Helical FOFO Snake Simulations

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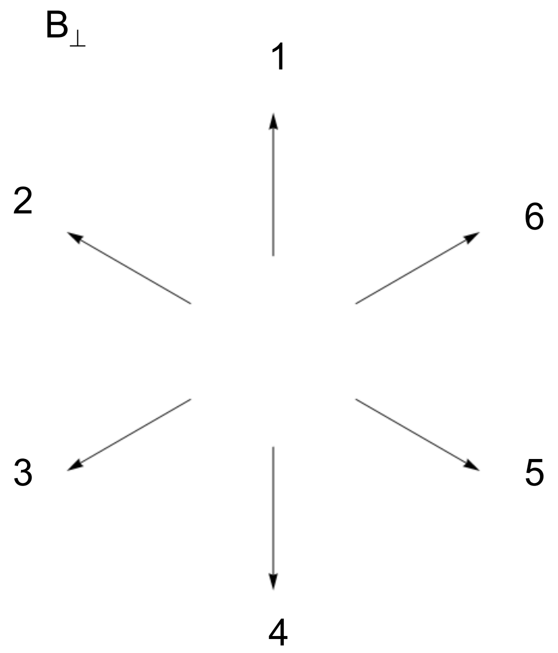
Basic Idea

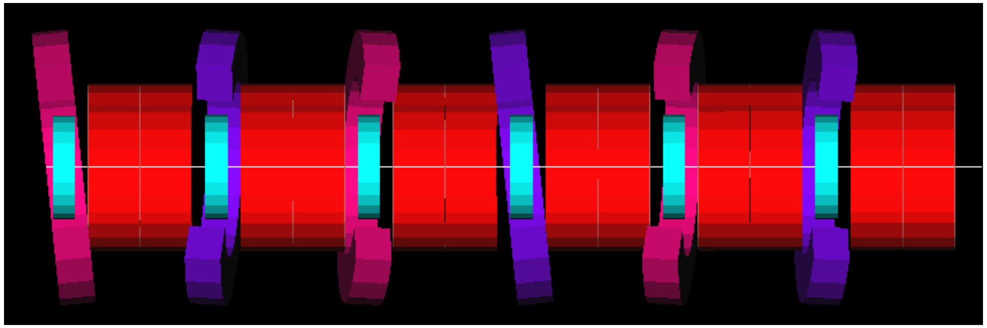
- Choose phase advance/period (6-cell period here) > $2\pi\times$integer to obtain $\alpha_p > 0$
- Create rotating $B_\perp$ field by tilting (or displacing) solenoids in rotating planes

$$x^*\cos(\phi_k)+y^*\sin(\phi_k)=0,\ k=1,2,\ldots$$

Example for 6-cell period:

Solenoid #	1	2	3	4	5	6
Polarity	+	-	+	-	+	-
Roll angle ϕ_k	0	$2\pi/3$	$4\ \pi/3$	0	$2\ \pi/3$	$4\ \pi/3$





Channel parameters:

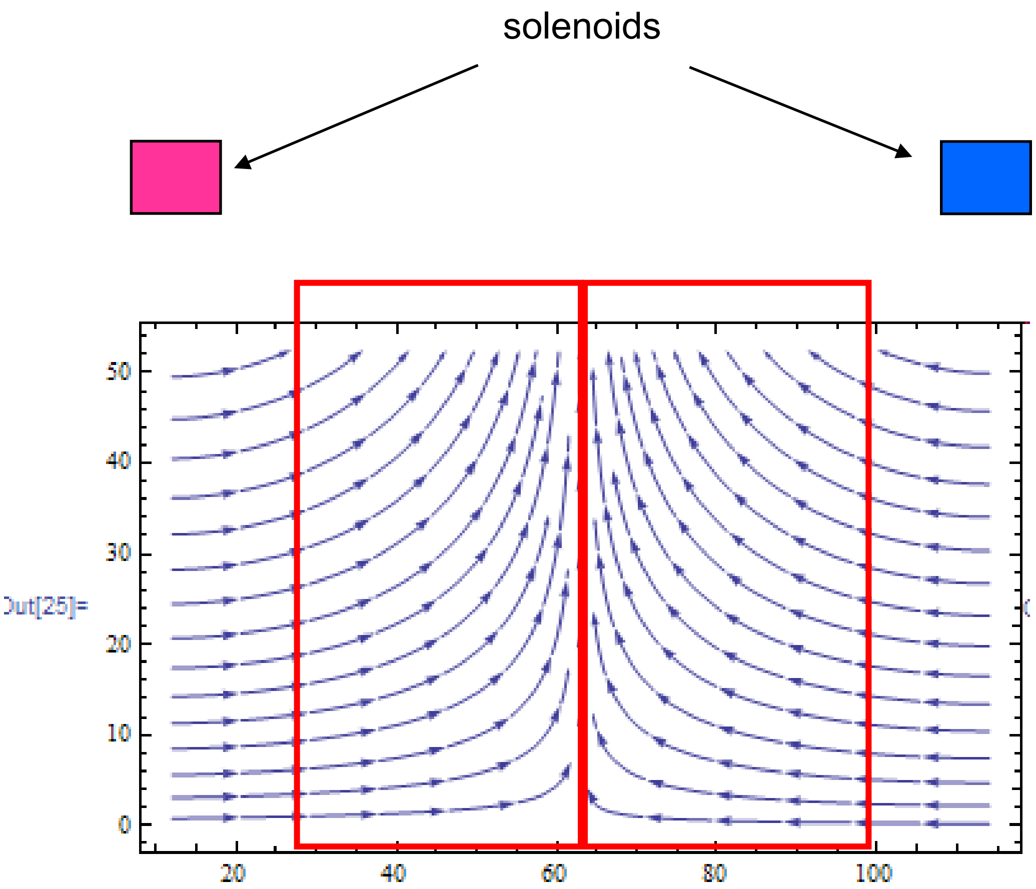
200 MHz pillbox RF 2x36cm, Emax=16MV/m

Solenoids: L=24cm, Rin=60cm, Rout=92cm,

Absorbers: LH2, total width (on-axis) 6x15cm,

Total length of 6-cell period 6.12m

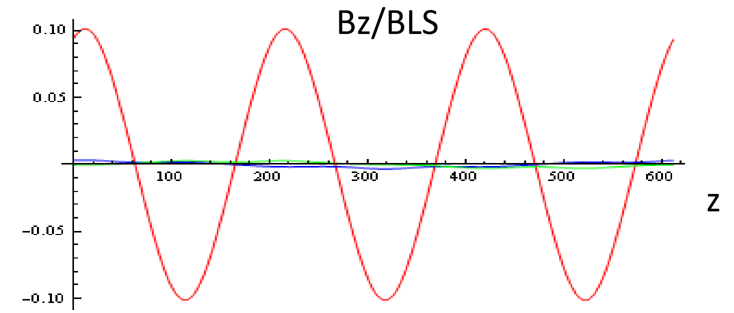
Why double cavities?

