Collider Ring—Nonlinear Compensations

Alex Bogacz

Muon Collider Ring at 1.5 TeV CM

Low Emittance MC Scenario:

Energy = 750 GeV

Normalized emittance = 2 μm rad

Relative momentum spread = ±0.005

Dipole bending field = 10 Tesla

Quadruople gradient = 250 Tesla/m

Number of IRs = 4

Betas at IR, $\beta^*_{x,y}$ = 10 mm

Peak Luminosity/IP = 7×10^{34} s^{-1} cm^{-2}

(maximum allowed by the tune shift limit

*New SBIR proposal with Muons Inc (submitted)

IR Collider Ring – Design Goals

Energy = 750 GeV

Low β^* = 1 cm

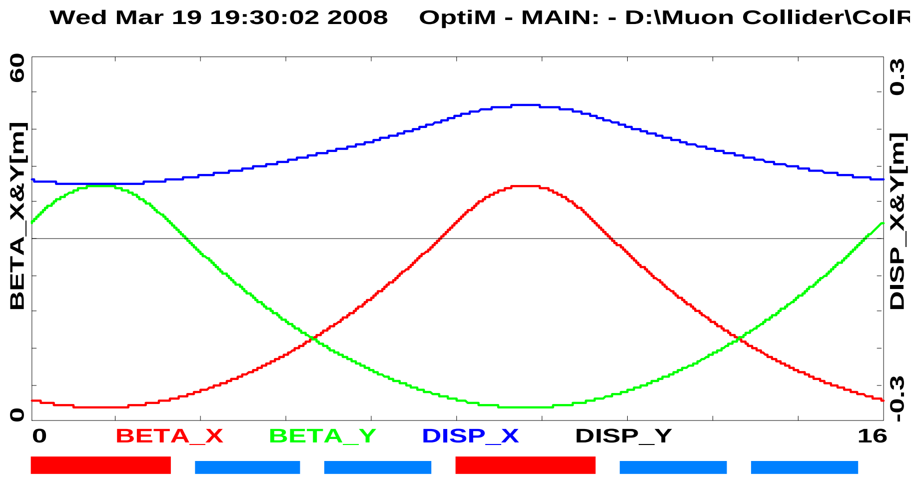
Small momentum compaction factor $\alpha_c \sim 10^{-5}$

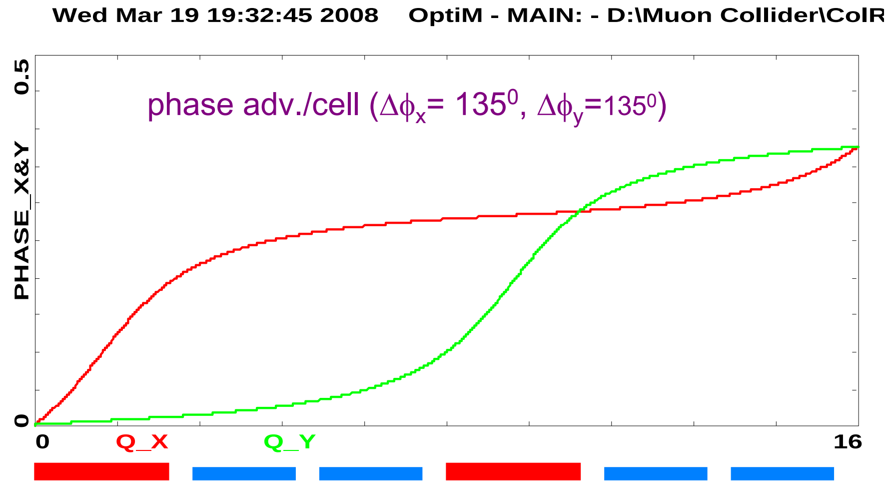
Small circumference ~ 4 km

Momentum acceptance ±0.005

Dynamic aperture (> 3σ at normalized emittance = 2 μm rad)

Periodic FODO cell – 135 deg. phase advance





Arc dipoles:

$B=100; => 10 Tesla
$E0=750000; => 750 GeV
$rho = 250 m

$N=4*50; => 200

$Lcell=1600; => 16 m

$ang=360/$Ndip; => 0.45 deg.

$Lb=$PI*$Hr*$ang/(180*$B); => 196.5 cm

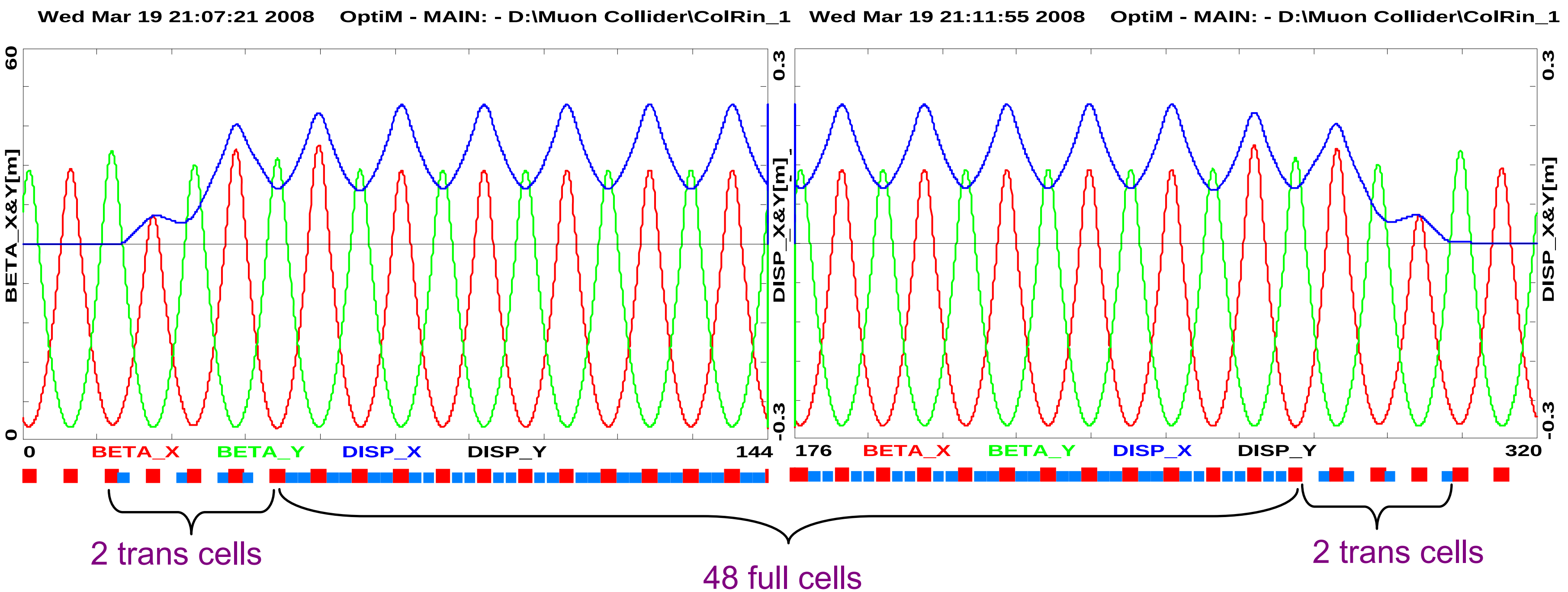
Arc quadrupoles:

L[cm]	G[kG/cm]
260	25.1
260	-25.1

$$M_{56} = -\int \frac{D_x}{\rho}\, ds$$

$$\alpha_p = \frac{L_{arcs}}{L}\,\frac{\pi^2}{N^2 \sin^2\left(\mu_{cell}/2\right)} = 2.2\times10^{-4}$$

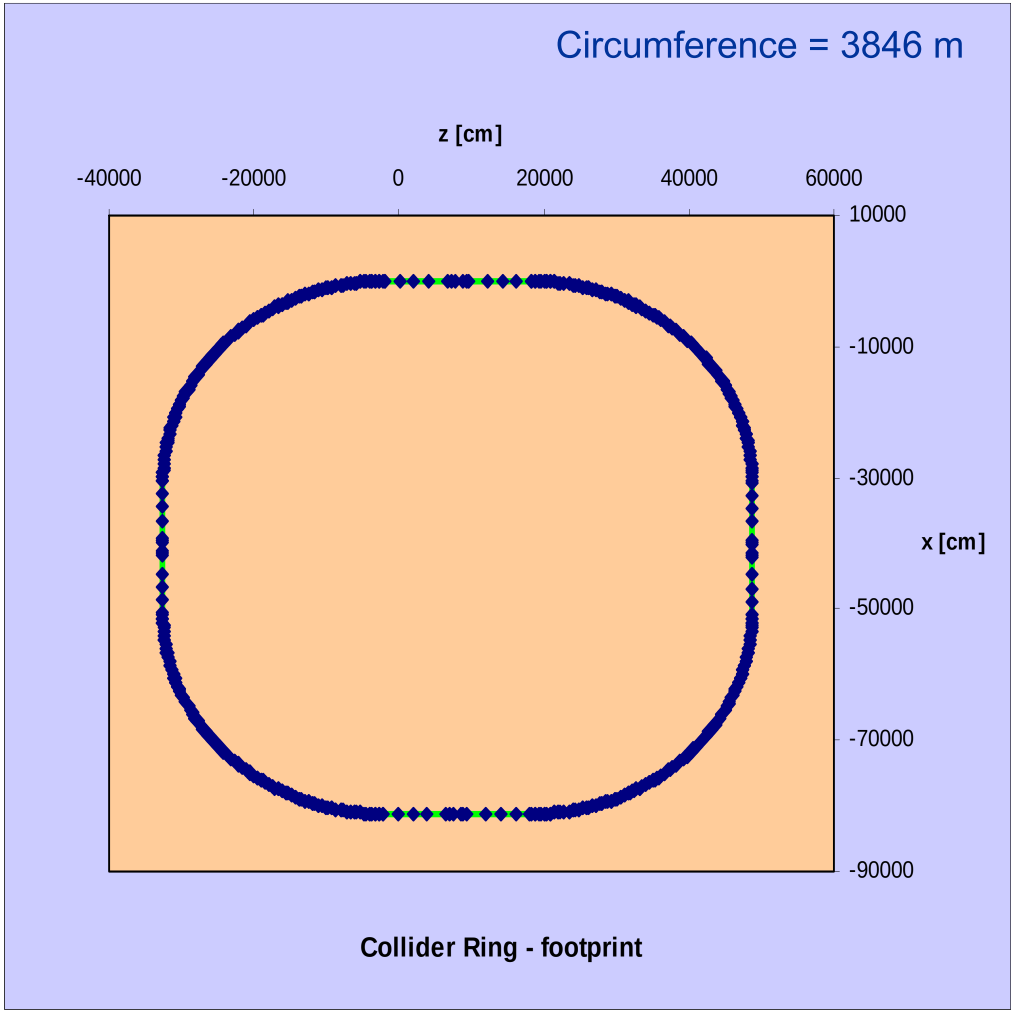
Quadrant Arc – Momentum compaction



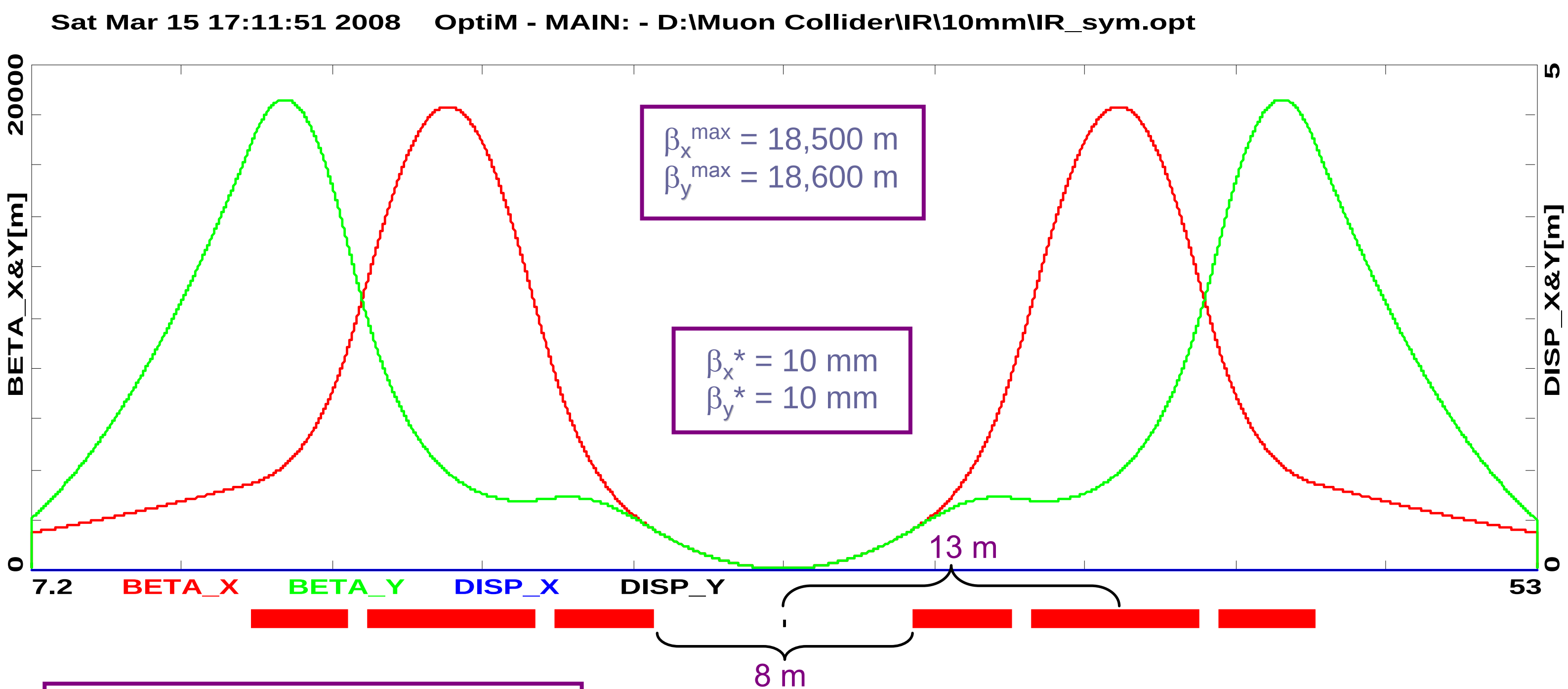
$$M_{56} = -\int \frac{D_x}{\rho} ds = -D_x^{dip} \int d\left(\frac{s}{\rho}\right) = -D_x^{dip} \int d\theta_{rad} = -D_x^{dip} \times \theta_{rad}^{tot}$$

