Non-Scaling Triplet FFAG Lattices for the Electron Demonstration Ring and Proton acceleration for the AGS upgrade

CONTENT:

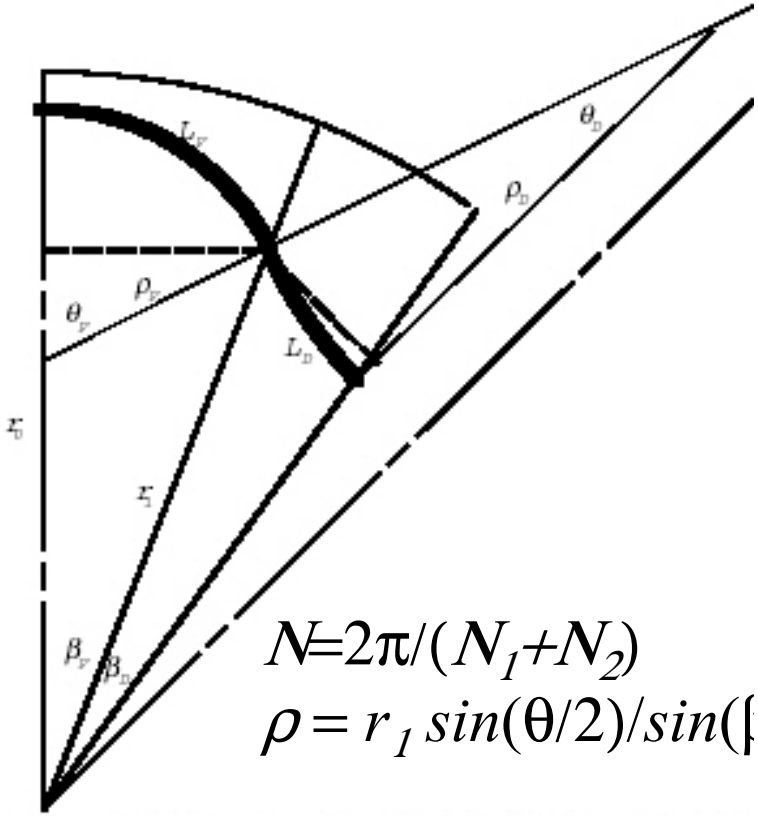
- **Electron Demonstration Ring: dp/p = +/–50%**
  - Assumptions and Goals for the Electron Demonstration Ring.
  - Following directives from the BNL workshop: (symmetric parabola)
  - Basic lattice properties.
  - Courant-Snyder functions dependence on momentum.
- **Proton Non-Scaling Design:**
  - Momentum Dependence of the lattice functions.
  - Larger orbit offsets during acceleration.

Electron Demonstration Ring: Assumptions and Goals

- Energy: acceleration from 10 MeV to 20 MeV.
- Dimensions: Small ring: circumference ~12-15 meters.
- Scaling factor from Muon ~300 meter ring to 15 m is about 20:
  - C~ 12-15 m, magnetic field 4T/20 = 0.02 T, dipole length 1m/20 = 5 cm, …
- Momentum Acceptance dp/p~+-50%
- Horizontal and vertical betatron tunes in the cell to be within 0.1 $< \nu_{x,y} < 0.4$.
- Path Length dependence on momentum is expected to be parabolic and the central energy needs to be close to the transition gamma γ_t.
- The RF cavity required drift space should be of the order of 10 cm.
- Optimize the magnets sizes and drift lengths.

What is the SCALING FFAG ?

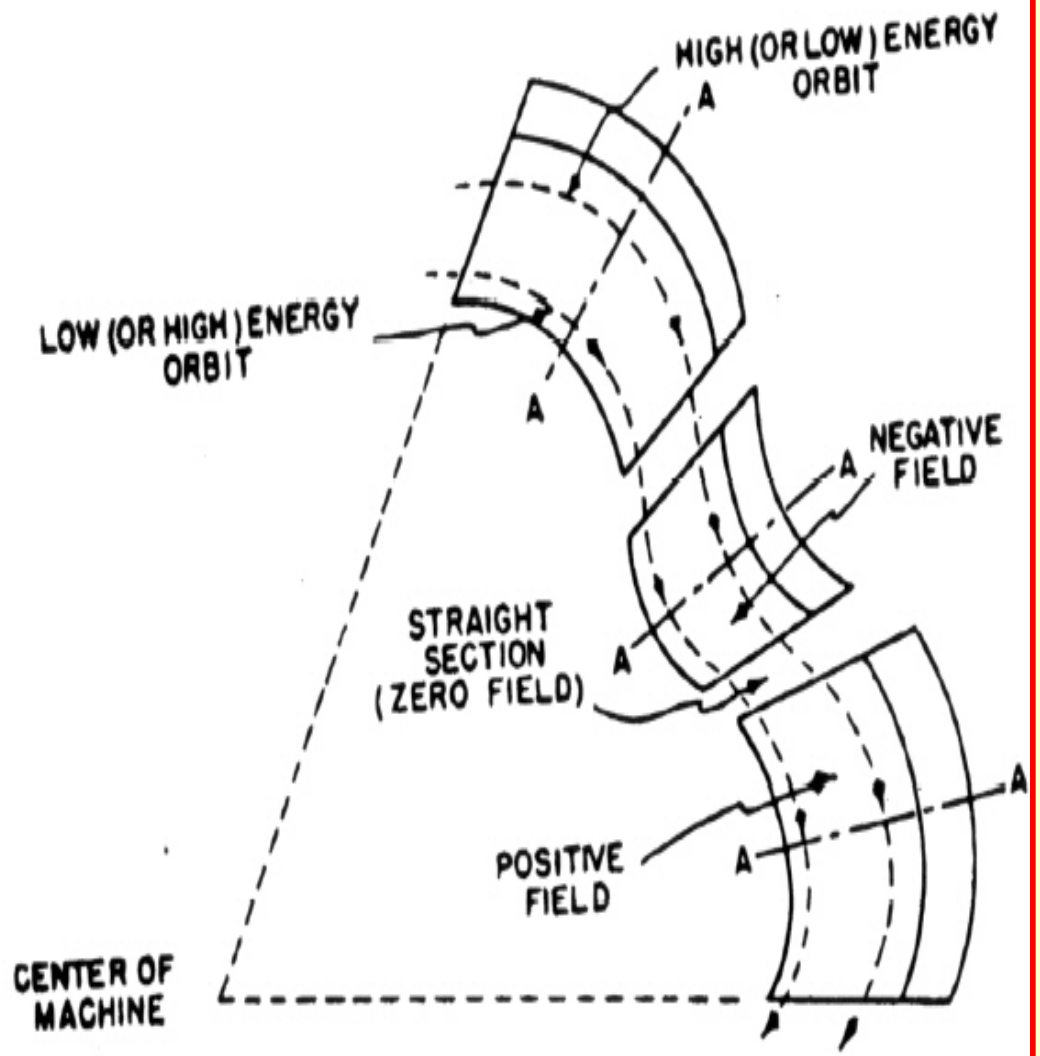
MURA-KRS-6
November 12, 1954
K. R. Symon: The FFAG SYNCHROTRON – MARK I



$$N = 2\pi/(N_1 + N_2)$$

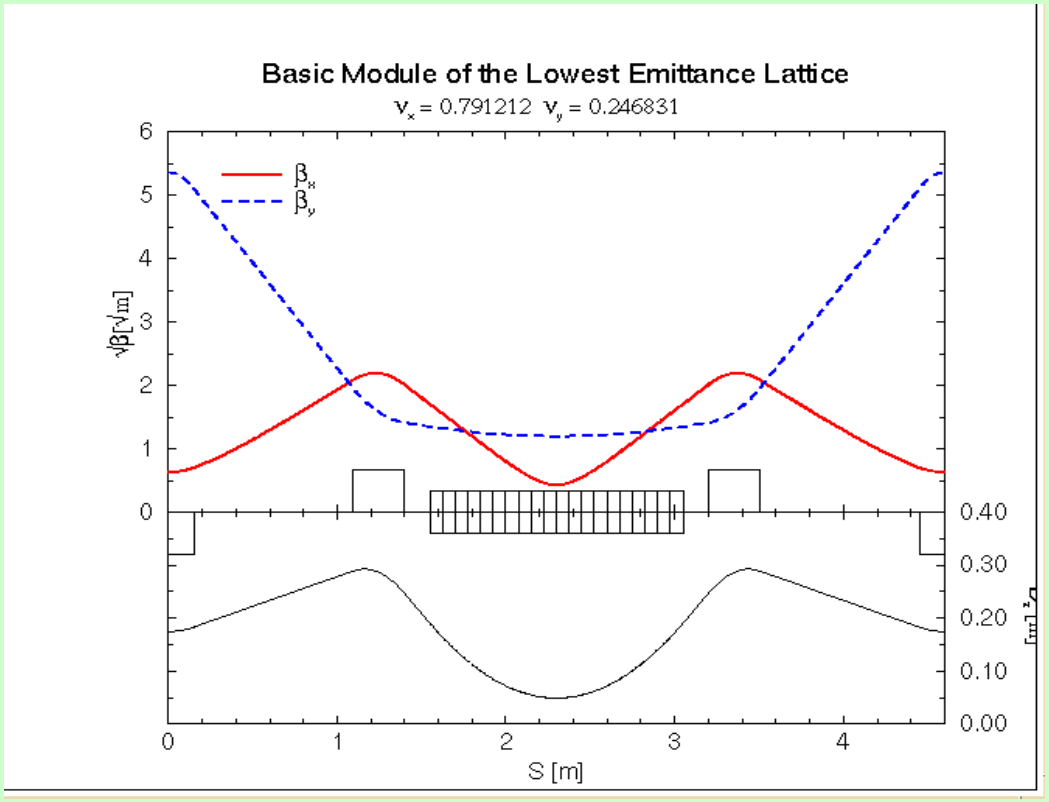
$$\rho = r_1 \sin(\theta/2)/\sin(\beta$$

Figure 1: Closed orbit of a triplet focusing FF
half cell; a half of F magnet, D magnet of one
half straight section, is depicted.



