_i + N s_i)} \quad 1/n$$

k_i = ratio for range#i $k_i \equiv (P_{\text{out.i}}/P_{\text{in.i}}) = (P_f/P_o)$

$$N_{\mu} / N_{\mu_0} = \prod_{i=1}^n (k_i)^{-\lambda m_o / g_{\text{avg.i}} c}$$

RLAs as polygons...

$$k \equiv k_i \equiv (P_{\text{out},i}/P_{\text{in},i}) = (P_f/P_o)^{1/n}$$

*Make all
steps the
same ratio*

$$N_\mu/N_{\mu_0} = \prod_{i=1}^n (k) \quad -\lambda m_o / g_{\text{avg},i} c$$

$$N_\mu/N_{\mu_0} = \left(P_f/P_o \right)^{\sum_i^n 1/g_{\text{avg},i}} \quad -\lambda m_o / c / n$$

RLAs as polygons...

$$N_{\mu} / N_{\mu_0} = \left(\frac{P_f}{P_o} \right)^{\lambda m_o / c / g_L} \left(\frac{\sum_i^n C_i / L_T}{n} \right)$$

straight linac

$$\begin{aligned} C_i / L_T &= (L_T + 2\pi r_i + 2(R - r_i) N \sin(\pi/N)) / L_T \\ &= 1 + 2\pi r_i / L_T + 2(R - r_i) / L_T N \sin(\pi/N) \end{aligned}$$

$$r_i = R (P_i / P_f) = R (k_i P_o) / P_f = R k^i / k^n = R k^{i-n}$$

$$C_i / L_T = 1 + 2\pi (R / L_T) k^{i-n} + 2(1 - k^{i-n}) (R / L_T) N \sin(\pi/N)$$

RLAs as polygons...

$$\begin{aligned} \mathbf{K} &\equiv \sum_{i=1}^n \mathbf{k}^{i-n} = \mathbf{k}^{-n} \sum_{i=1}^n \mathbf{k}^i = \mathbf{k}^{-n}(\mathbf{k}-\mathbf{k}^{n+1})/(\mathbf{k}-1) \\ &= (\mathbf{k}^{1-n}-\mathbf{k})/(\mathbf{k}-1) \end{aligned}$$

$$(\sum_1^n C_i / L_T) / n =$$

$$1 + 2\pi(R/L_T) \mathbf{K}/n + 2(1-\mathbf{K}/n)(R/L_T) \mathbf{N} \sin(\pi/\mathbf{N})$$



RLAs as polygons...

$$\mathbf{K} \equiv \sum_i^n k^{i-n} = (k^{1-n} - k)/(1 - k)$$

$$C_i / L_T = 1 + 2\pi(R/L_T)k^{i-n} + 2(1 - k^{i-n})(R/L_T) N \sin(\pi/N)$$

$$\sum_1^n C_i / L_T / n$$

$$= 1 + 2\pi(R/L_T)\mathbf{K}/n + 2(1 - \mathbf{K}/n)(R/L_T) \mathbf{N} \sin(\pi/\mathbf{N})$$

$$= 1 + \underbrace{2(\pi - \mathbf{N} \sin(\pi/\mathbf{N}))}_{\text{straights}}(R/L_T) \mathbf{K}/n + \underbrace{2(R/L_T) \mathbf{N} \sin(\pi/\mathbf{N})}_{\text{arcs}}$$

$$1 + 2(\pi - N \sin(\pi/N))(R/L_T) \mathbf{K/n} + 2(R/L_T) N \sin(\pi/N)$$

$$N/N_0 = \left[\begin{matrix} (P_f/P_0) \\ -\lambda m_0/c/g_L \end{matrix} \right]$$