$$\alpha = -M_{56}/L = \frac{2\pi \times 1.5\ m}{3846\ m} = 2.5 \times 10^{-3}$$

Collider Ring layout



IR – Linear lattice

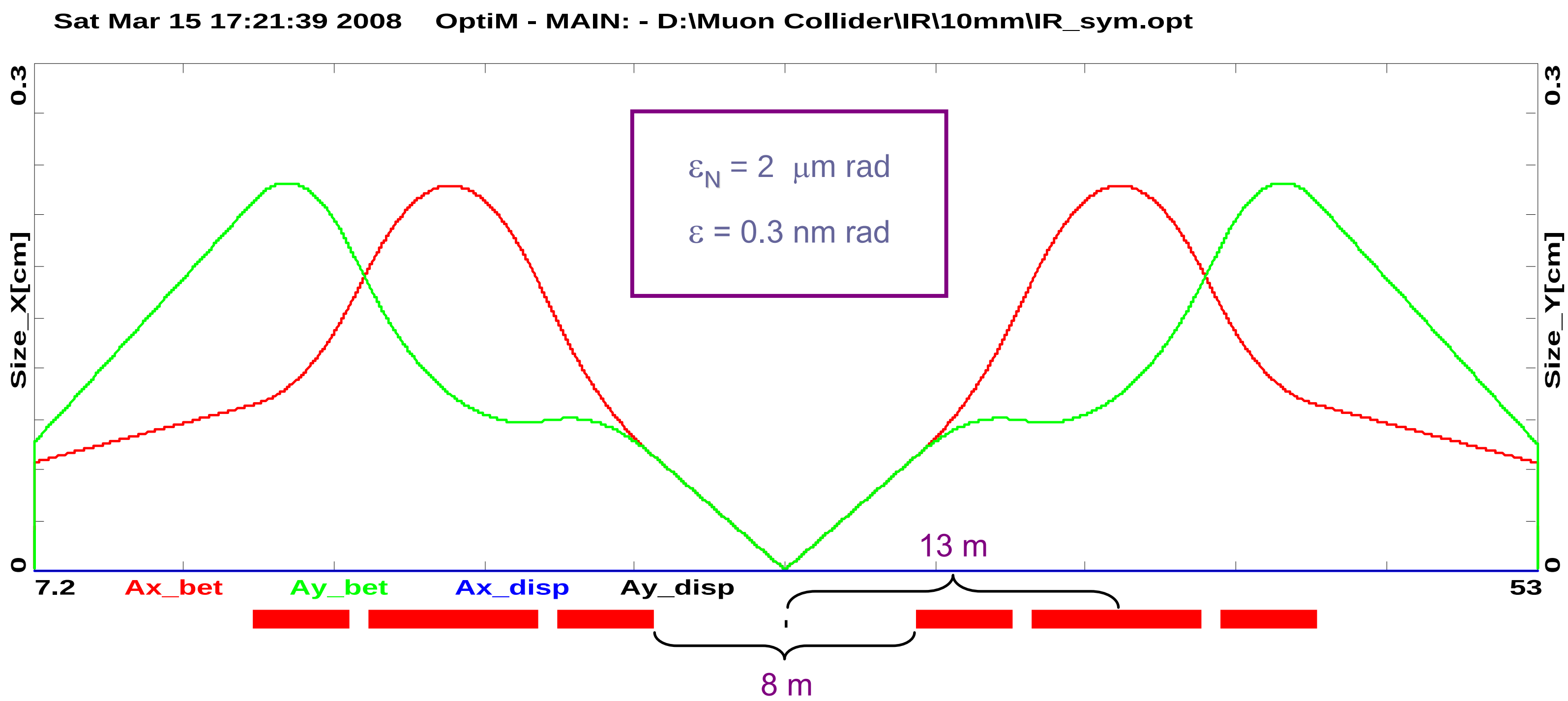


$$\beta(s) = \beta^* + \frac{s^2}{\beta^*}$$

$$\beta^{max} = \beta^* + \frac{f_{tripl}^2}{\beta^*}, \quad f_{tripl}^2 \approx \beta^{max}\beta^*$$