The minimum emittance lattice:

- The minimum emittance lattice requires reduction of the function H:
  - The normalized dispersion amplitude corresponds to the $\langle H \rangle^{1/2}$
  - Conditions are for the minimum of the betatron function β_x and dispersion function D_x to have small values at the middle of the dipole (combined function dipole makes it even smaller).

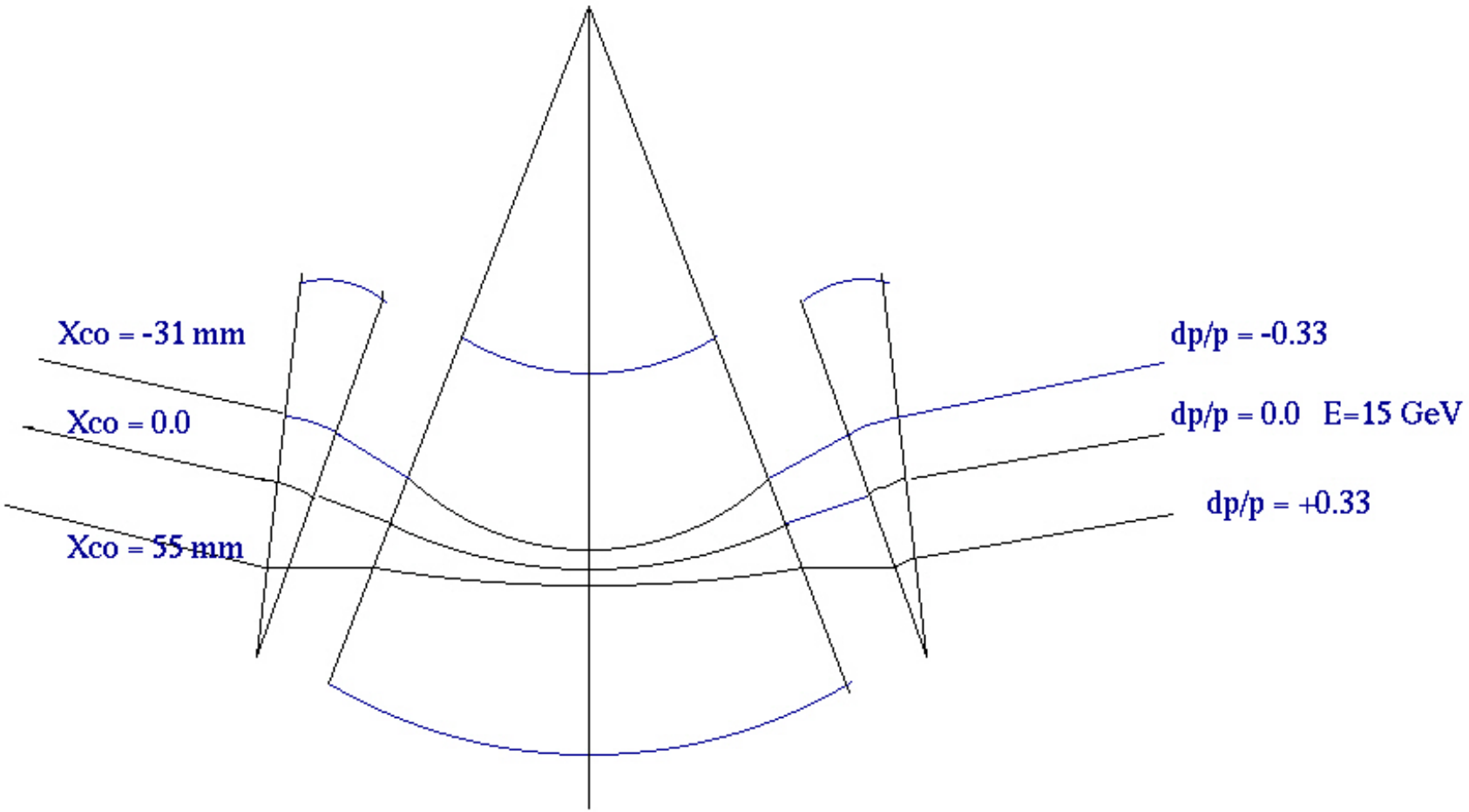


$$\beta_{min} = Ld/2\sqrt{15}$$

$$D_{xmin} = \theta * Ld/24$$

Non Scaling Minimum Emittance FFAG

D. Trbojevic



Scaling or non- scaling FFAG, Minimum emittance lattice or FODO?

Scaling FFAG properties:

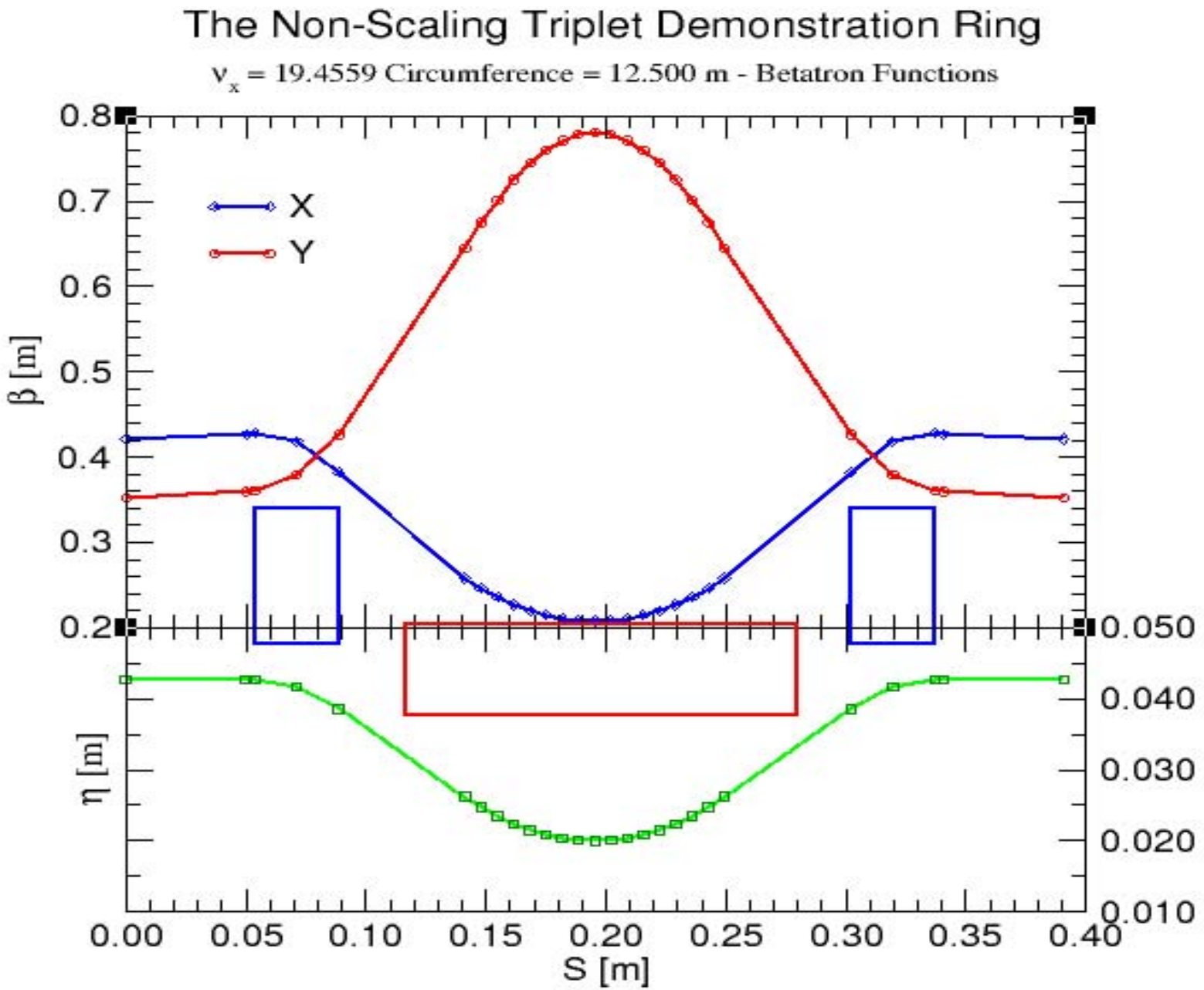
- Zero chromaticity.
- Orbits parallel for different energies.
- Large momentum acceptance.
- Relatively large circumference.
- Relatively large physical aperture.
- RF:large aperture-follows the energy.
- Tunes are fixed for all energies.
- Negative momentum compaction.
- Orbits of the high energy particles are at high field, low energy particles at low field.

