Ring Models for Emittance Exchange  
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- Develop simple curved Lithium lens rings for 6D Cooling with Emittance Exchange.
- Compare cooling performance of Curved Lithium lens Rings with various radii, \( \beta \) at 1 cm, 10 cm, a RF gap /10 cm curved Li lens
Model Parameters

- Liquid Lithium
- Total Current
- Arc length

- RF Gap
- Vacuum length

- Radius

- Gradient
- Freq.
- Phase

- μ
- p_z

- 10 cm – 0.5 mm
- 0.5 mm
- 100 MHz
- 20 deg
- 250 MeV/c
Transverse Emittance

$\beta = 1 \text{ cm}$

$\beta = 10 \text{ cm}$
Transverse Emittance

$\beta = 1 \, \text{cm}$

$\beta = 10 \, \text{cm}$
$\varepsilon_x, \varepsilon_y$ difference

$\beta = 1$ cm

$\beta = 10$ cm
Transmission/Longitudinal Emittance

\[ \beta = 1 \text{ cm} \quad \beta = 10 \text{ cm} \]
Beam profile

$\beta = 1 \text{ cm}, \ R=8 \text{ cm}$

$\beta = 10 \text{ cm}, \ R=64 \text{ cm}$
Beam profile

$\beta = 1 \text{ cm}, R=8 \text{ cm}$

$\beta = 10 \text{ cm}, R=64 \text{ cm}$
Magnetic Field models

**By**

ROD Field + By
BROD Field at y = 0

**Bx**

ROD Field + By
BROD Field at x = 0
Summary

- Radius dependence of the Cooling is observed in the larger beta models.
  \[ \sigma_x / \text{Radius} \sim 1 \text{ cm}/32 \text{ cm} \]
- \( \varepsilon_z \) saturation was observed in \( \beta = 10 \text{ cm}, \text{Radius 32 cm case} \).
- Can we see Emittance Exchange dynamically?
  Strength of Emittance Exchange as a function of \( z \).
- Use the model in a single path cooling channel.