Name	L[cm]	G[kG/cm]
DD1	290	-20.1
FF	510	18.2
DD2	290	-23.5

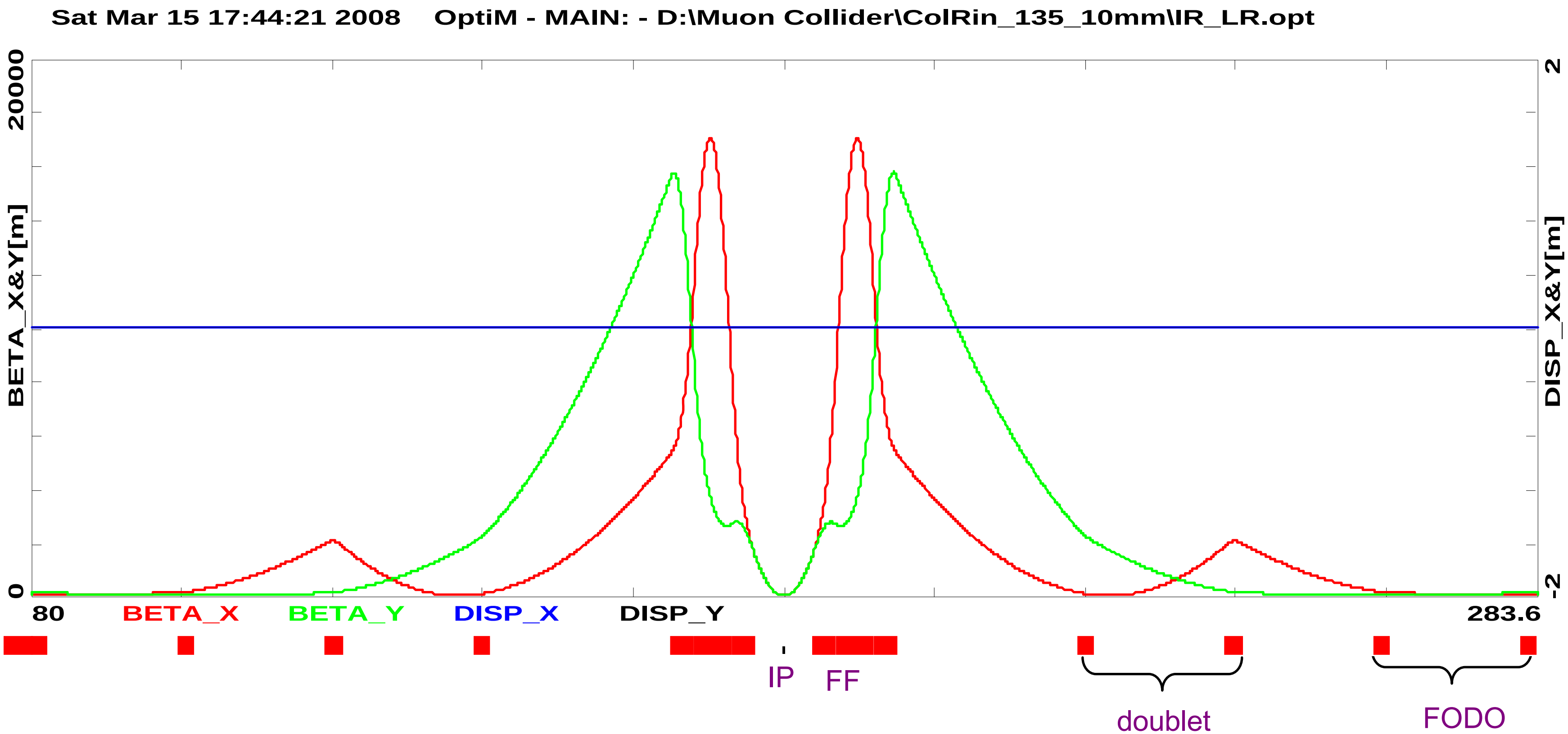
IR – Beam Envelopes (σ_{rms})



β_x^{max} = 18,500 m
β_y^{max} = 18,600 m

IR - matching to the Ring

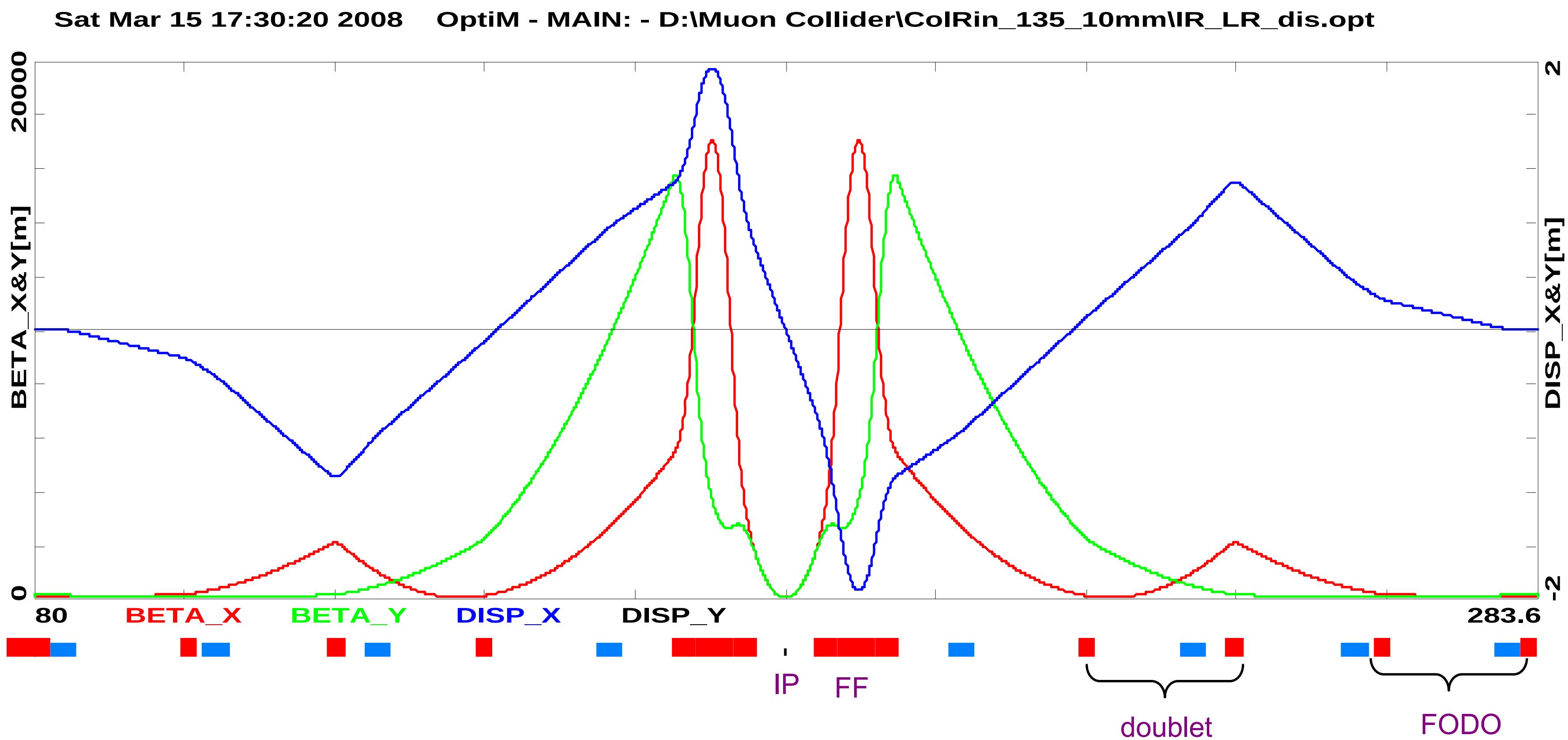
β_x^{max} = 18,500 m
β_y^{max} = 18,600 m