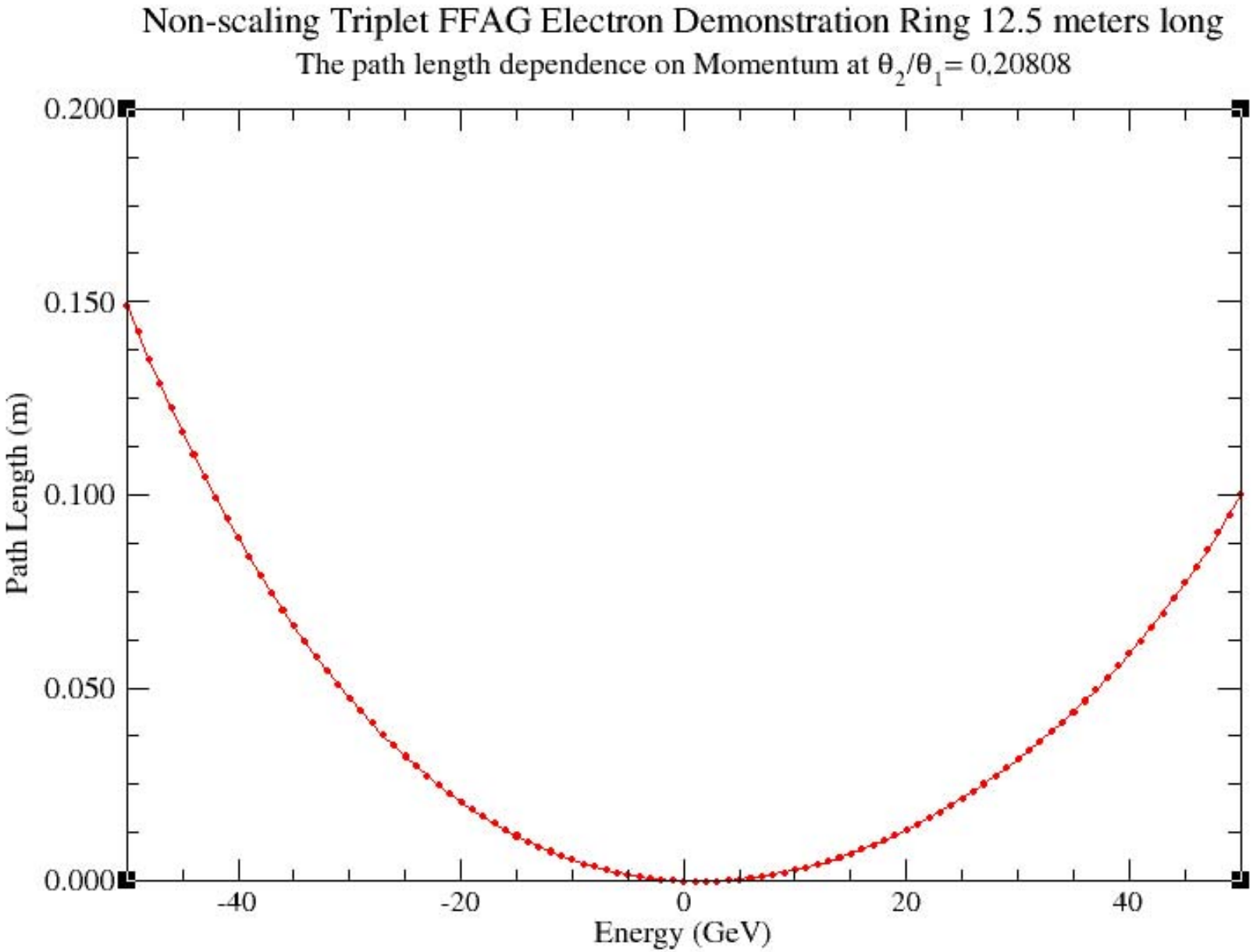
Minimum emittance FFAG properties:

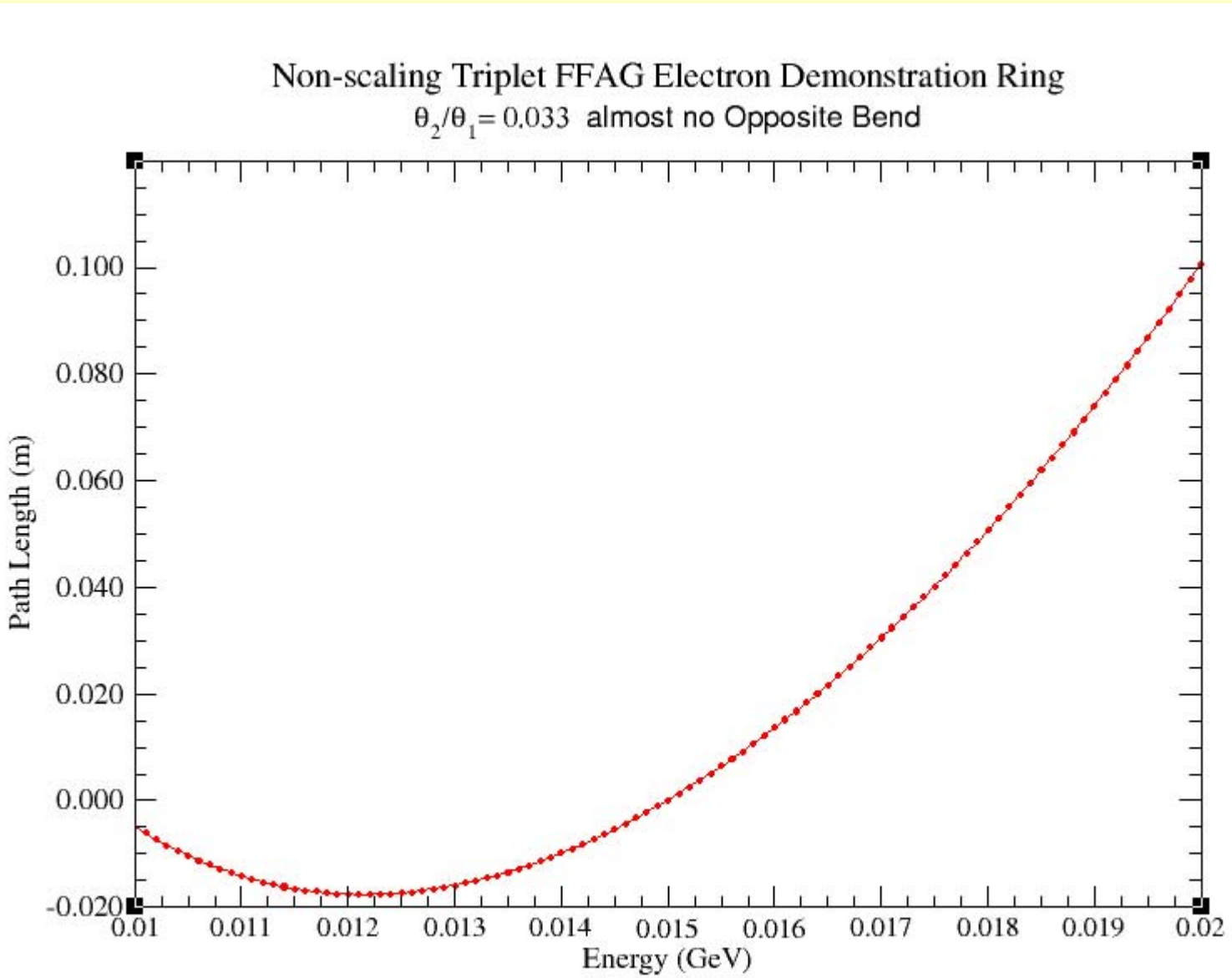
- Chromaticity is changing.
- Orbits not parallel.
- Large momentum acceptance.
- Relatively small circumference.
- Relatively small physical aperture.
- RF:small aperture-at the crest.
- Tunes move 0.4->0.1 in basic cell.
- Momentum compaction changes.
- Orbits of the high energy particles are at high field, low energy particles at low field.

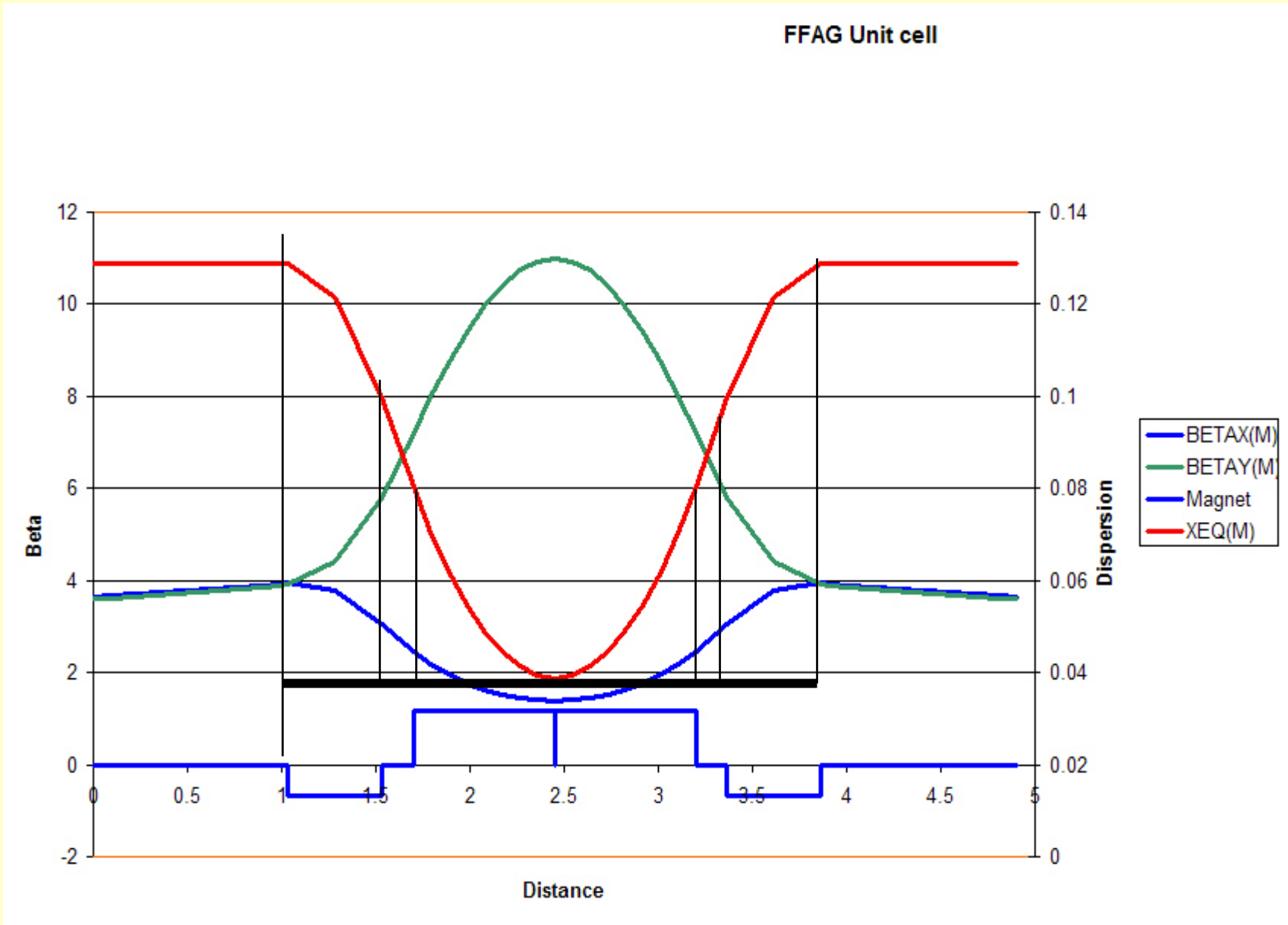
FODO or minimum emittance lattice?