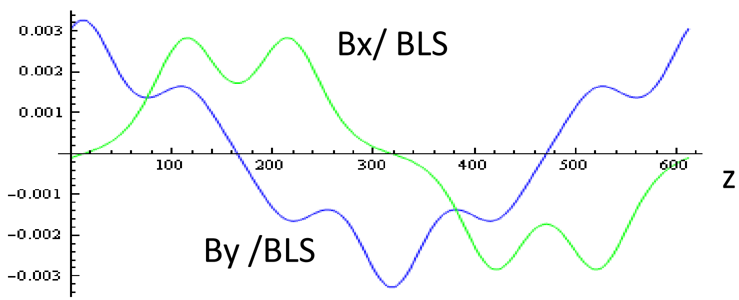


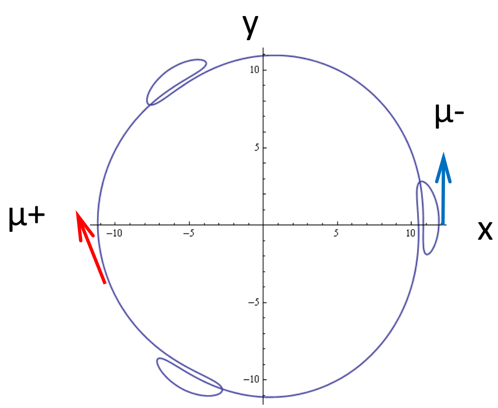
As discussed by D. Neuffer, there is hope not to lose much of gradient due to magnetic field with such configuration

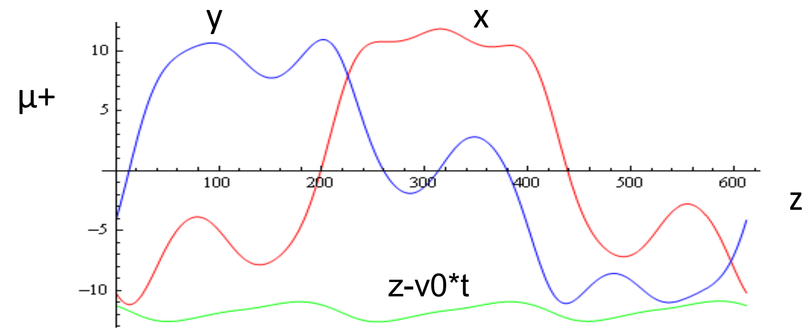
Periodic Helical Orbits

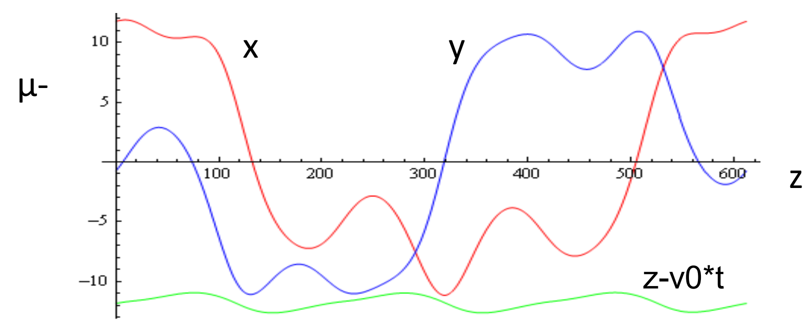
20mrad pitch angle, BLS=25.2 for p=200MeV/c

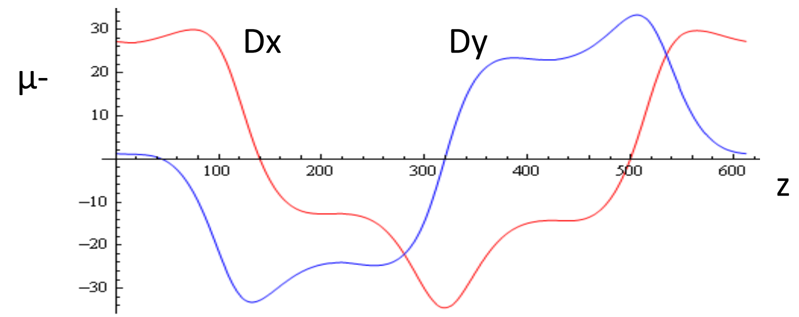






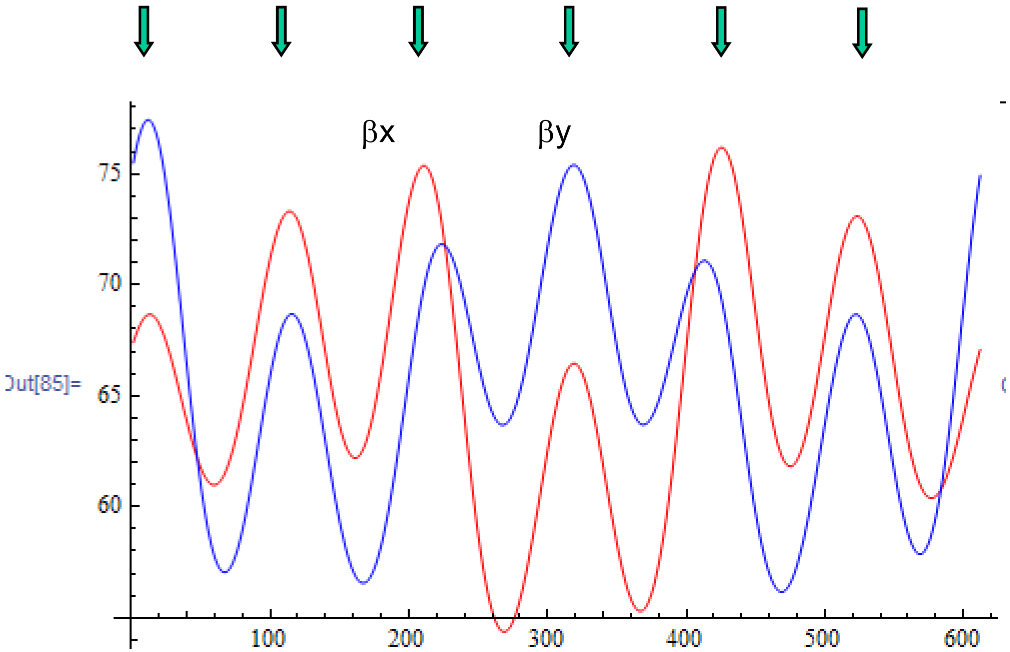






Helical FOFO snake – good for cooling both μ+ and μ-!