β_x^* = 10 mm
β_y^* = 10 mm

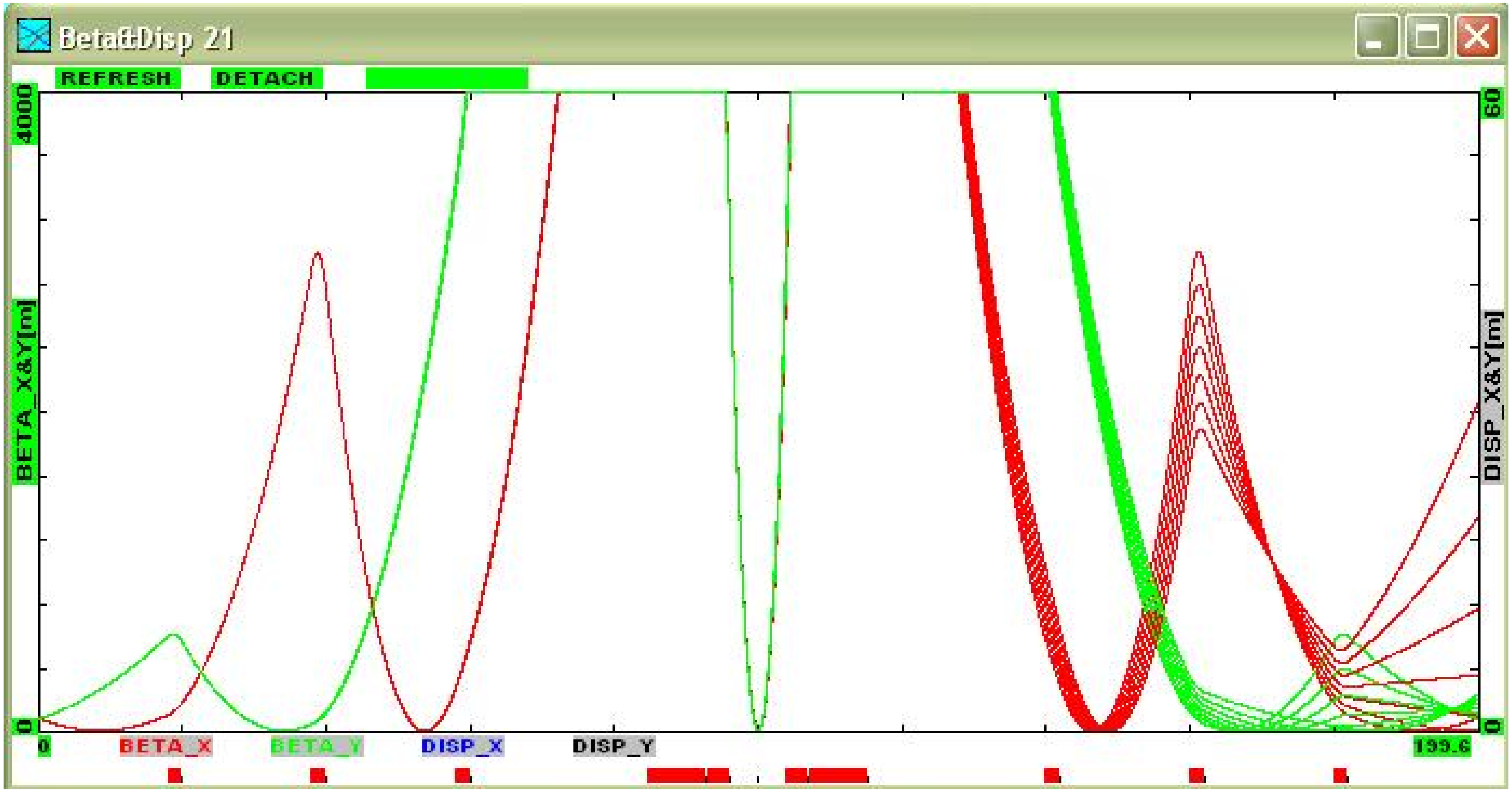
IR – Dispersion wave (negative M_{56})

β_x^{max} = 18,500 m
β_y^{max} = 18,600 m



β_x^* = 10 mm
β_y^* = 10 mm

IR - beta chromaticity



$\Delta p/p = \pm 0.005$

Beta Chromaticity

- IR beta functions strongly vary for off momentum particles
- It is measured by the beta chromaticity functions:

$$a_x = \frac{\partial}{\partial \delta_p}\alpha_x - \frac{\alpha_x}{\beta_x}\frac{\partial}{\partial \delta_p}\beta_x,$$

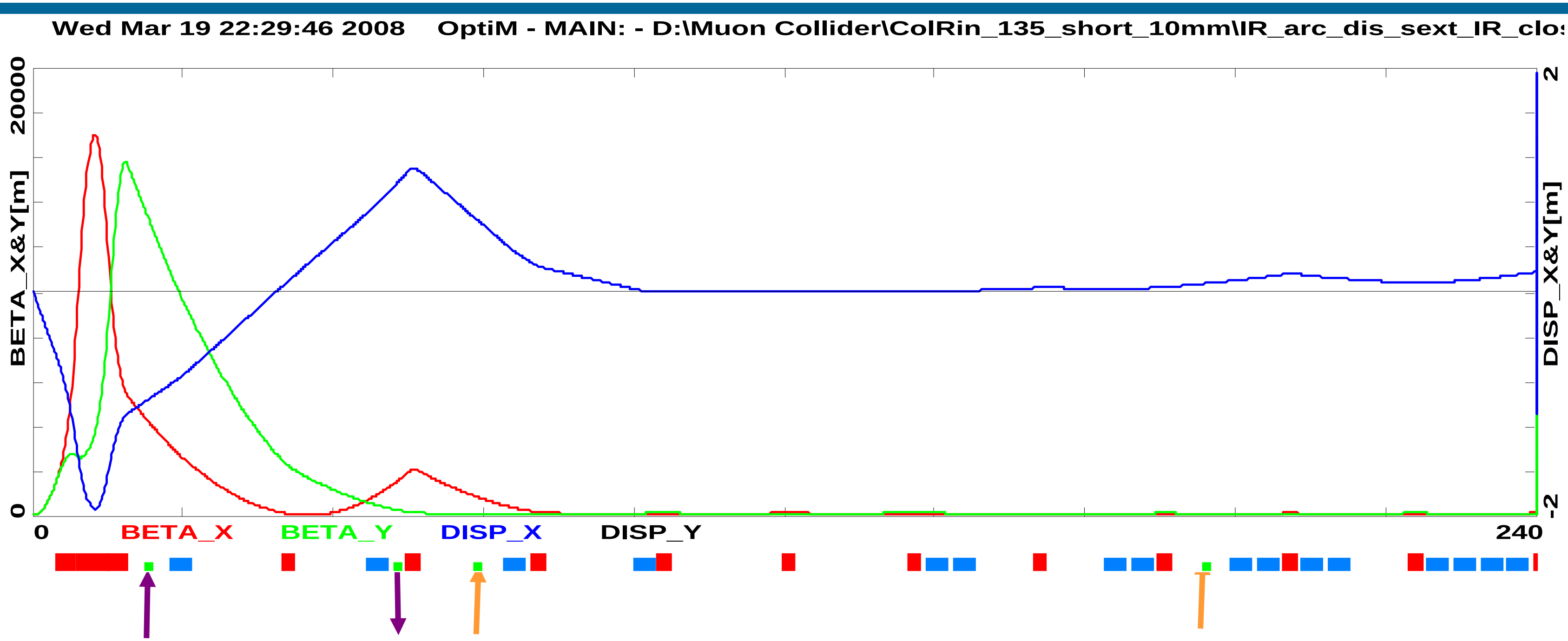
$$b_x = \frac{1}{\beta_x}\frac{\partial}{\partial \delta_p}\beta_x,$$