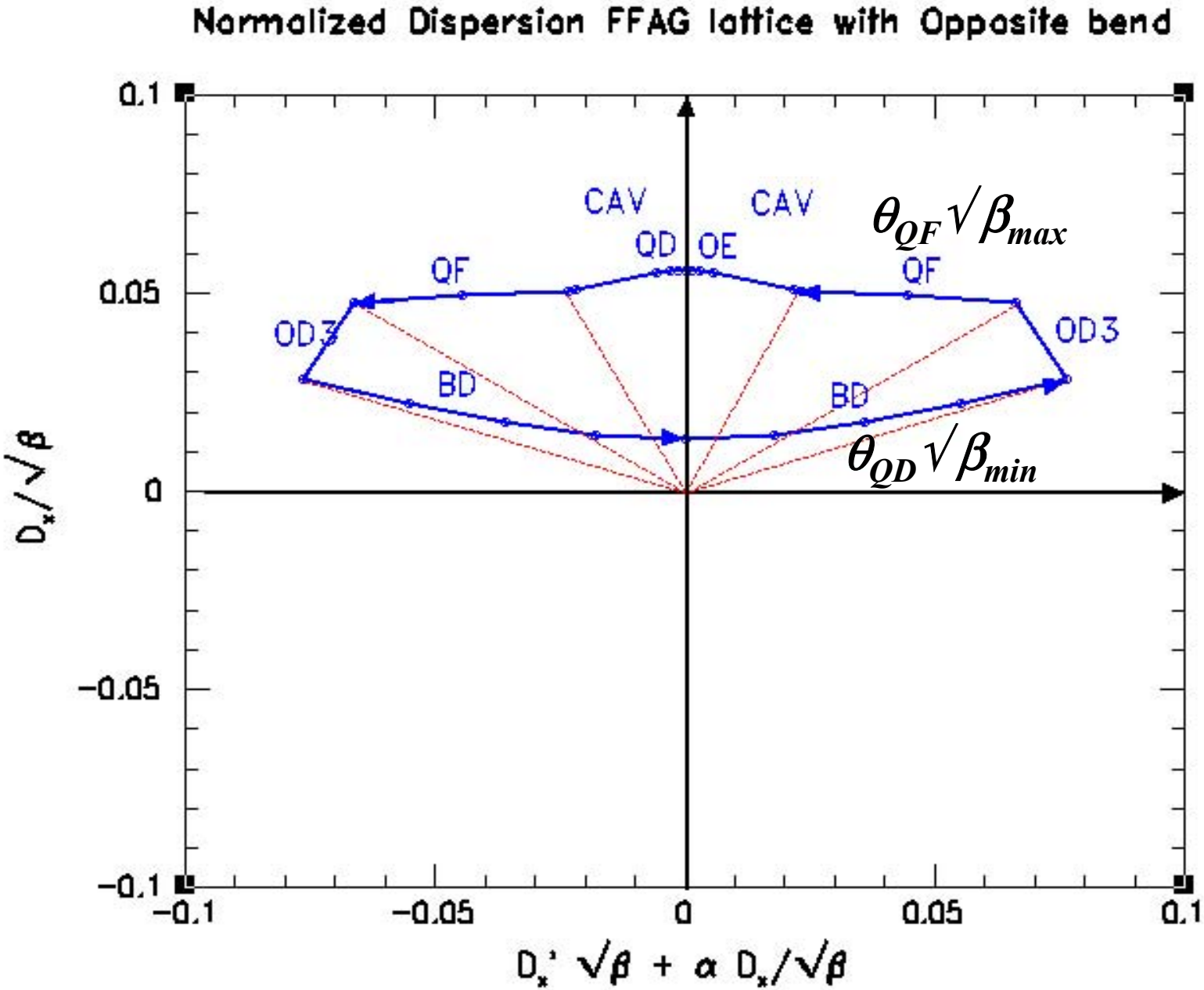
- For the same magnet properties larger circumference and larger X_{co}.
- For the same dispersion [$\Delta x = D_x * dp/p$] and the same magnet smaller field and larger circumference.
- The FODO has larger available free space.

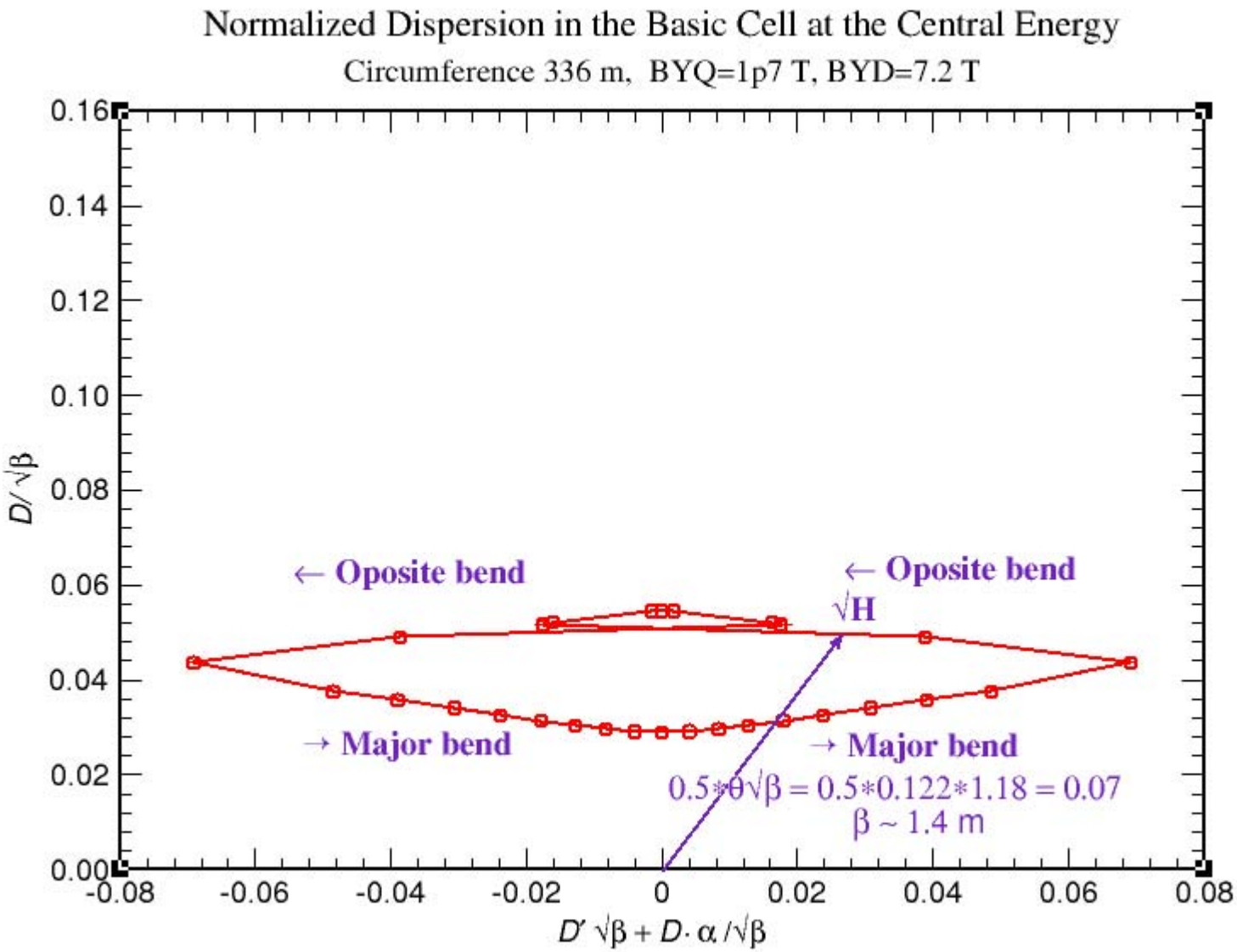

The Non-Scaling Triplet Demonstration Ring
ν_x = 19.4559 Circumference = 12.500 m - Betatron Functions
X
Y
β [m]
η [m]
S [m]
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.050
0.040
0.030
0.020
0.010
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40


Non-scaling Triplet FFAG Electron Demonstration Ring 12.5 meters long
The path length dependence on Momentum at θ2/θ1= 0.20808
0.200
0.150
0.100
0.050
0.000
Path Length (m)
-40
-20
0
20
40
Energy (GeV)


Non-scaling Triplet FFAG Electron Demonstration Ring
θ2/θ1= 0.033 almost no Opposite Bend
Path Length (m)
0.100
0.080
0.060
0.040
0.020
0.000
-0.020
0.01
0.011
0.012
0.013
0.014
0.015
0.016
0.017
0.018
0.019
0.02
Energy (GeV)