Beta-functions & tunes



at the absorber locations

< β > ~ 70cm

- compare with MICE's

< β > ~ 45cm

The best results with 7mrad pitch angle, no absorber wedge angle:

mode	I	II	III
tune	1.239+0.012i	1.279+0.007i	0.181+0.002i
ε_eq (mm)	3.2	4.5	6.9

ImQ=0.007 ⇒ cooling rate d log ε / dz = 2×2π/L ImQ = 1/70m

There is difficulty in equalization of damping rates of the transverse modes

Bmax=2.3T ⇒ j=58A/mm^2, Itot=4.4MA/solenoid

Transmission

Tracking simulations:

- "True" action vars of 1771 particles evenly distributed in tetrahedron

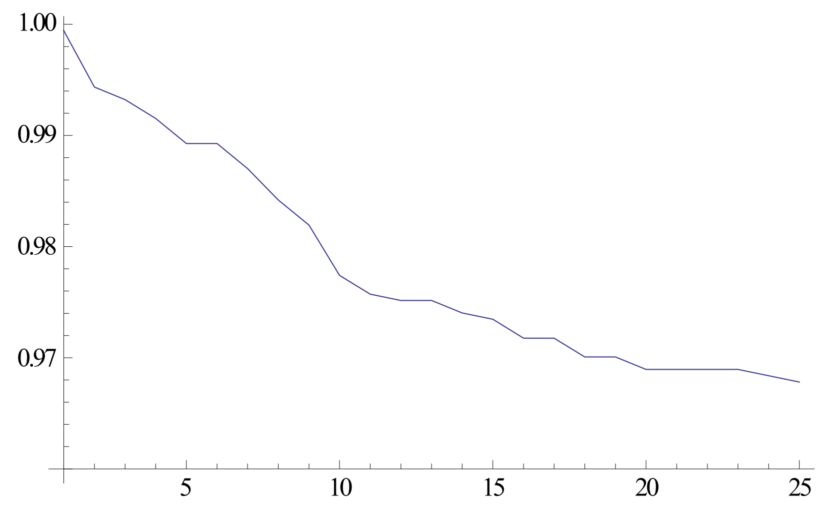
  $(J_I + J_{II})/2.6 + J_{III}/4 < 1$ (cm)

- Phases chosen at random
- No decay nor stochastic processes

Courant-Snyder invariant =2J,

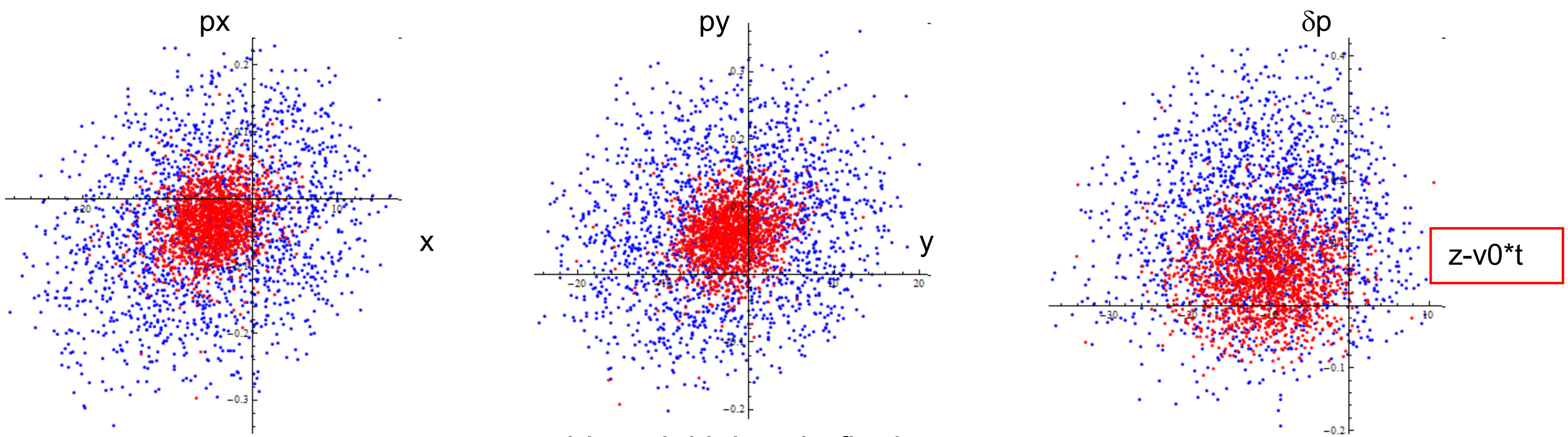
to compare with normalized emittance multiply by $\beta\gamma \approx 2$:

$\beta\gamma \times$CSImax$\approx$10cm or 2.2σ for εN=2cm



Survival after 25 periods (153m) 97%

Phase space distributions



blue - initial, red - final

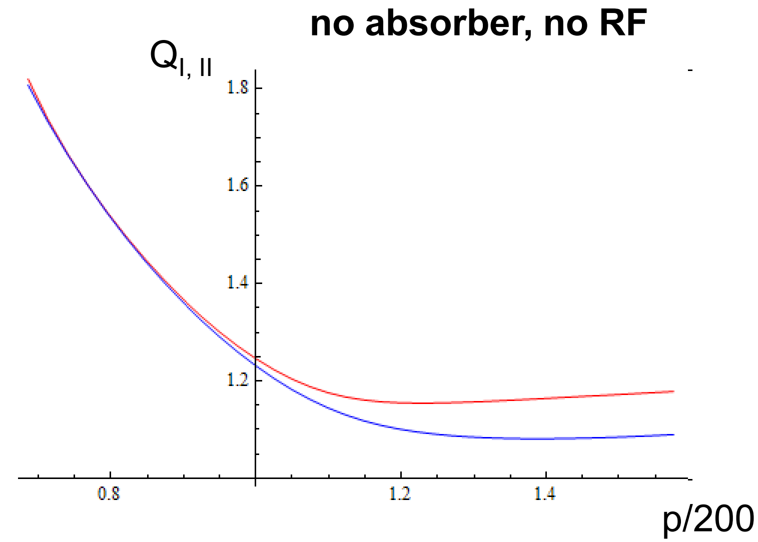
$$\delta_p = \frac{\gamma - \gamma_0}{\beta_0^2 \gamma_0},$$

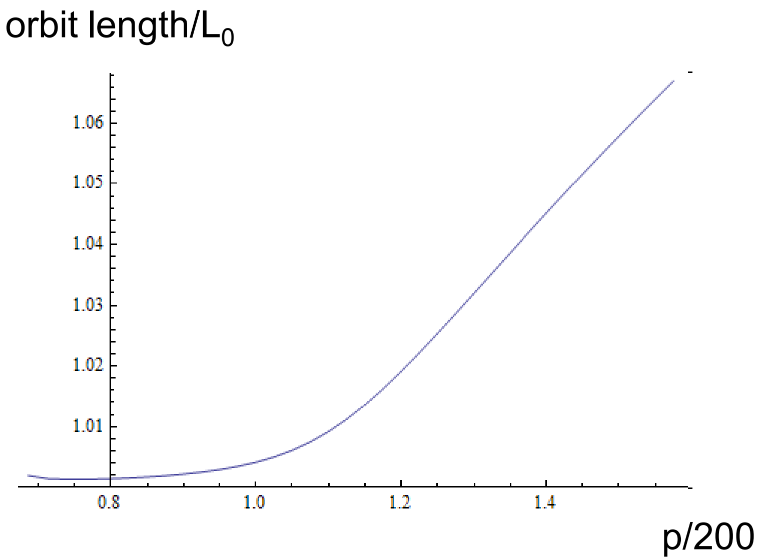
$$\frac{p}{p_0} = 1 + \delta_p - \frac{\delta_p^2}{2\gamma_0^2} + \dots$$

"Emittances" (cm)	initial	final
6D	10.3	0.07
Trans. average	1.99	0.29
Longitudinal	3.75	1.46

Why momentum acceptance is so large (>60%) in the resonance case?

Tune "repulsion" from integer resonance





Nice surprise:

Large 2nd order chromaticity due to nonlinear field components keeps both tunes from crossing the integer !

Momentum compaction factor:

$$\alpha_p \approx 0.1 \; < \; 1/\gamma_0^2 \approx 0.22$$

- in contrast to classical HCC with homogeneous absorber where

$$\alpha_p > 2/\gamma_0^2$$

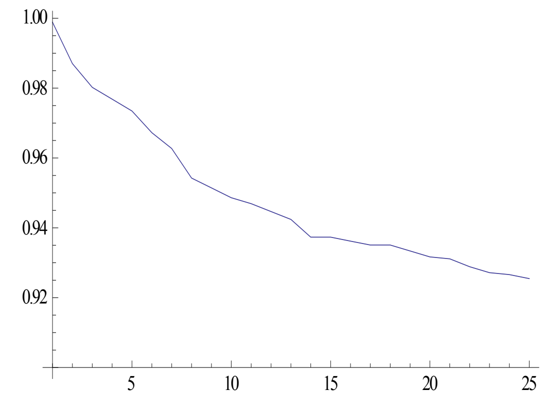
to ensure longitudinal damping.

Equalization of transverse modes damping rates

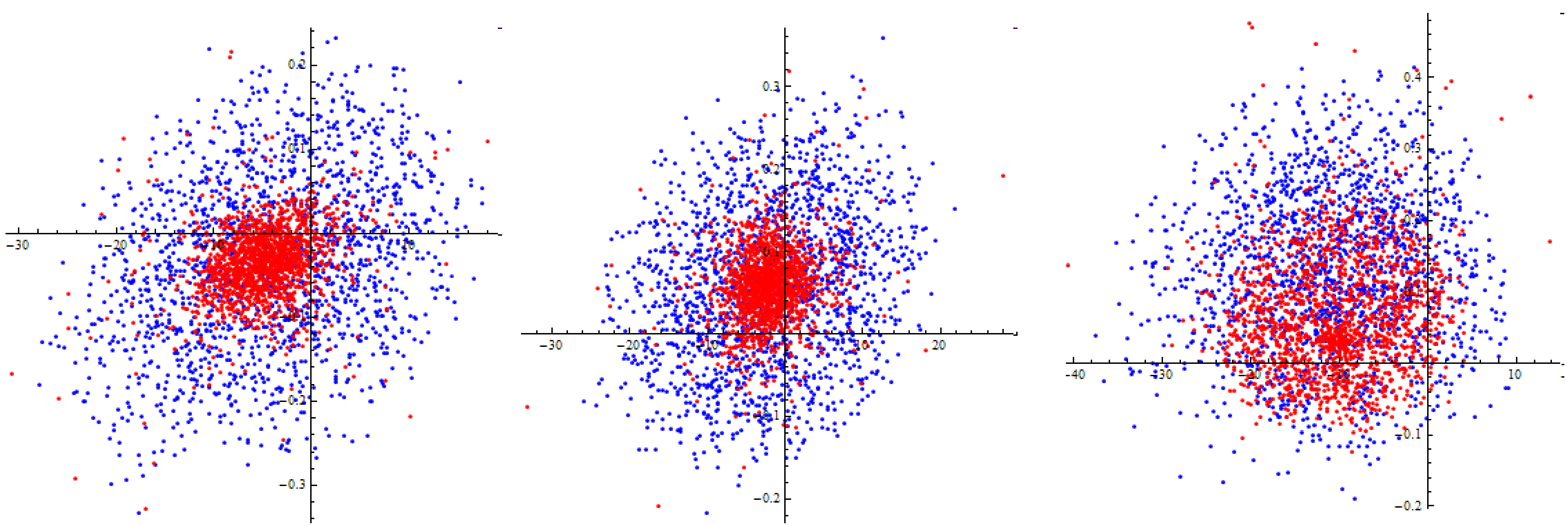
Bob Palmer proposed to add ~ constant solenoidal field to better mix the transverse modes and equalize their damping rates. This can be achieved by powering e.g. negative solenoids with slightly lower current. With just 1.6% difference in currents

mode	I	II	III
tune	1.211+0.0100i	1.301+0.0108i	0.196+0.0003i
ε_eq (mm)	3.8	3.2	36.5

