PIC & REMEX
(Parametric-Resonance Ionization Cooling and Reverse Emittance Exchange for Muon Colliders)

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PIC & REMEX

• Introduction to PIC and REMEX
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  • Technical Issues
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PIC & REMEX: Motivation

- It is important for a muon collider to have a high luminosity $L$, on the order of $10^{35}$.
- $L$ is proportional to $(N_{\mu^+}N_{\mu^-})/\varepsilon_N$, so a lower normalized emittance requires fewer muons to achieve a given $L$.
- As muon-collider designs evolve, there is an increasing focus on high-luminosity designs that feature lower beam emittances and fewer muons, on the order of $10^{11}$ per bunch.
PIC: Technical Issues

• New, advanced designs for muon colliders feature ionization cooling, high-pressure RF cavities, and helical cooling channels to provide very fast muon beam cooling

• Unfortunately, ionization cooling as it is currently envisioned is limited at an equilibrium value and will not cool transverse beam sizes sufficiently well to provide high luminosity without high muon intensities

• PIC is a new method to provide much smaller transverse beam emittances
IC Reaches an Equilibrium

- The equation describing the rate of cooling is a balance between cooling (first term) and heating (second term):

\[
\frac{d\varepsilon_n}{ds} = -\frac{1}{\beta^2} \frac{dE_\mu}{ds} \frac{\varepsilon_n}{E_\mu} + \frac{1}{\beta^3} \frac{\beta_\perp (0.014)^2}{2E_\mu m_\mu X_0}
\]

- Here \(\varepsilon_n\) is the normalized emittance, \(E_\mu\) is the muon energy in GeV, \(dE_\mu/ds\) and \(X_0\) are the energy loss and radiation length of the absorber medium, \(\beta_\perp\) is the transverse beta-function of the magnetic channel, and \(\beta\) is the particle velocity.
PIC is a Solution

Excite \( \frac{1}{2} \) integer parametric resonance (in Linac or ring)
- Similar to \( \frac{1}{2} \)-integer slow beam extraction
- Weak lenses every half period drive a parametric resonance, and the elliptical phase space motion becomes hyperbolic
- Use \( xx' = \text{const} \) to reduce \( x \), increase \( x' \)
- Use IC to reduce \( x' \)

Detuning issues being addressed (chromatic and spherical aberrations, space-charge tune spread). Simulations underway.

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Figure 1: Schematic of a snake phase cooling (SPC) section, where wedge absorbers are placed at symmetric locations relative to the dispersion function, which has a period half that of the betatron function. Chromatic aberration correction sextupoles are placed at the maxima of the dispersion functions. The wedge absorbers require smaller dispersion for EMEX to control the momentum spread during the cooling process.
REMEX: Motivation

• The muon beam is cooled at a few hundred MeV/c, and it is then accelerated to the TeV/c range for an energy frontier collider.

• Consequently, the relative momentum spread $\Delta p/p$ is greatly reduced during this acceleration process, and in fact it becomes smaller than is required for a collider.

• Reverse Emittance Exchange (REMEX) is a concept to take advantage of this situation, in order to reduce further the transverse beam emittance.
REMEX is a Solution

• Thin wedge absorbers are used to exchange the transverse and longitudinal emittances, in order to provide an additional reduction in the transverse emittance.

• The longitudinal emittance grows, but it remains low enough for a muon collider
Benefits of PIC and REMEX

- After IC is used to cool the beam in a muon collider, PIC can further reduce the transverse emittance by about an order of magnitude
  - PIC reduces $x$, at the expense of increasing $x'$
  - IC is then used to reduce $x'$
- REMEX can reduce the transverse emittance by one more order of magnitude – the longitudinal emittance is increased, but that does not matter
- Together, PIC and REMEX can reduce the transverse emittance by about two orders of magnitude during the very last stages of beam cooling. This emittance reduction is crucial to the performance of an energy frontier collider.
Further Developments of REMEX

- Optimization of REMEX/coalescing: three stages to increase transverse brightness, at the expense of increasing longitudinal emittance
  - 1st Stage REMEX: after 6-D cooling using helical and PIC techniques, wedge absorbers provide REMEX up to the limit of RF acceptance
  - 2nd Stage REMEX: the beam is accelerated to an intermediate energy, now with reduced $\Delta p/p$, for another stage of REMEX (which can take place during the acceleration process). This process can continue until about 2.5 GeV, at which time energy straggling in the absorber becomes too large.
Further Development of REMEX

• Optimization of REMEX (continued)
  • BUNCH COALESCING follows two stages of REMEX: the beam is accelerated to about 20 GeV, to achieve smaller $\Delta p/p$ and longer muon lifetime. 6 to 15 bunches in a batch are then kicked into a special ring for coalescing into one intense bunch of short time but large momentum spread. This newly-formed bunch is then extracted for further acceleration.