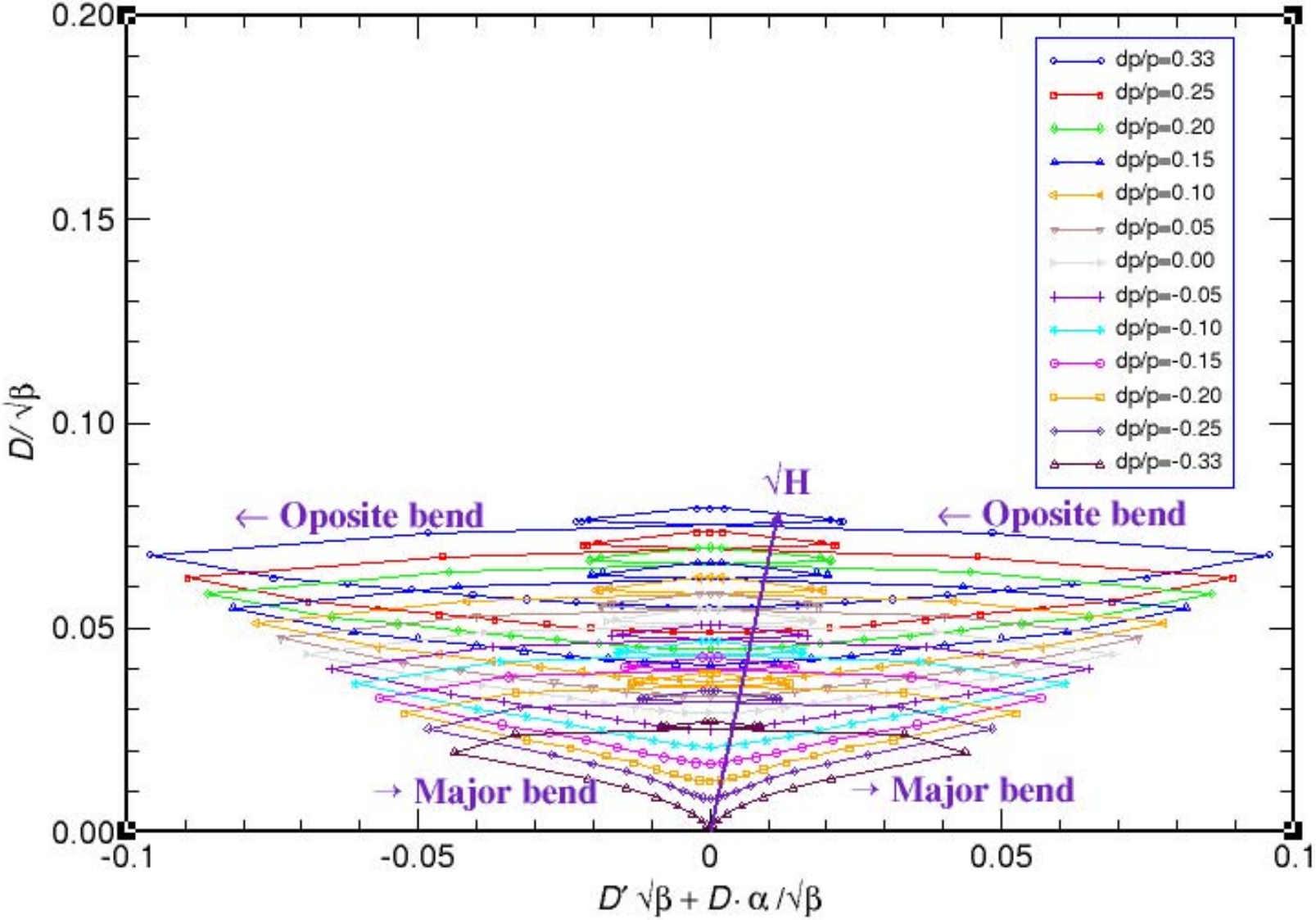

FFAG Unit cell
Beta
Dispersion
Distance
12
10
8
6
4
2
0
-2
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
BETAX(M)
BETAY(M)
Magnet
XEQ(M)

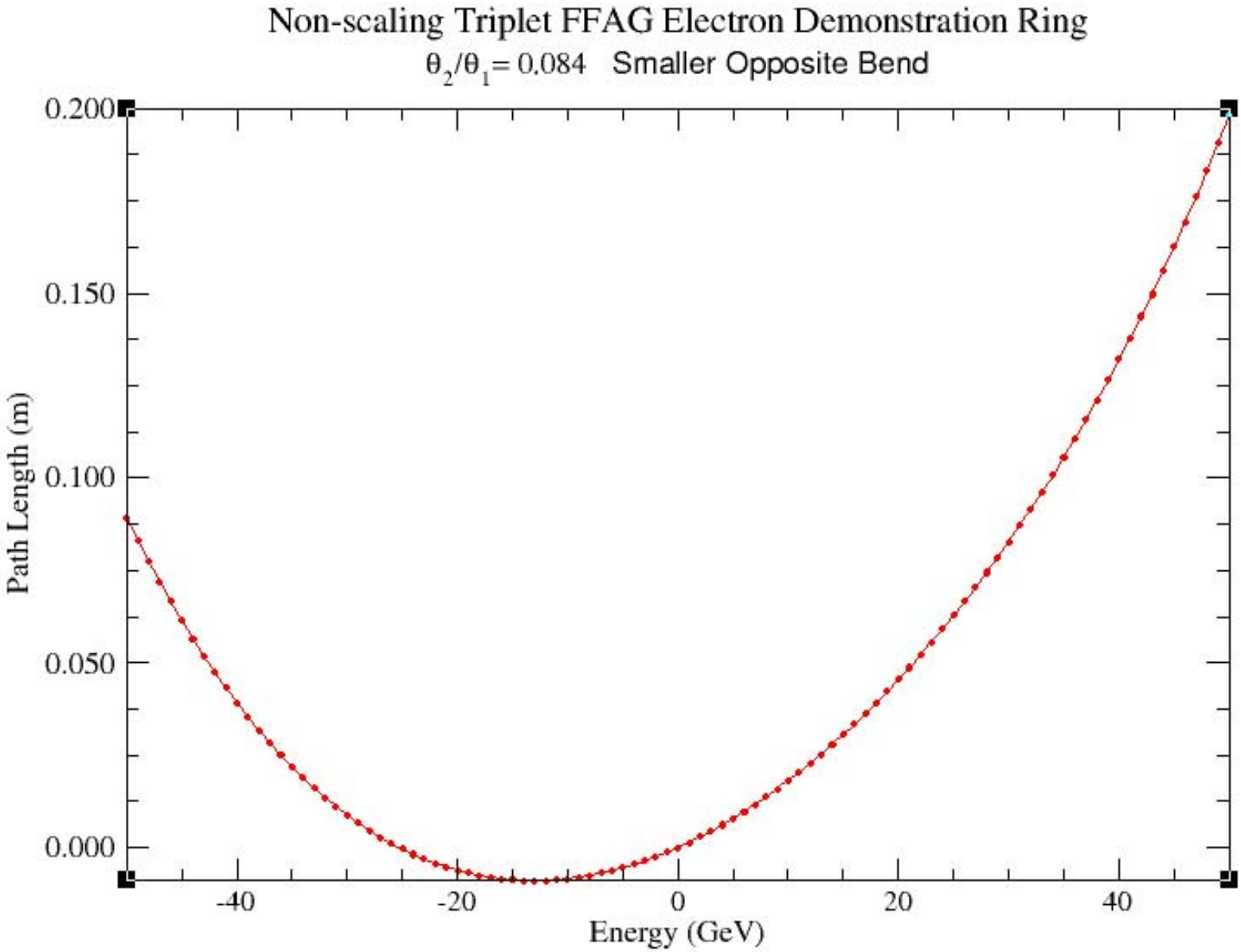

Normalized Dispersion FFAG lattice with Opposite bend
CAV
CAV
QD
OE
QF
QF
OD3
OD3
BD
BD
$\theta_{QF}\sqrt{\beta_{max}}$
$\theta_{QD}\sqrt{\beta_{min}}$
0.1
0.05
0
−0.05
−0.1
−0.1
−0.05
0
0.05
0.1
$D_x/\sqrt{\beta}$
$D_x'\sqrt{\beta} + \alpha D_x/\sqrt{\beta}$

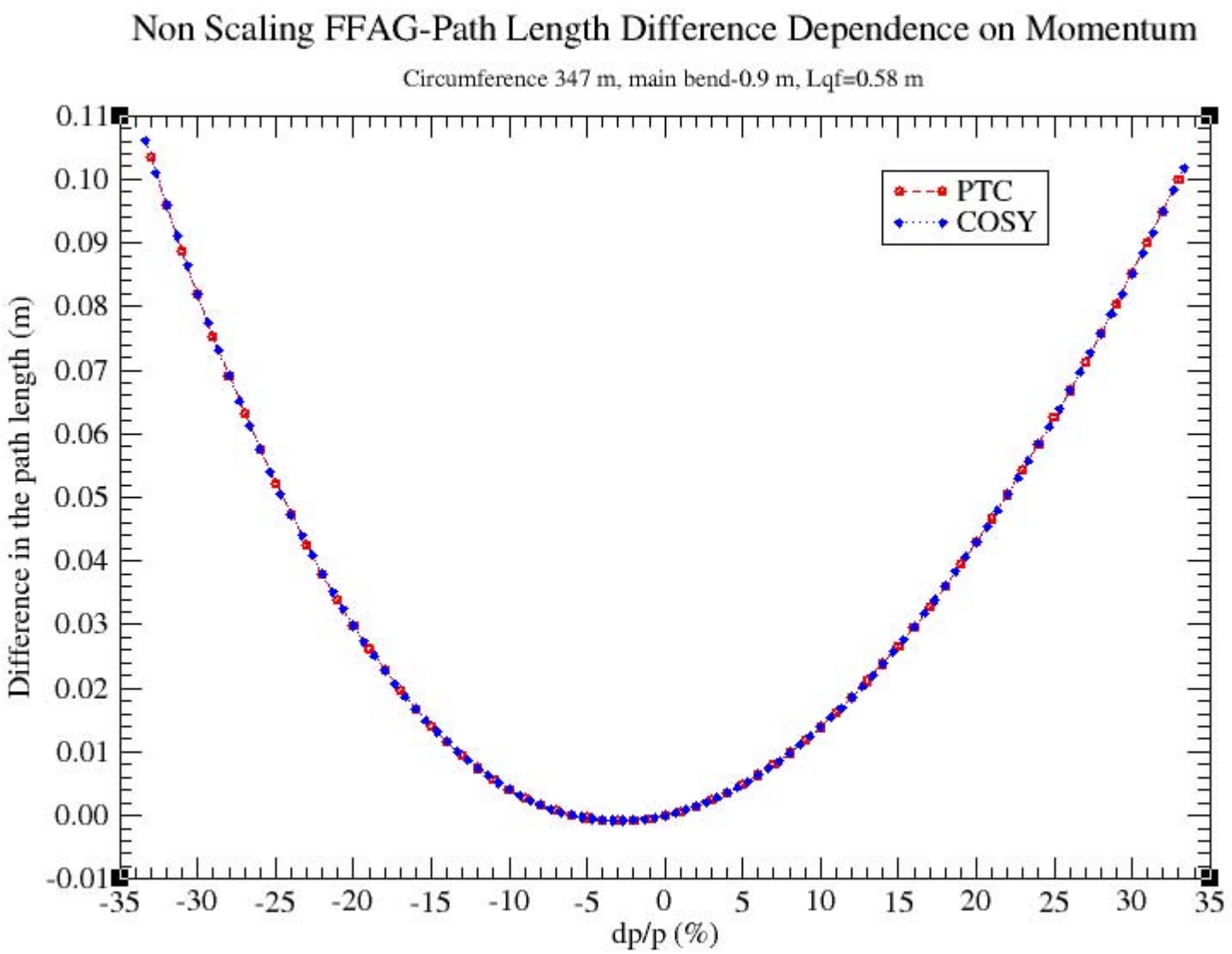

Normalized Dispersion in the Basic Cell at the Central Energy
Circumference 336 m, BYQ=1p7 T, BYD=7.2 T
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
D/√β
← Oposite bend
← Oposite bend
√H
→ Major bend
→ Major bend
0.5*θ√β = 0.5*0.122*1.18 = 0.07
β ~ 1.4 m
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
D′ √β + D· α /√β