(fast transverse cooling w/o longitudinal blowup was the intent)



Unfortunately this compromised the transmission while cooling does not look better :



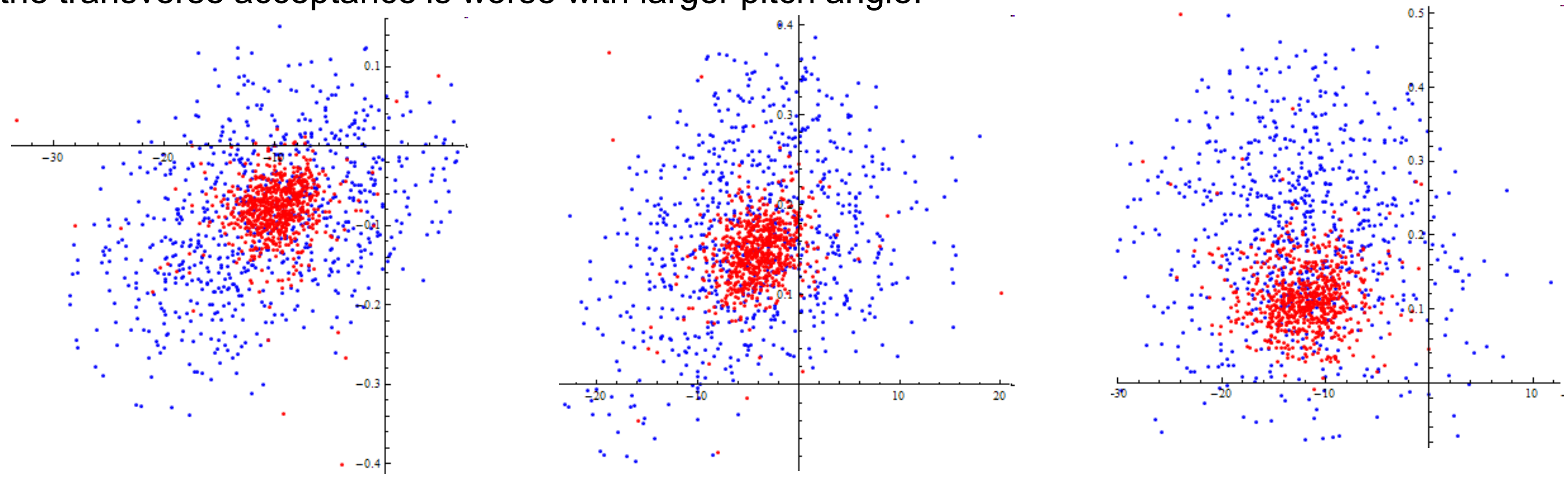
Tapering

The damping rates can be equalized at large pitch angles, e.g.

increase pitch angle (20mrad here), tunes (via Bz) and introduce absorber wedge angle (0.1 rad conical angle on either side)

mode	I	II	III
tune	1.330+0.0084i	1.423+0.0095i	0.153+0.0058i
ε_eq (mm)	4.7	4.2	2.4

but the transverse acceptance is worse with larger pitch angle:



This suggests the following strategy:

- start with a small pitch angle cooling mostly transversely
- as the transverse emittance shrinks, increase pitch angle
- at the final stage the transverse emittance stays ~constant, cooling mostly longitudinal

By reducing overall dimensions (using 1-cell RF) it is possible to lower equilibrium emittances to ~3mm for 200MHz channel.

Helical snake for final 6D cooling

By increasing B-field strength it is possible to get phase advance >180°/cell and small β-function at the solenoid center $\Rightarrow$ much smaller emittance.

Tune/period > odd_integer for resonant orbit excitation

Puzzle:

2-cell period (planar snake), Q>1
6-cell period, Q>3
} $\alpha_p < 0$

4-cell period, Q>3
6-cell period, Q>5
} $\alpha_p > 0$

Channel parameters (4-cell period):

800 MHz pillbox RF 2x8cm, Emax=32MV/m

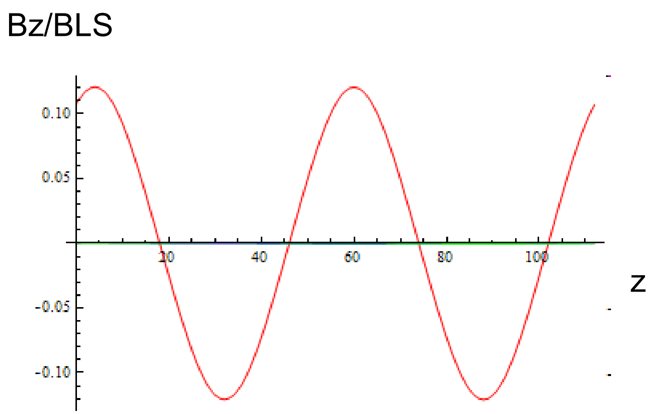
Solenoids: L=8cm, Rin=16cm, Rout=26cm,
Inclination: vertical +3mrad, horizontal +3mrad, vertical -3mrad, horizontal -3mrad

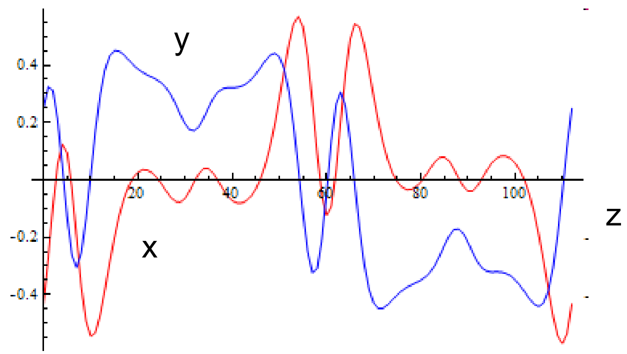
Absorbers: LiH, total width (on-axis) 4x1.2cm, no wedge angle