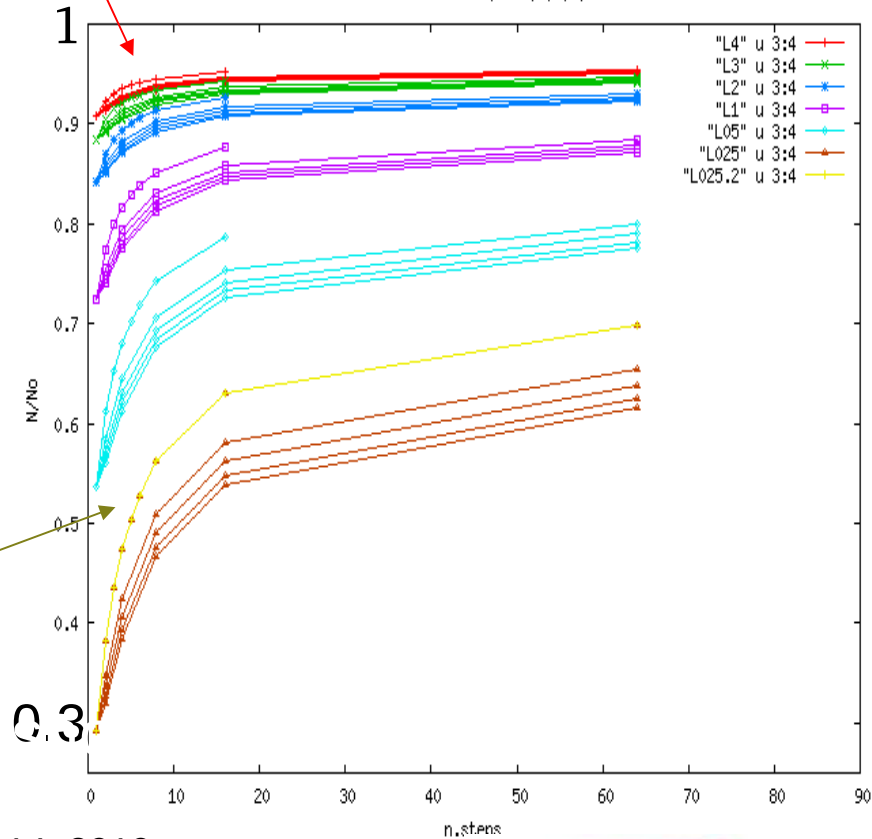
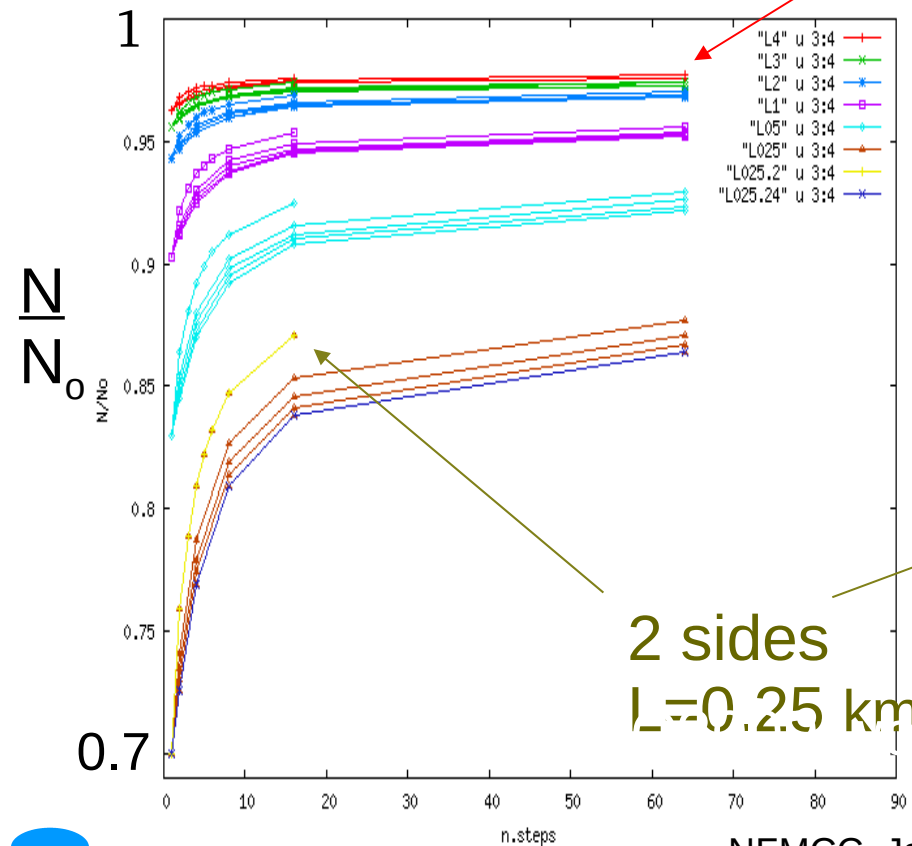
750GeV