Normalized Dispersion in the Basic Cell During Acceleration

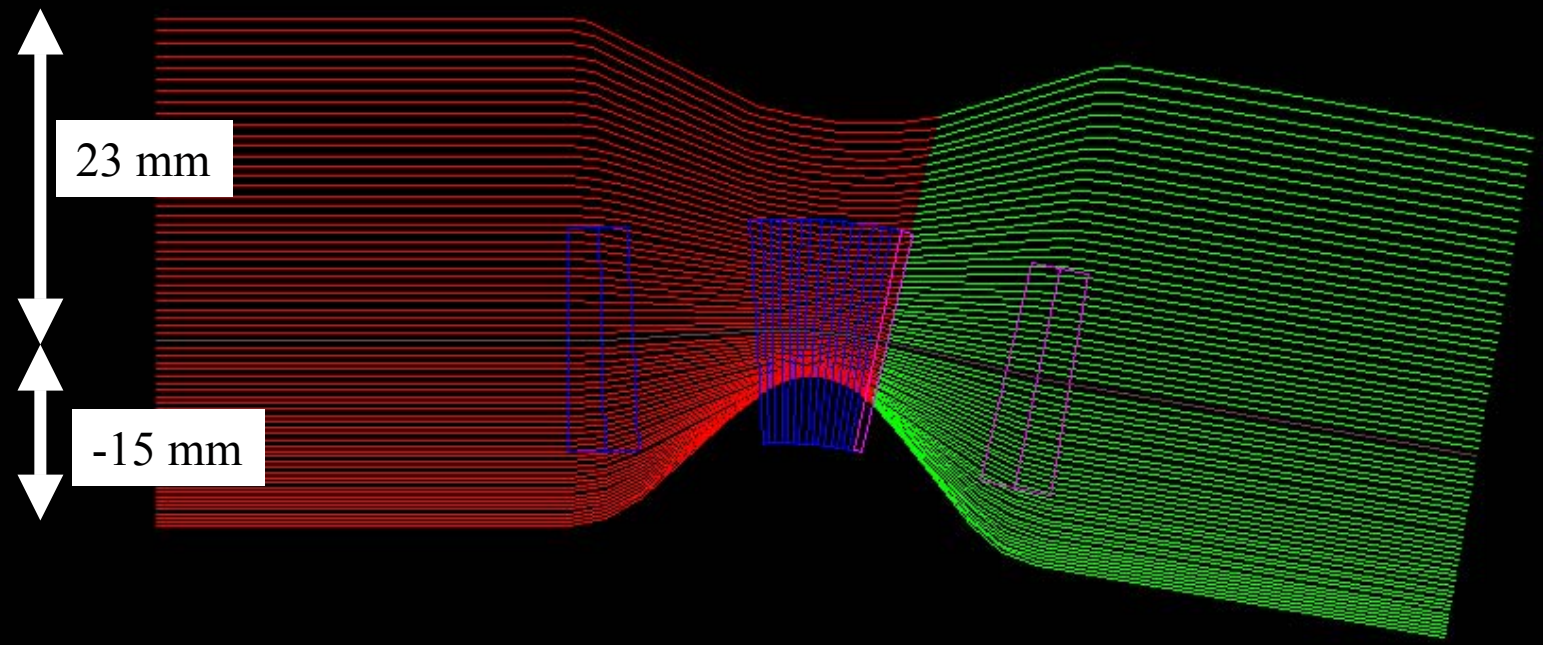
Circumference 336 m, BYQ=1p7 T, BYD=7.2 T