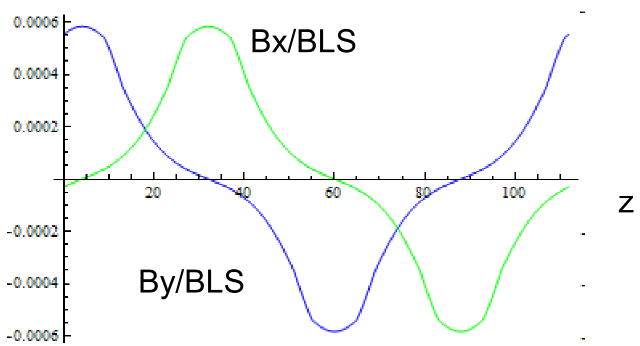
Total length of 4-cell period 1.12m

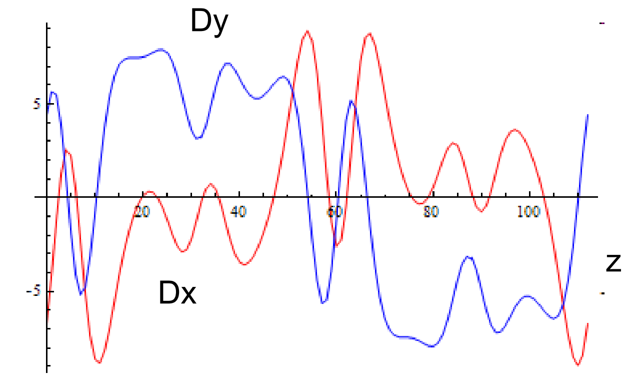
Helical snake for final 6D cooling

BLS=154T for p=100MeV/c ⇒ Bz_max=18.5T

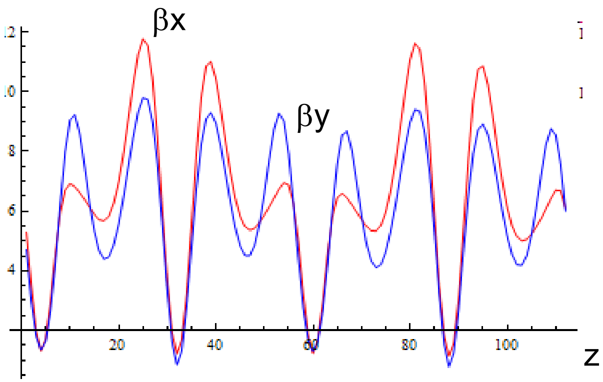






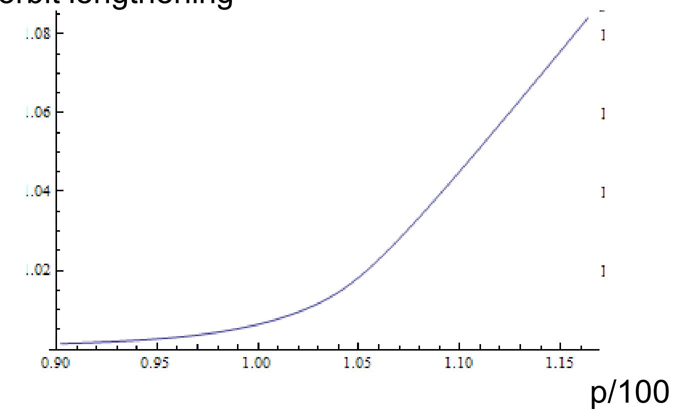


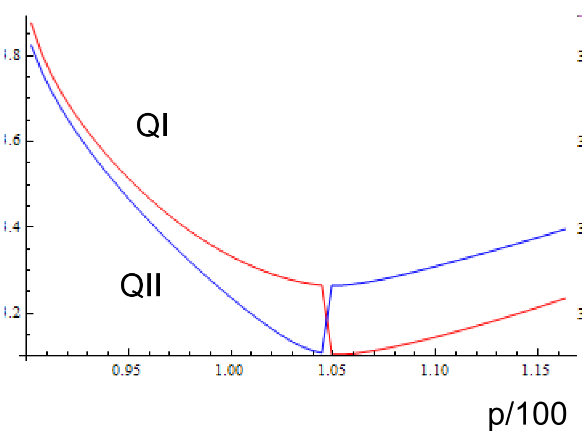
Helical snake for final 6D cooling

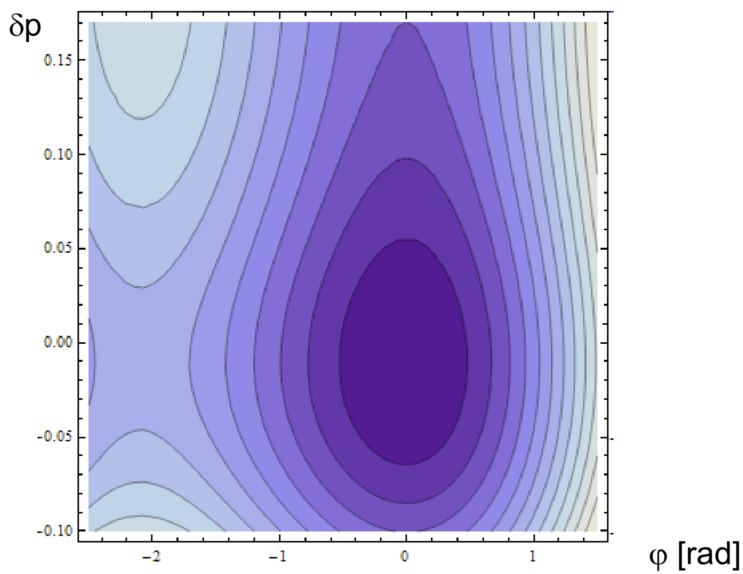


βmin ~1.6cm

orbit lengthening







Longitudinal acceptance limited by nonlinearity, not by insufficient RF bucket height

Helical snake for final 6D cooling

3 mrad pitch angle, no absorber wedge angle:

mode	I	II	III
tune	3.391+0.0066i	3.280+0.0141i	0.217-0.0047i
ε_eq (mm)	0.20	0.08	-

3 mrad pitch angle is not enough to obtain longitudinal cooling, larger pitch will reduce separatrix energy width.