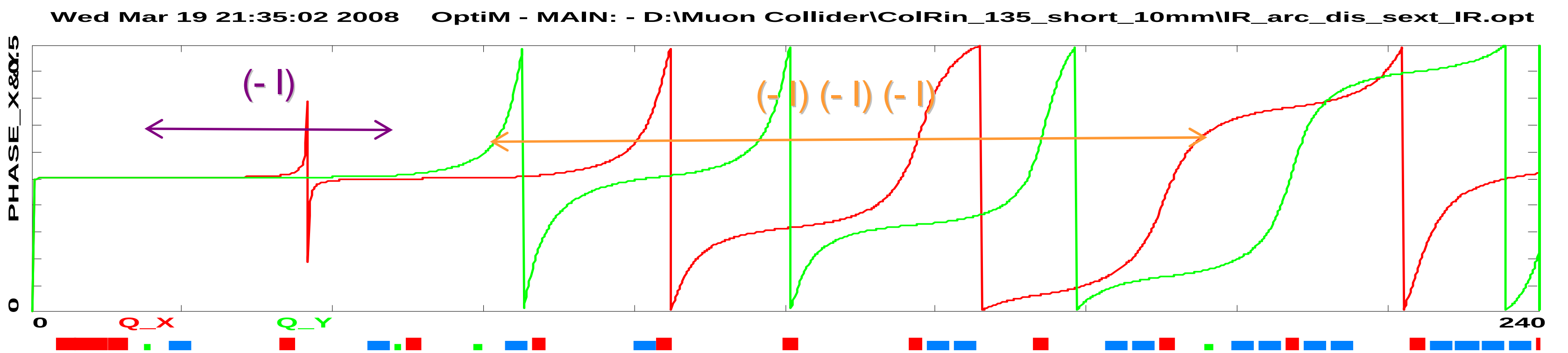
or by is the so called 'envelope' dispersion

$$w_x = \sqrt{a_x^2 + b_x^2},$$

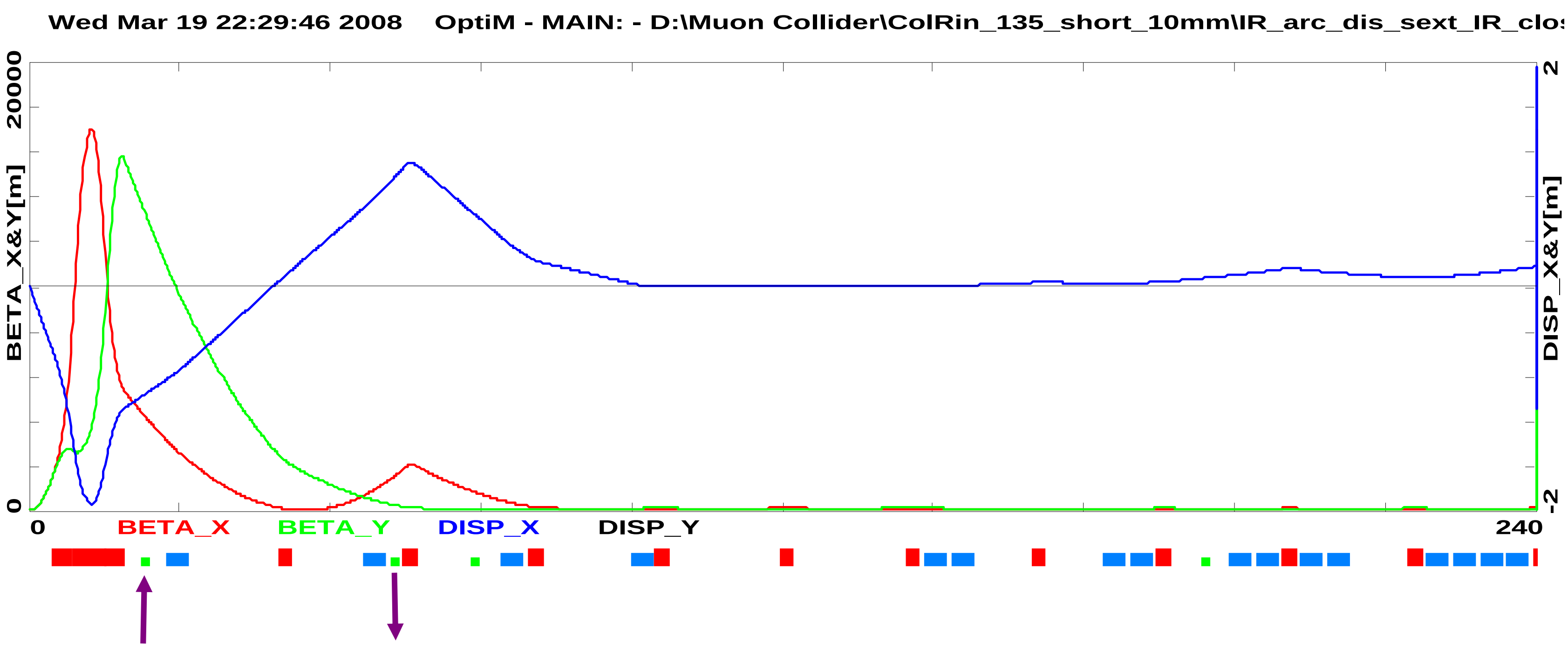
- Typical values of the w-functions ~ 100, need to be ~10
- Could be corrected with sextupoles placed in the Matching region where dispersion is generated in a controlled fashion
  - dipoles outside the IR so that $D = 0$ but $D' \neq 0$ at the IP