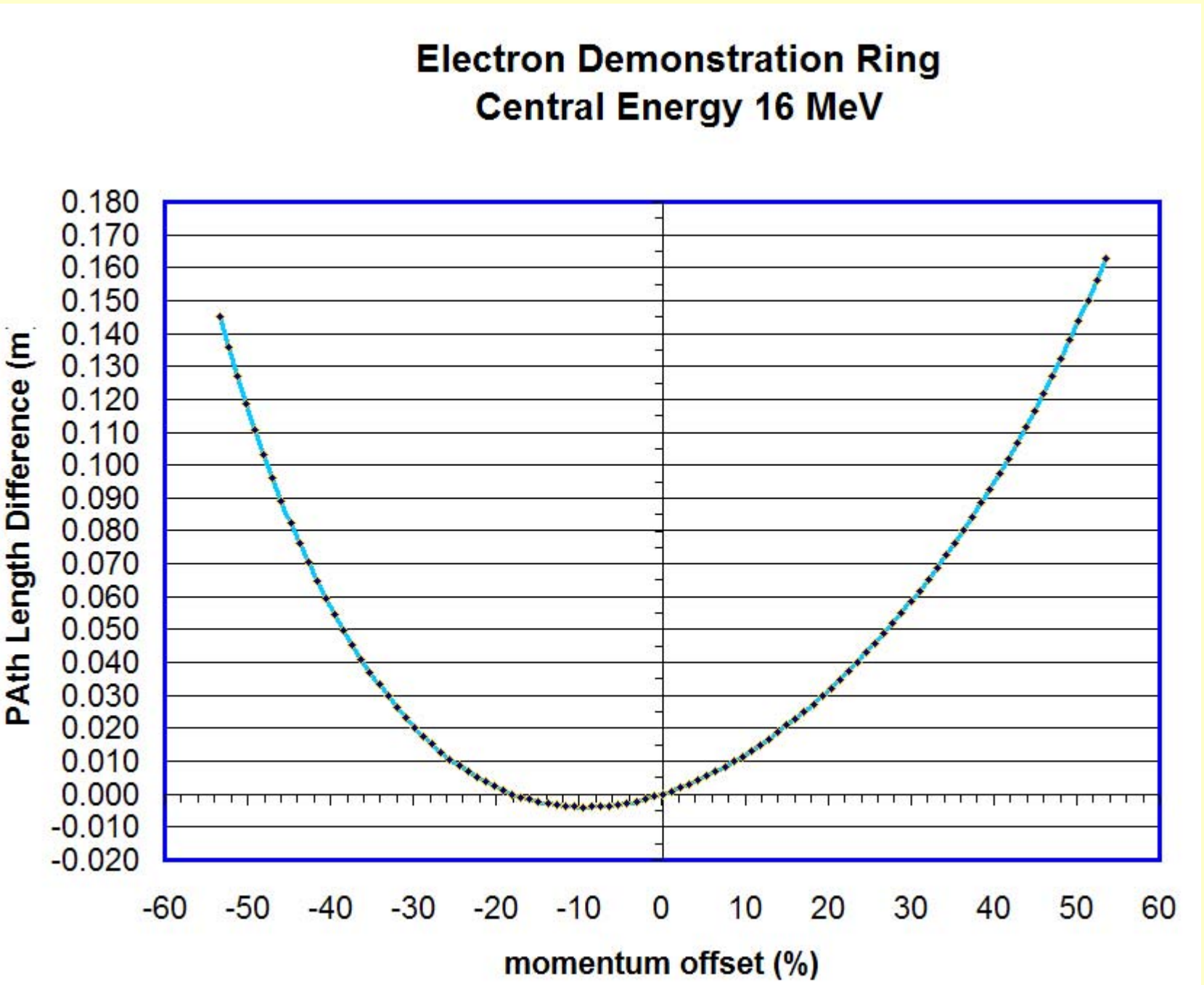


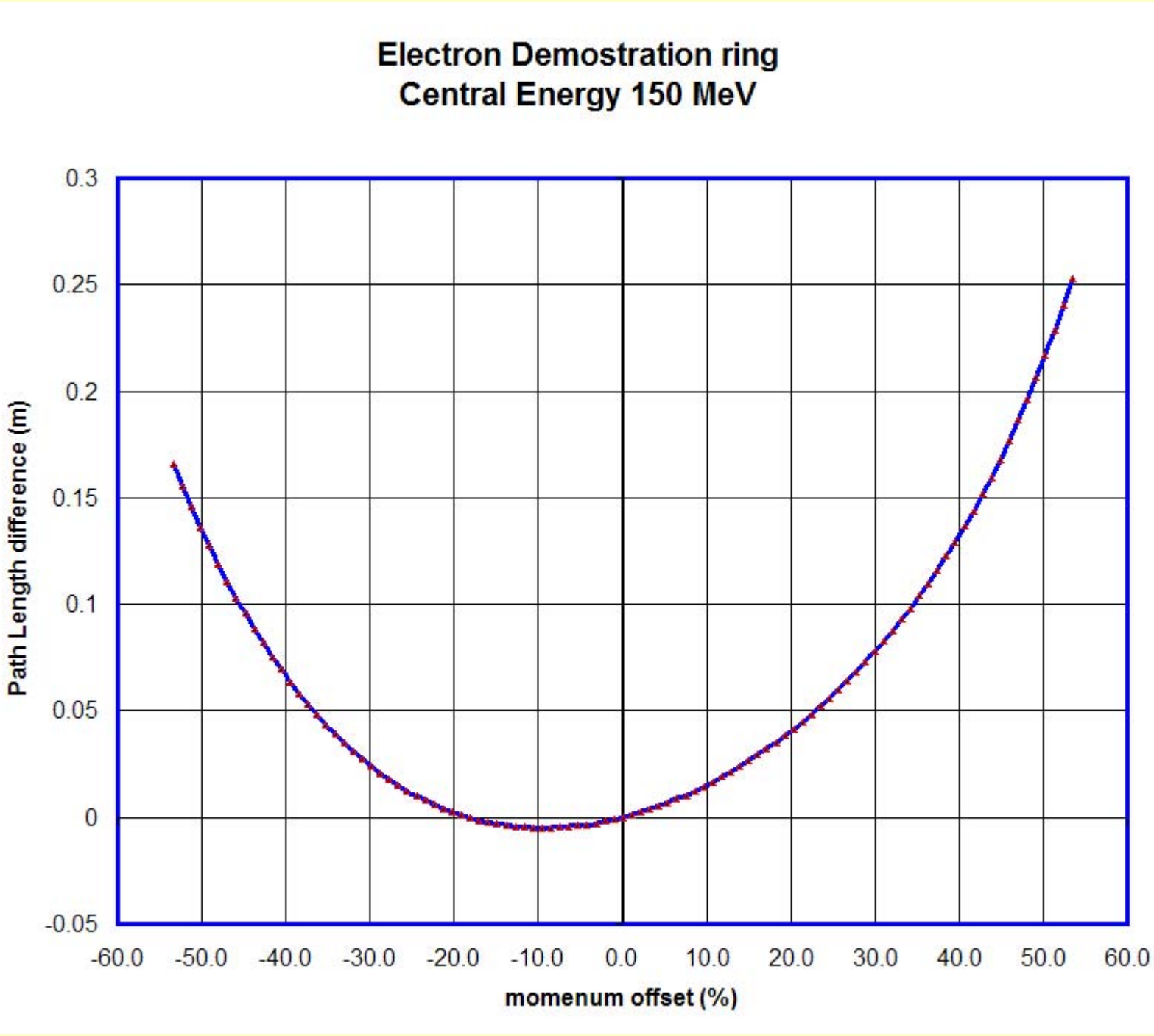

Non-scaling Triplet FFAG Electron Demonstration Ring
θ2/θ1 = 0.084 Smaller Opposite Bend
0.200
0.150
0.100
0.050
0.000
Path Length (m)
-40
-20
0
20
40
Energy (GeV)

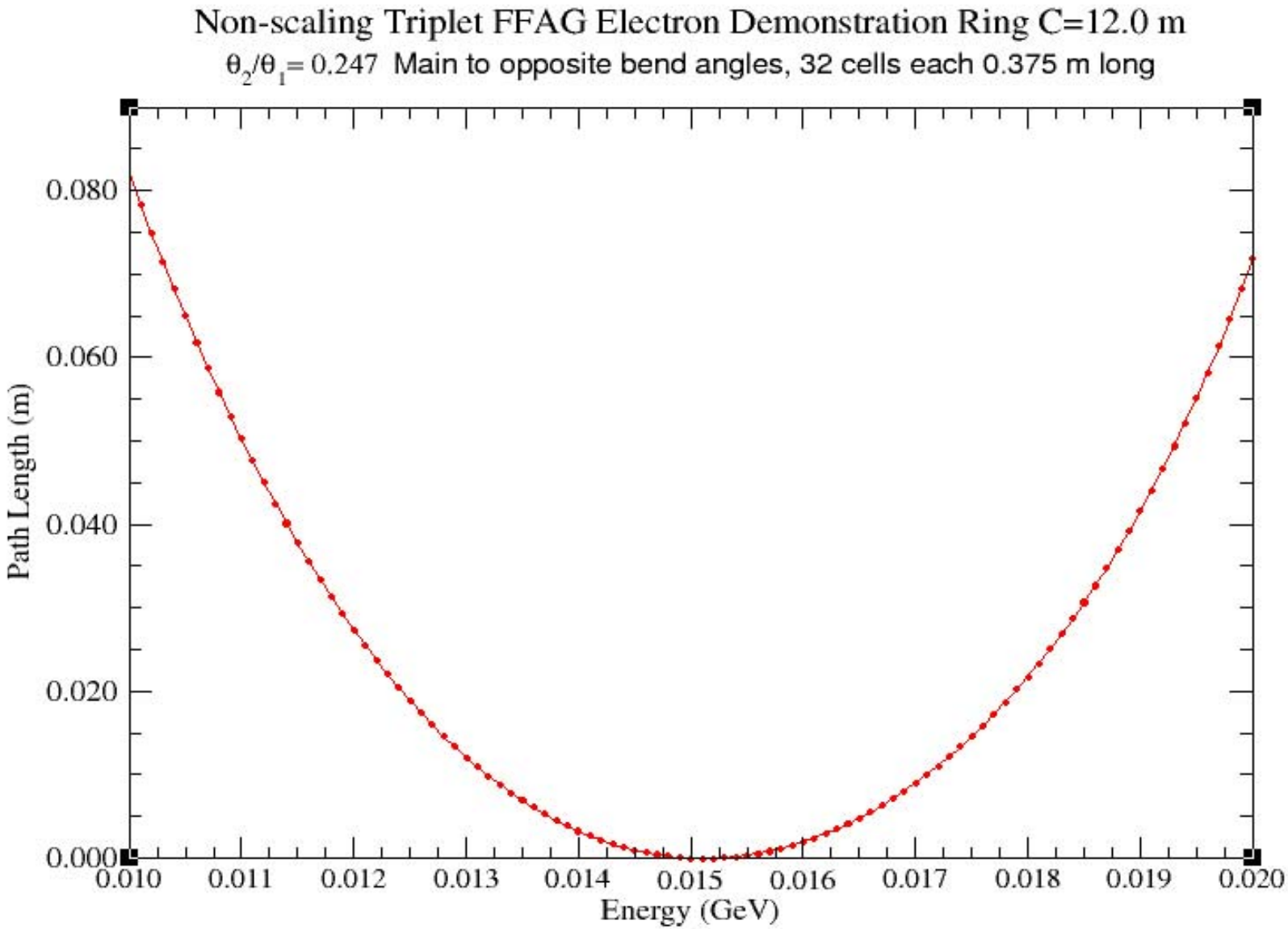

Non Scaling FFAG-Path Length Difference Dependence on Momentum
Circumference 347 m, main bend-0.9 m, Lqf=0.58 m
PTC
COSY
Difference in the path length (m)
dp/p (%)
0.11
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
-0.01
-35
-30
-25
-20
-15
-10
-5
0
5
10
15
20
25
30
35

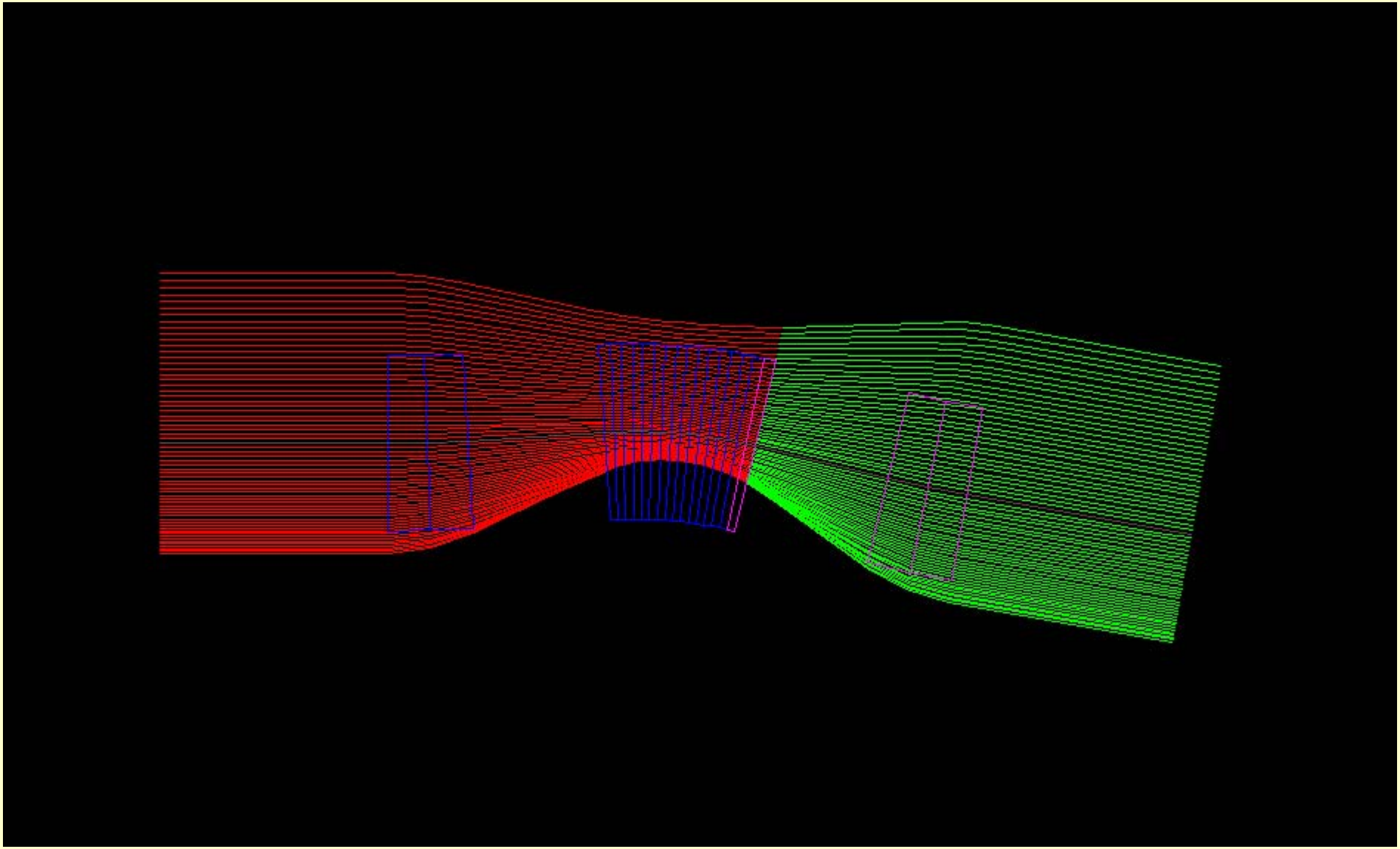
BNL workshop produced 13m circumference
Electron Demonstration non-scaling FFAG