30-750GeV 0.5-4km linac, N=2,3,4,6,24 sides

L=4 km

2000GeV

30-2000GeV 0.5-4km linac, N=2,3,4,6,24 sides

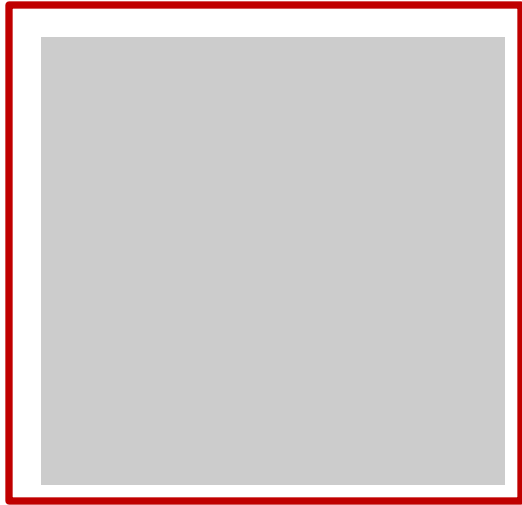


NFMCC, Jan. 14, 2010



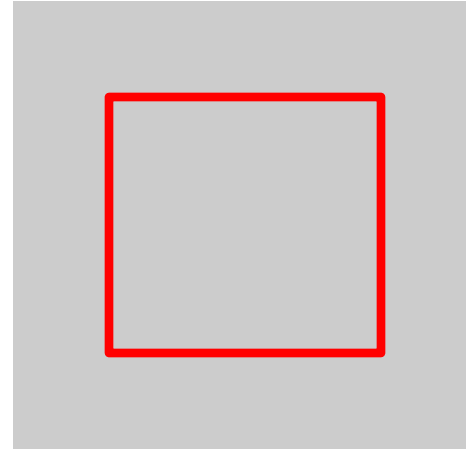
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$N=4, L_T=4, n=1$

~FNAL



$N=4, L_T=4, n=16$



Costs? - linac

- ILC \sim 500 GeV, 31.5 MV/m, 16km \sim \$20B
 \sim \$40M/GeV
- 12 GeV CEBAF \sim \$50M/GeV, \sim 10MV/m
- Study IIb RLA \sim \$40M/GeV \sim 5.3 MV/m



Costs? - arc

~ stored energy ~ aperture * length * B_{\max}

- (nc) FFAG ~ 2.369m 3.508 T ~ \$150K ~ \$63k/m
- Study IIb FFAG ~ \$28k/m
- D.Summers (this week) ~ \$14k/m