Beta Chromaticity correction with sextupoles



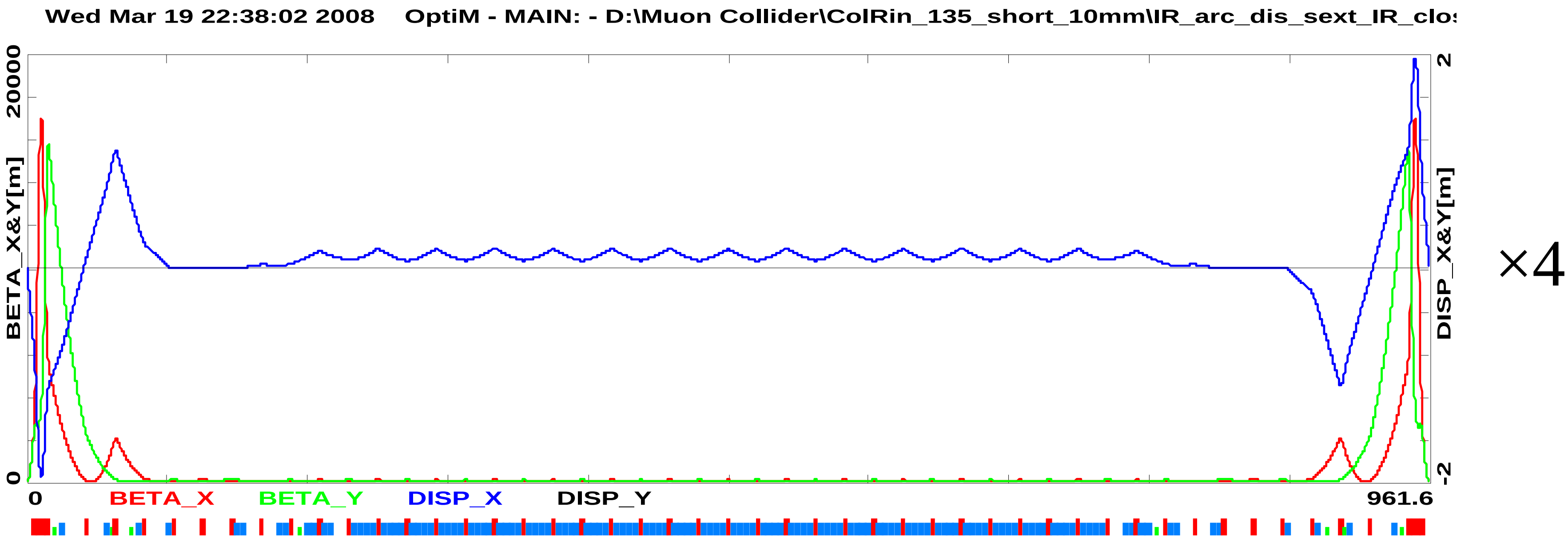


Beta Chromaticity correction with sextupoles



sS1	L[cm]=90	S[kG/cm/cm)]=0.1	Tilt[deg]=0
sS2	L[cm]=90	S[kG/cm/cm)]=-0.1	Tilt[deg]=0

T_{126}/T_{226} and T_{346}/T_{446} correction with sextupoles



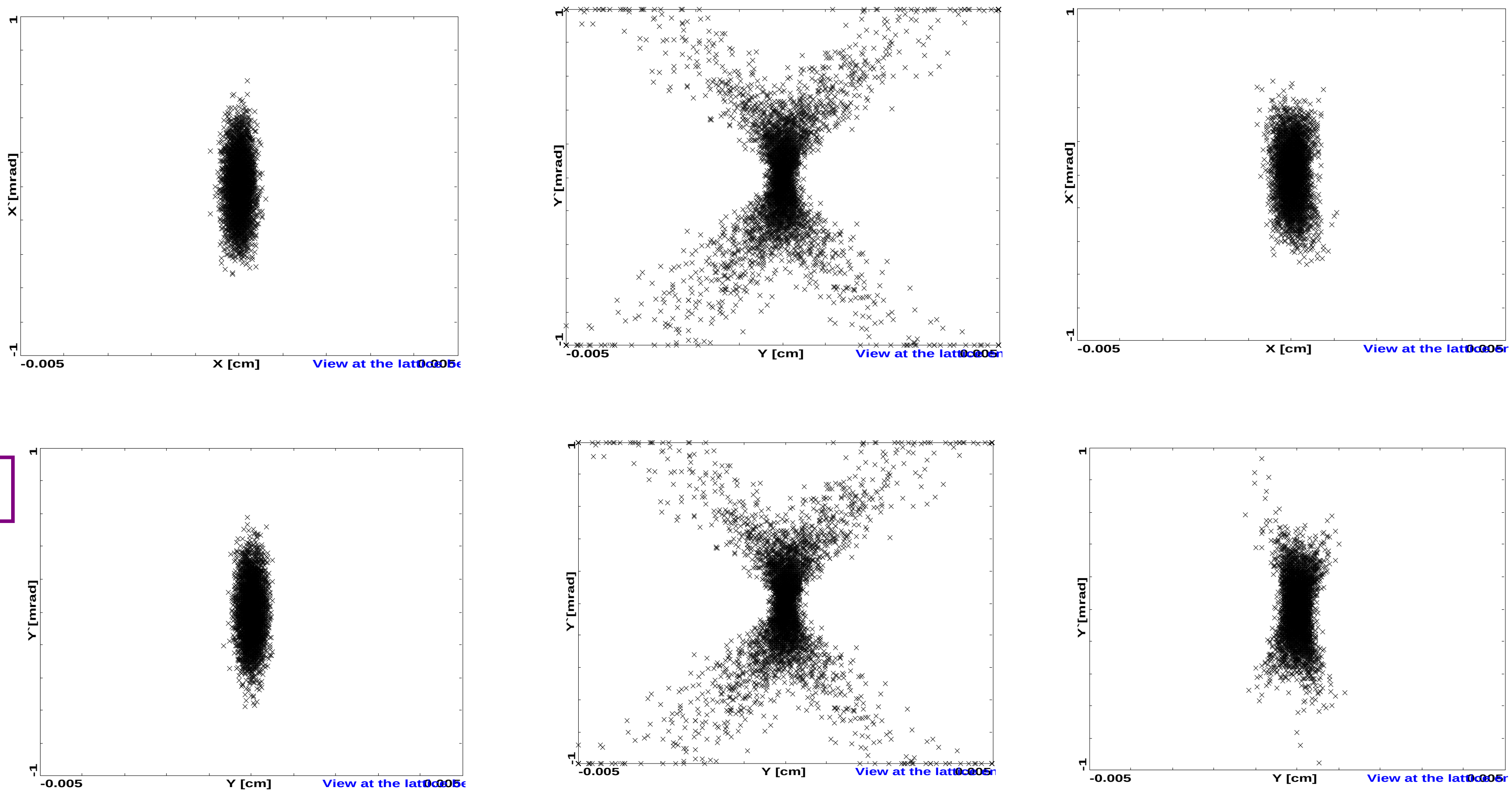
ELEGANT (Michael Borland) studies

T_{126}/T_{226} and T_{346}/T_{446} correction with sextupoles

initial | 1 turn uncorrected | 1 turn: S1=0.1 S2=-0.1

x x'