• Optimization issues will be hot topics at the upcoming LEMC Workshop on February 12-16, 2007. e.g.
  • use of different RF frequencies to let the bunches get longer either before or during the acceleration stages of REMEX
  • Forming and cooling bunches suitable for coalescing
Cooling Channel Studies

- Beamline Designs (ppt from Alex Bogacz)
- Particle-Tracking Simulations (ppt from David Newsham)
- Higher-Order Effects (ppt from Alex Bogacz)
Prototype Lattice

• Lattice requirements:
  • Integer/Half-integer betatron phase advance per cell
  • One dispersion period per cell

  Momentum = 300 MeV/c
  Dispersion at the absorber = ±22 cm
  Minimum betas at the absorber (h/v) = 1/1 cm
  Maximum dispersion = 80 cm
  Maximum betas (h/v) = 5/5 m
  Periodic cell length = 2 m
  Betatron phase adv/cell (h/v) = \( \pi/2\pi \)

• Lattice implementation:
  • Solenoid focusing around the absorber – axially symmetric \( \beta^* \) region
  • Two pairs of dipoles with opposing bends – ‘dispersion flip’
  • Quadrupole compensation for the dipole edge focusing – balanced betas
Lattice Layout

Solenoid

Sector Dipole

Quad

Absorber Plane
**Periodic Cell – Magnets**

### 'inward half-cell'

- **Solenoids:**
  - $L = 22\text{ cm}$, $B = 105\text{ kG}$

- **Dipoles:**
  - $L = 20\text{ cm}$, $B = 25\text{ kGauss}$
  - $\text{Ang} = \frac{L \times B}{HR} = 0.4996\text{ rad}$
  - $\text{ang} = \frac{\text{Ang} \times 180}{\pi} = 28.628\text{ deg}$

- **Quadrupoles:**
  - $L = 8\text{ cm}$, $G = 1.79754\text{ kG/cm}$
  - $L = 8\text{ cm}$, $G = -0.3325\text{ kG/cm}$
  - $L = 8\text{ cm}$, $G = 1.79754\text{ kG/cm}$

### 'outward half-cell'

- **Solenoids:**
  - $L = 22\text{ cm}$, $B = 105\text{ kG}$

- **Dipoles:**
  - $L = 20\text{ cm}$, $B = 25\text{ kGauss}$
  - $\text{Ang} = \frac{L \times B}{HR} = 0.4996\text{ rad}$
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**betatron phase adv/cell (h/v) = $\pi/2\pi$**
Periodic Cell – Optics

betatron phase adv/cell (h/v) = $\pi/2\pi$
Entr/Exit into the Snake channel – Dispersion suppression

- entrance cell
- periodic cells
- exit cell
PIC/REMEX Channel – Optics

- beam extension
- disp. anti-suppr.
- n periodic PIC/REMEX cells (n=2)
- disp. suppr.
- beam extension
- RF cavity
- skew quad

snake - footprint

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Detuned Cell for Initial Simulations

- Resonant cell
  - Tune per cell \( \frac{Q_x}{Q_y} = 0.964/0.545 \)

- Detuned cell
  - Tune per cell \( \frac{Q_x}{Q_y} = 0.700/0.720 \)

\[ \begin{array}{cccc}
\hline
0 & 0.5 & 1 & 1.5 \\
\hline
0 & 0.6 & 1 & 1.2 \\
\end{array} \]
Detuned Single-Cell Lattice: Particle-Tracking Simulations with G4BL

Must Use Small Solenoid Apertures to Avoid End Effects
Average Motion in “Center Line”

Understand the Corners
Detuned Single-Cell Beta

15 mm aperture

200 mm aperture

Detuned OptiM Result
Initial Beam Profile

15 mm aperture

200 mm aperture
Initial Phase Space

15 mm aperture

200 mm aperture

Phase Space $Z = 0$ mm

X, Y, Position [mm]