Costs – estimate for next slide

Linac - ~ \$50M/GeV

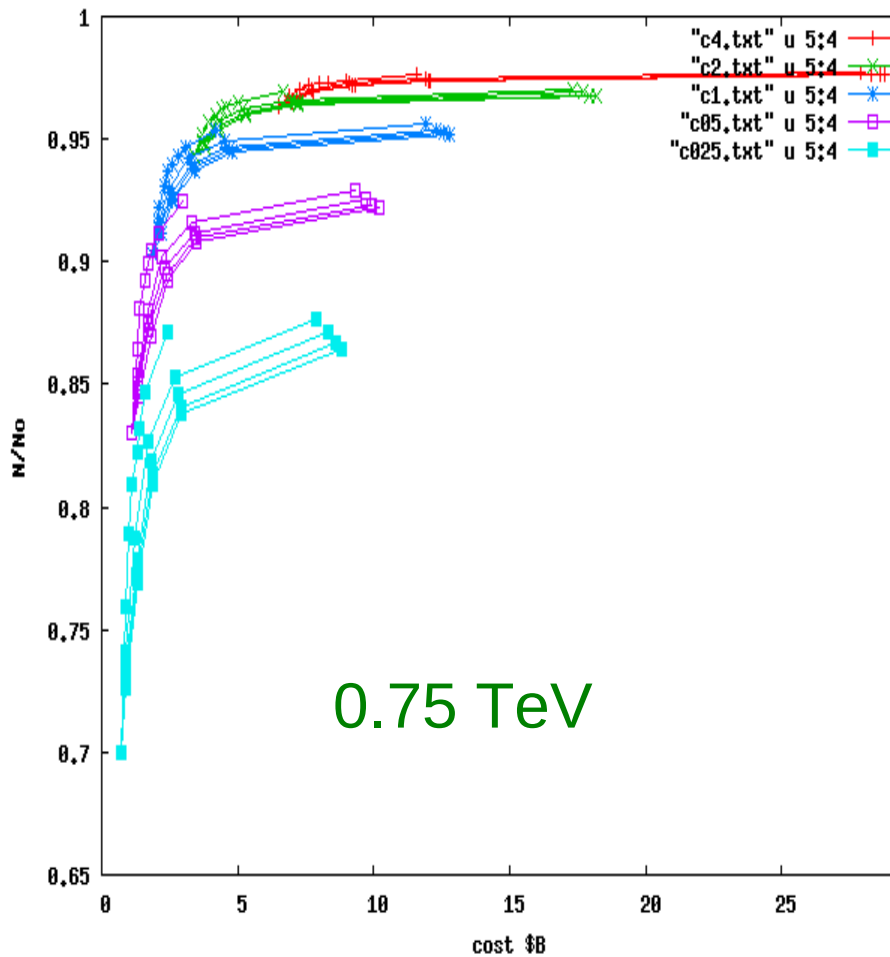
Arc ~ \$60K/m

Straight ~ \$6K/m

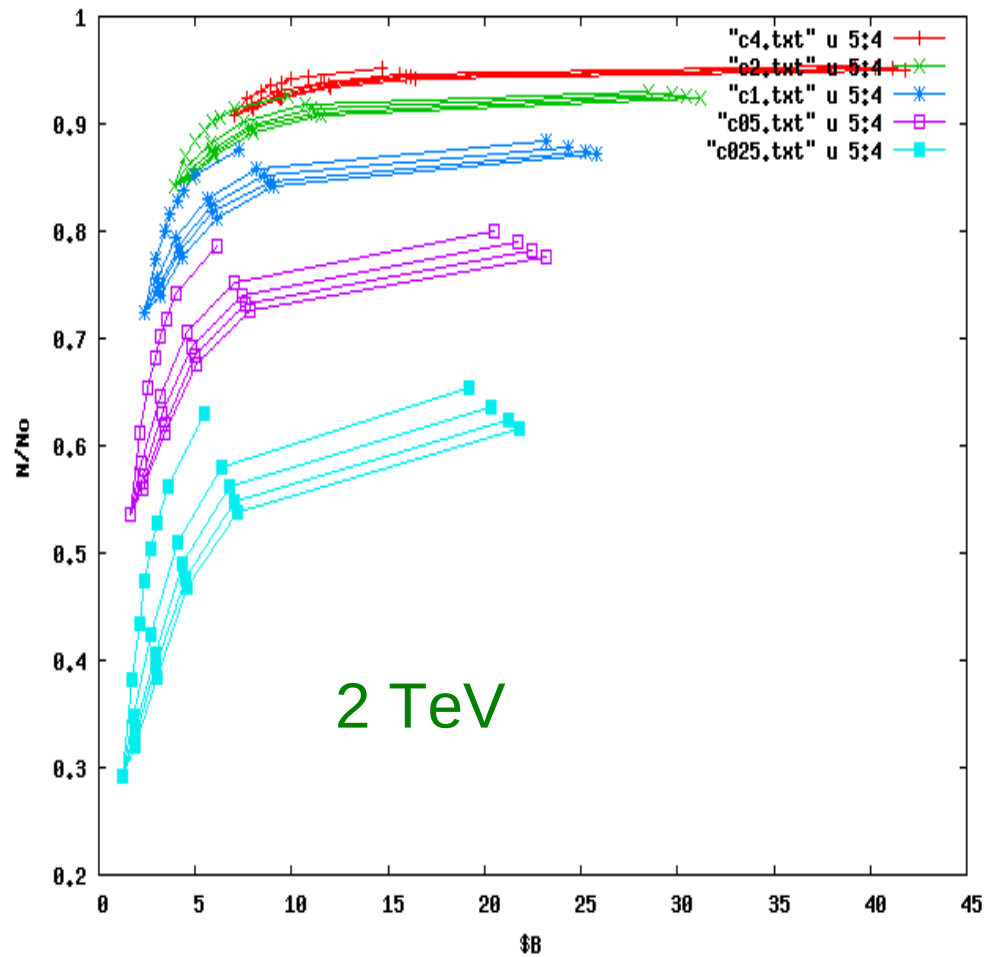


Muon survival vs. cost

0.75TeV muon survival vs cost



2TeV muon survival vs cost



Muon survival vs. cost - conclusions

Lots (1-2 km) of linac

4-5 arcs better than one

shape doesn't matter much

expensive