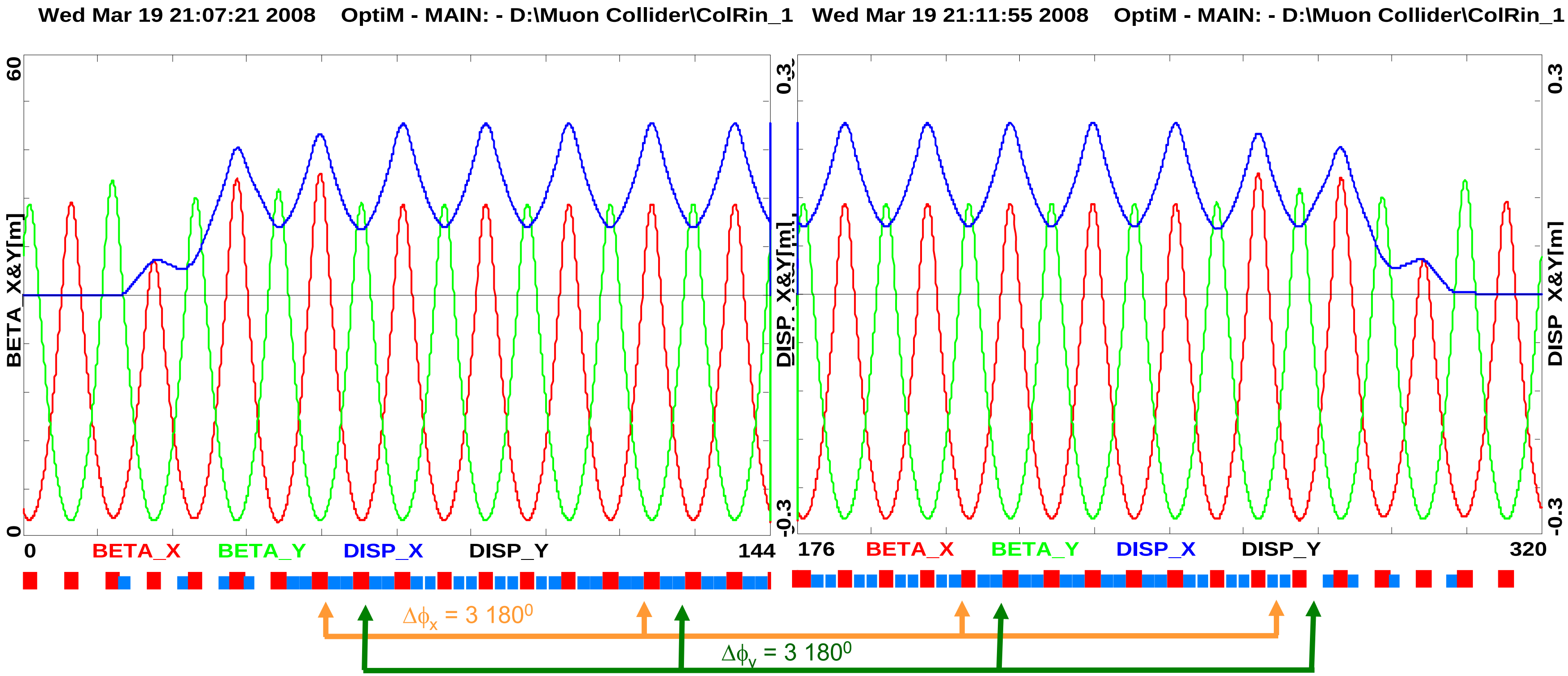
y y'



Chromatic Aberrations and Mitigation schemes

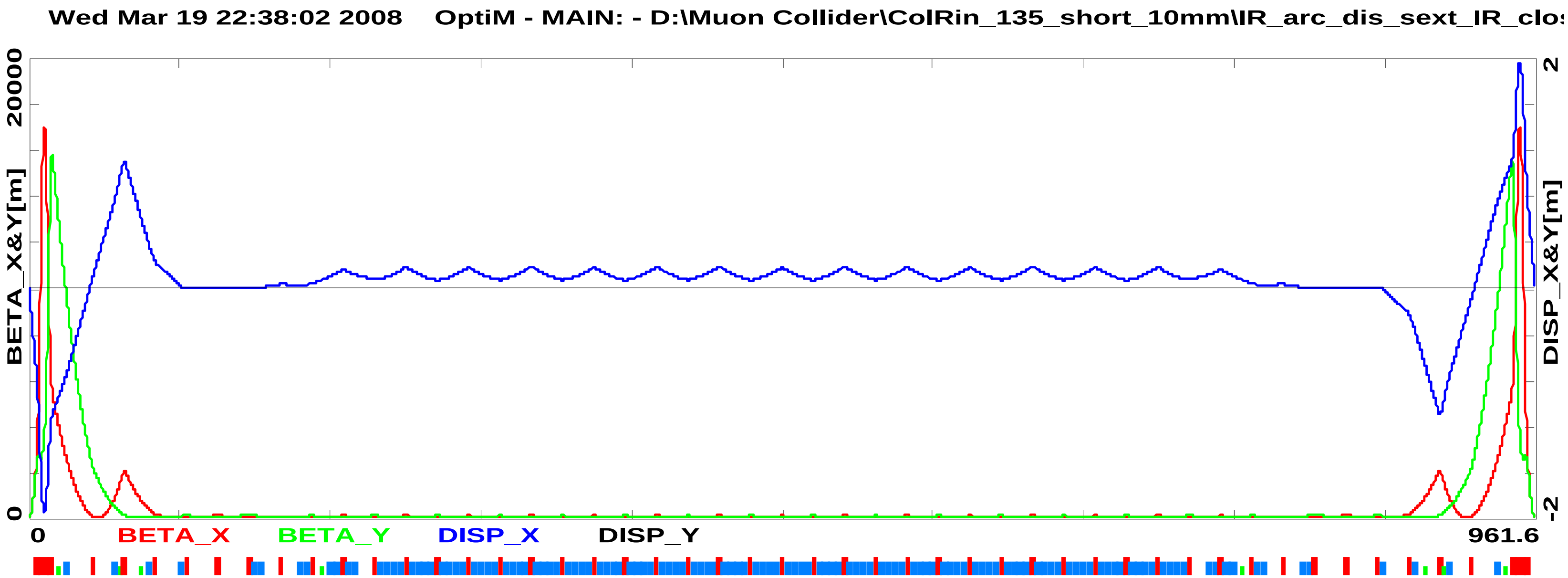
- Chromatic aberrations
  - Beta chromaticity in the IR
  - Second order chromaticity and momentum compaction
- Mitigation schemes
  - Localized Beta chromaticity correction with sextupoles in the IR-to-Arc matching sections
  - Chromaticity correction in the Arcs (two families of sextupoles)
  - Dynamic Aperture – octupoles in the IR quads

Chromaticity Compensation with two families of Sextupoles



Cancellation of geometric aberrations generated by sextupoles through 'pairing' them with a minus identity transformation between them

Dynamic Aperture studies – Multi turn tracking



×4

ELEGANT (Michael Borland) studies

Dynamic Aperture Considerations

- Large cross-detuning makes the dynamic aperture small – octupole corrections may be necessary
- Due to larger dispersion at IR sextupoles the requires sextupole gradient is lower reducing adverse 2nd order effects
- The 2nd order dispersion will be corrected with sextupoles in the matching section/Arcs

Summary

- Proposed Optics design for the Collider Ring and IR - Linear Lattice
  - Periodic dispersion achromat arcs
  - Beta chromaticity corrections with sextuoples outside the FF region
  - Natural chromaticity compensation with 2 families of orthogonal sextupoles
  - Compact matching from IR to the arcs
  - Ring parameters:

$\Delta p/p = 0.003$

Energy = 750 GeV

Total Length=3880 m

Tunes: Qx=13.8043 Qy=10.2107

Chromaticity: nuxp=-4011.85 nuyp=-2393.76

Momentum compaction ~ 10^{-5}