X, Y, Momentum

Phase Space $Z = 0$ mm

X, Y, Position [mm]

X, Y, Momentum
Detuned 10-Cell Beta

15 mm aperture

150 mm aperture
Results of Simulations of Detuned Lattice

Care must be taken in using G4Beamline to simulate channels designed with OptiM:

- Lumped linear elements (transfer matrices) vs. “real” objects
- Accurate injection is required to avoid beam loss
- Linear optics must be understood before adding absorbers
- The effects of fringe fields and end fields must be understood. The Detuned Lattice was too compact, and only small-aperture solenoids could be simulated successfully, because of end-field issues.
- Next: try simulations of more “spread out” channel, where elements are spaced further apart and interact less. Then more realistic solenoid apertures may be used.
Simulations of a Spread-Out Cell Have Begun

betatron phase adv/cell (h/v) = \pi/2\pi
Higher-Order Effects

• The purpose of the PIC and REMEX concepts is to reduce greatly the transverse emittance of the muon beam, at the very end of the beam cooling process in a collider
• Therefore, it is imperative that the final beam spot is not spoiled by any higher-order effects
• A number of effects must be considered:
  • Chromatic and geometric aberrations
  • Space charge effects
  • Effects of beamline elements – fringe fields, straggling in absorbers, etc.
Optical Aberrations

• Notation: second-order aberrations may be described in terms of a tensor $T$ such that
  \[ v_i^f = T_{ijk} v_j^i v_k^i \]
  where $v = (x, x', y, y', t, \delta)$

• From beamline and spectrometer design, it is known that two of the most important tensor elements (out of a total of 216 elements) are $T_{126}$ and $T_{122}$
Chromatic Aberrations

- $T_{126}$ is a chromatic aberration, which depends on the initial value of $v_6 = \delta$
- In a ring, this tensor element is the chromaticity, and it is usually corrected using carefully placed sextupole magnets
Geometric Aberrations

• $T_{122}$ is a spherical aberration, which is one type of geometric aberration

• The magnitude of this aberration, at the end of a section of beamline, depends on the square of the initial value of $x'$

• This aberration arises unavoidably from the use of dipole magnets, but beamlines with the proper reflection symmetry exhibit an exact cancellation of this aberration

• Simulations indicate that this cancellation can indeed be effectively employed in a PIC/REMEX cooling channel
‘Snake’ vs. ‘Chicane’
Layout

‘snake’

‘chicane’
Initial Transverse Phase-space

$\varepsilon_{x,y} = 100 \text{ mm mrad geometric emittance at } p = 100 \text{ MeV/c}$

Very small momentum spread, to remove chromatic aberrations

**Initial phase-space**

- BunchLength[cm] = 1.0
- $dP/P = 0.000001$
- $\beta_X_1[cm] = 1.36483$, $\beta_X_2[cm] = 0.$
- $\alpha_X_1 = 0.0$, $\alpha_X_2 = 0.0$
- $\beta_Y_1[cm] = 0$, $\beta_Y_2[cm] = 1.36448$
- $\text{Emit}_1[cm] = 1.\text{e-2}$, $\text{Emit}_2[cm] = 1.\text{e-2}$
- $\text{Disp}_X[cm] = 0$, $\text{Disp}_Y[cm] = 0$
- $\text{Disp}_Xpr[cm] = -59.8339$, $\text{Disp}_Ypr[cm] = 0$

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‘Snake’ vs. ‘Chicane’ – Tracking

x - y

x – x’ y – y’

chicane
Cancellation of Geometric Aberration

• The speculation that $T_{122}$ is the most important geometric aberration in this cooling channel is supported by the simulation results.

• That is, the reflection symmetry of the “chicane” cooling channel cancels this specific geometric aberration and greatly reduces the total geometric aberrations observed.
Summary

- The PIC and REMEX concepts have been invented and developed to provide the crucial final beam cooling in a muon collider. Their use can greatly reduce the final beam emittance and can permit the construction of a high-luminosity collider that requires fewer muons.

- Significant progress has been made in the design of these cooling channels and in the corresponding particle-tracking simulations.

- Reflection symmetry in the cooling channel design can be used effectively to cancel the largest geometric aberration in the beamline optics.