Again, there is difficulty in equalization of damping rates of the transverse modes.

Difference in + and - solenoid currents helps, but spoils acceptance.

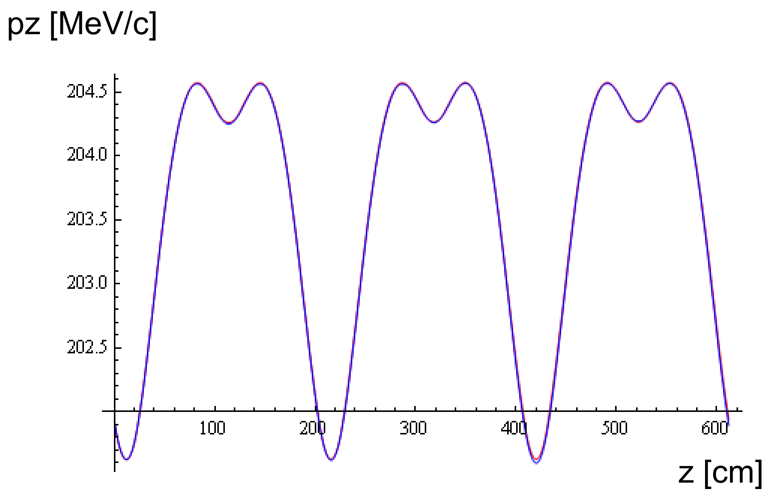
I tried to superimpose fields of different periodicity and structure, but with no success so far.

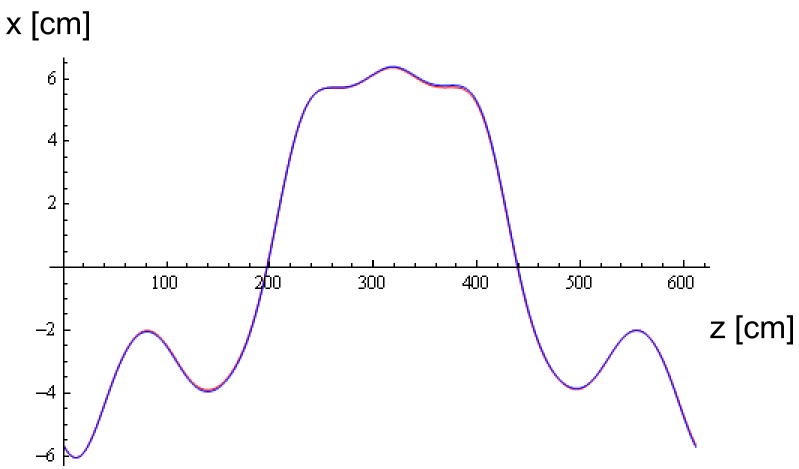
Still, the hope is the last to die!

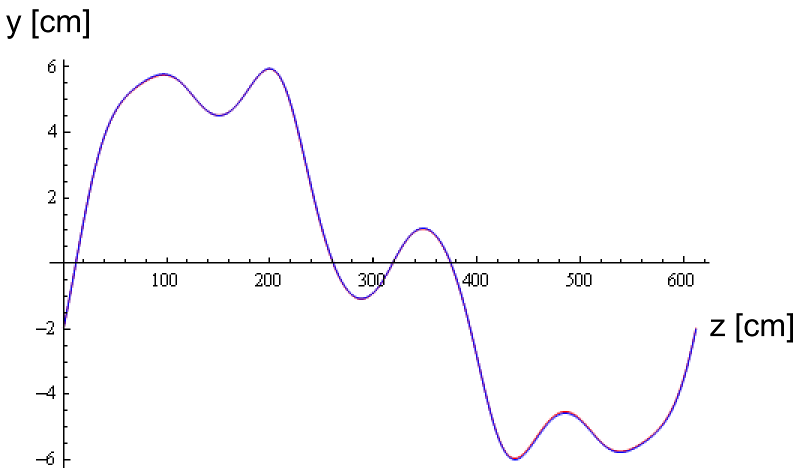
Test with G4BL

No absorbers & RF

MICCD: magnetic field via vector-potential series expansion in x,y (up to 5th order)
G4BL: superposition of field-maps







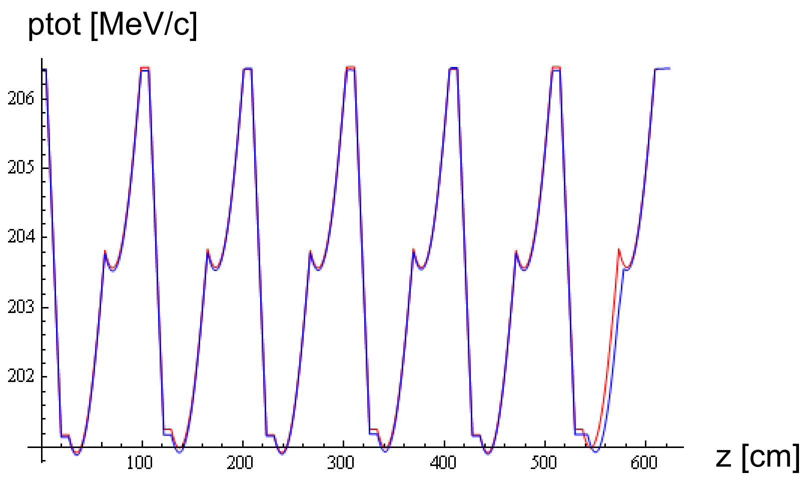
Periodic orbit:
MICCD – red, G4BL – blue

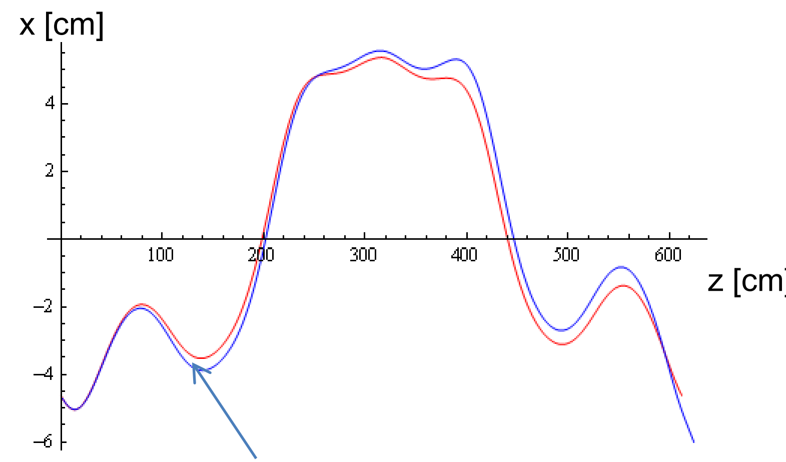
In the following I used G4BL with field-map computed with MICCD

Test with G4BL

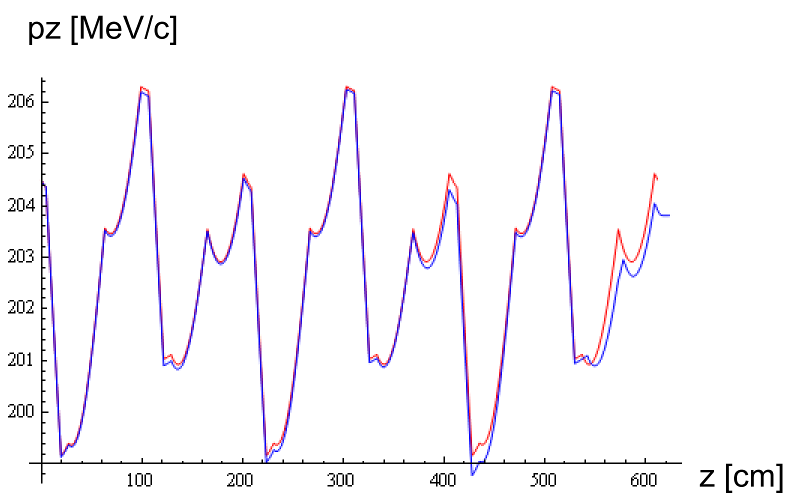
Absorbers & RF in

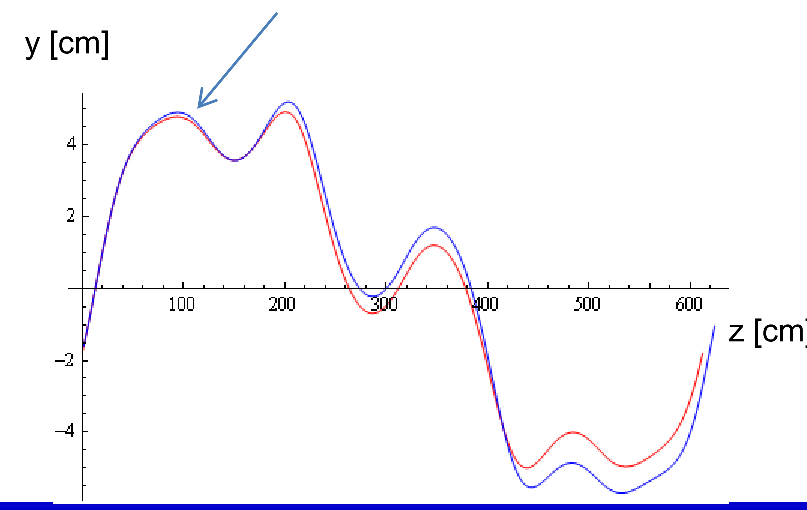
MICCD: RF phase evenly incremented
G4BL: RF phase adjusted w.r.t. the reference particle





something wrong with RF!

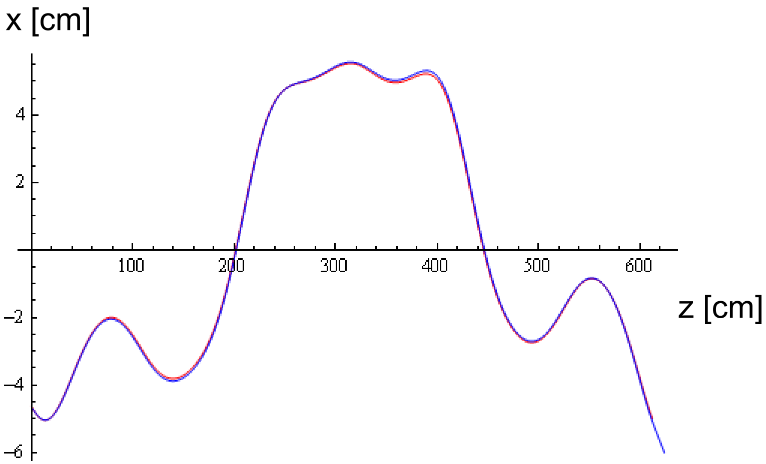


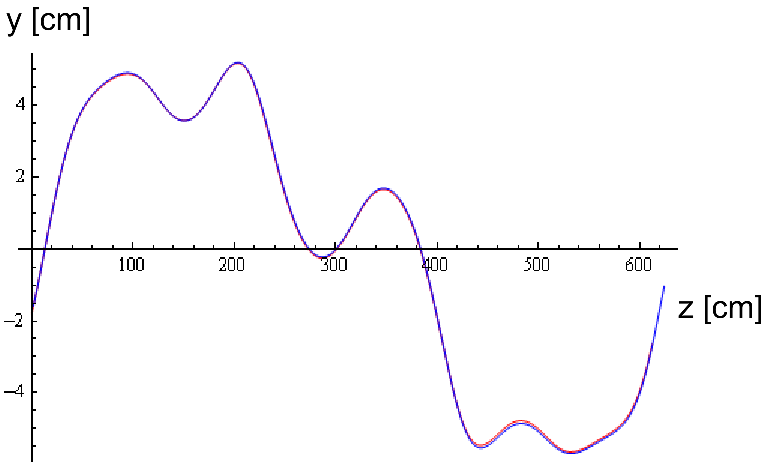


Test with G4BL

RF magnetic field

MICCD: Bphi and Ez from the same Az
G4BL: Bphi and Ez computed separately





Red: MICCD with Bphi=0
Blue: G4BL

The reason of the discrepancy is the practical absence of magnetic RF field in G4BL.

However small, this field is important: it makes mapping over an RF cavity symplectic.

Its absence may result in artificial cooling or heating.