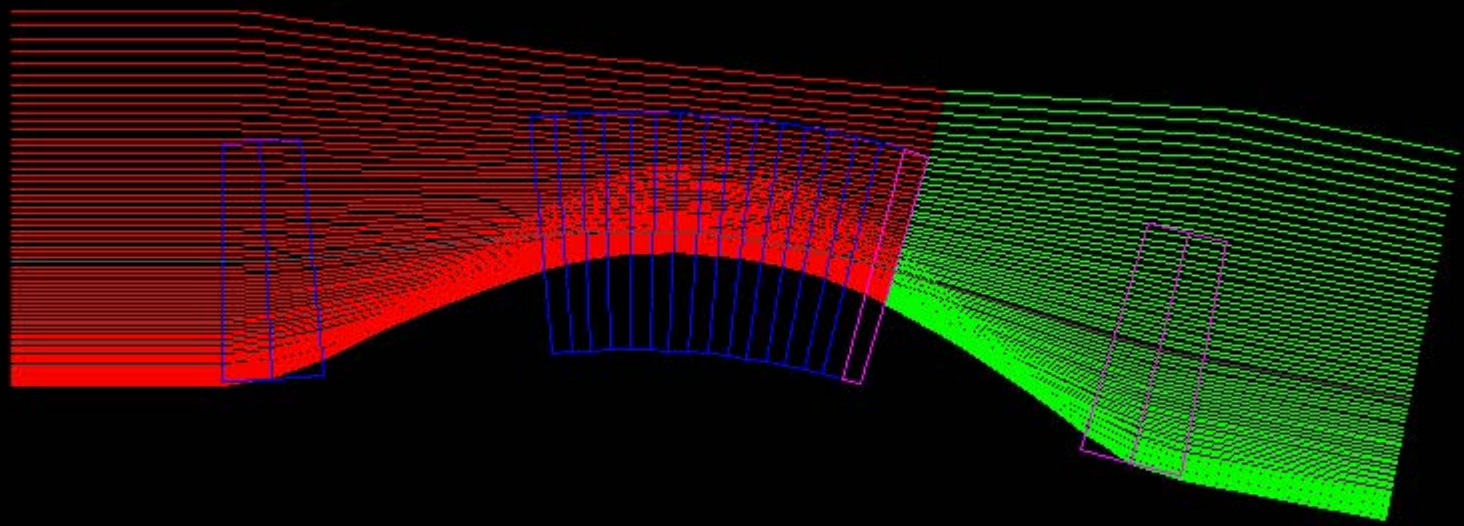

Electron Demonstration Ring
Central Energy 16 MeV
PAth Length Difference (m
0.180
0.170
0.160
0.150
0.140
0.130
0.120
0.110
0.100
0.090
0.080
0.070
0.060
0.050
0.040
0.030
0.020
0.010
0.000
-0.010
-0.020
-60
-50
-40
-30
-20
-10
0
10
20
30
40
50
60
momentum offset (%)


Electron Demostration ring
Central Energy 150 MeV
Path Length difference (m)
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-60.0
-50.0
-40.0
-30.0
-20.0
-10.0
0.0
10.0
20.0
30.0
40.0
50.0
60.0
momenum offset (%)

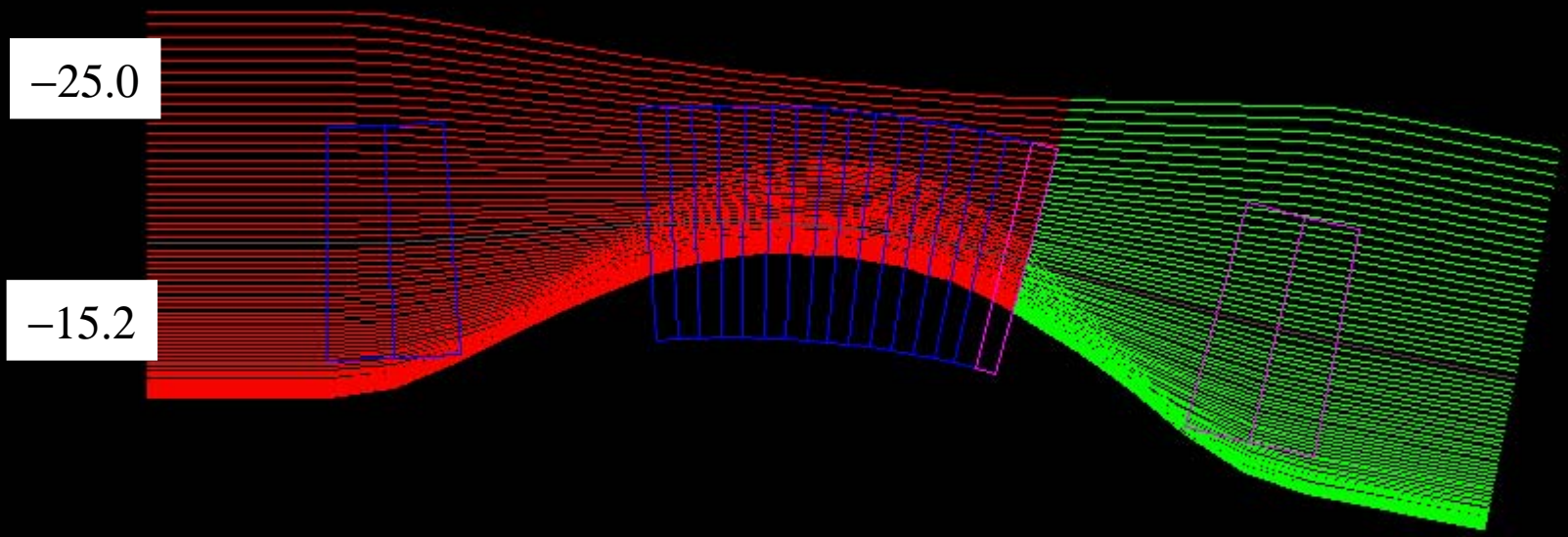

Non-scaling Triplet FFAG Electron Demonstration Ring C=12.0 m
θ2/θ1= 0.247 Main to opposite bend angles, 32 cells each 0.375 m long
Path Length (m)
0.080
0.060
0.040
0.020
0.000
0.010
0.011
0.012
0.013
0.014
0.015
0.016
0.017
0.018
0.019
0.020
Energy (GeV)



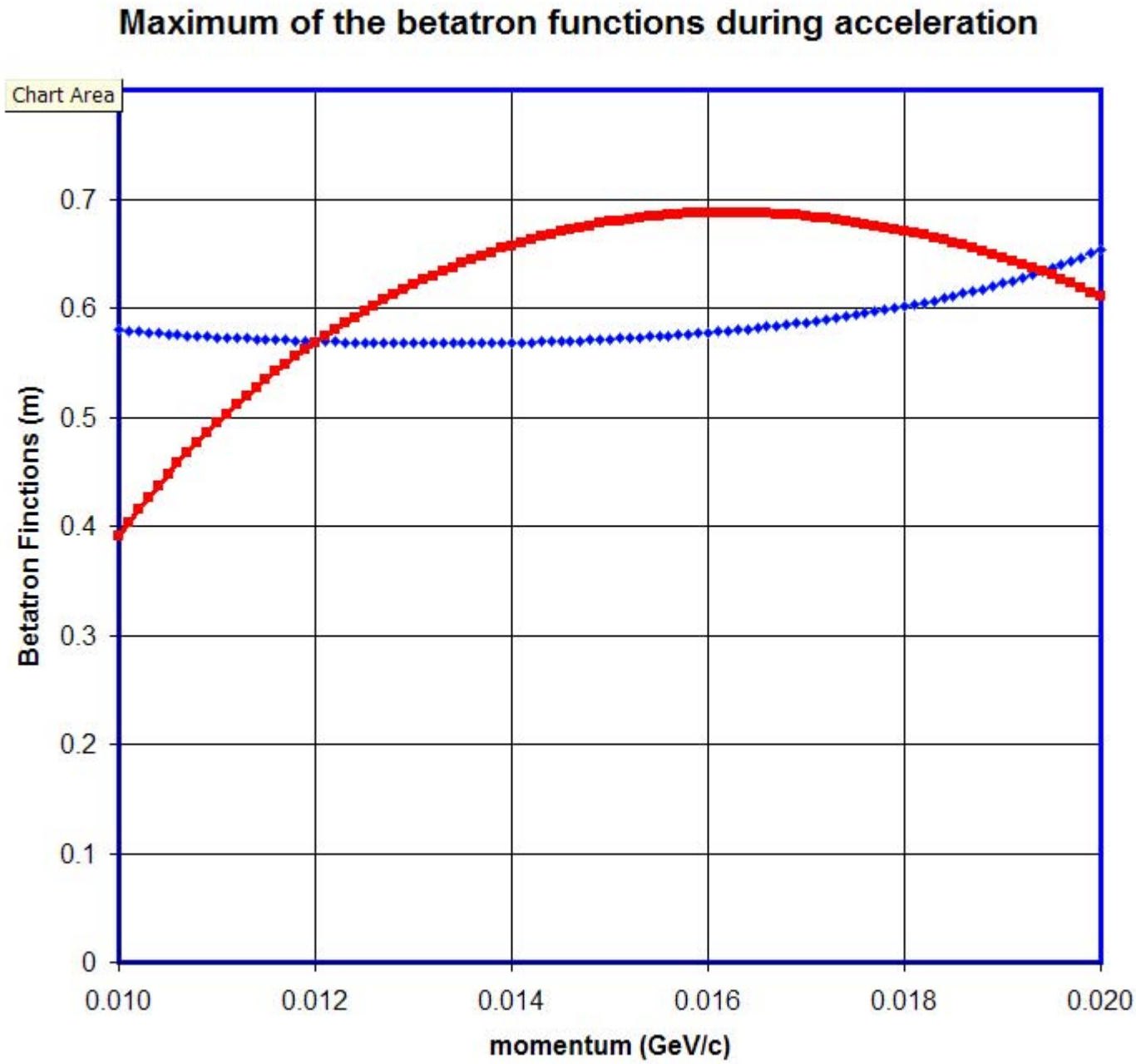
Circumference 12 m

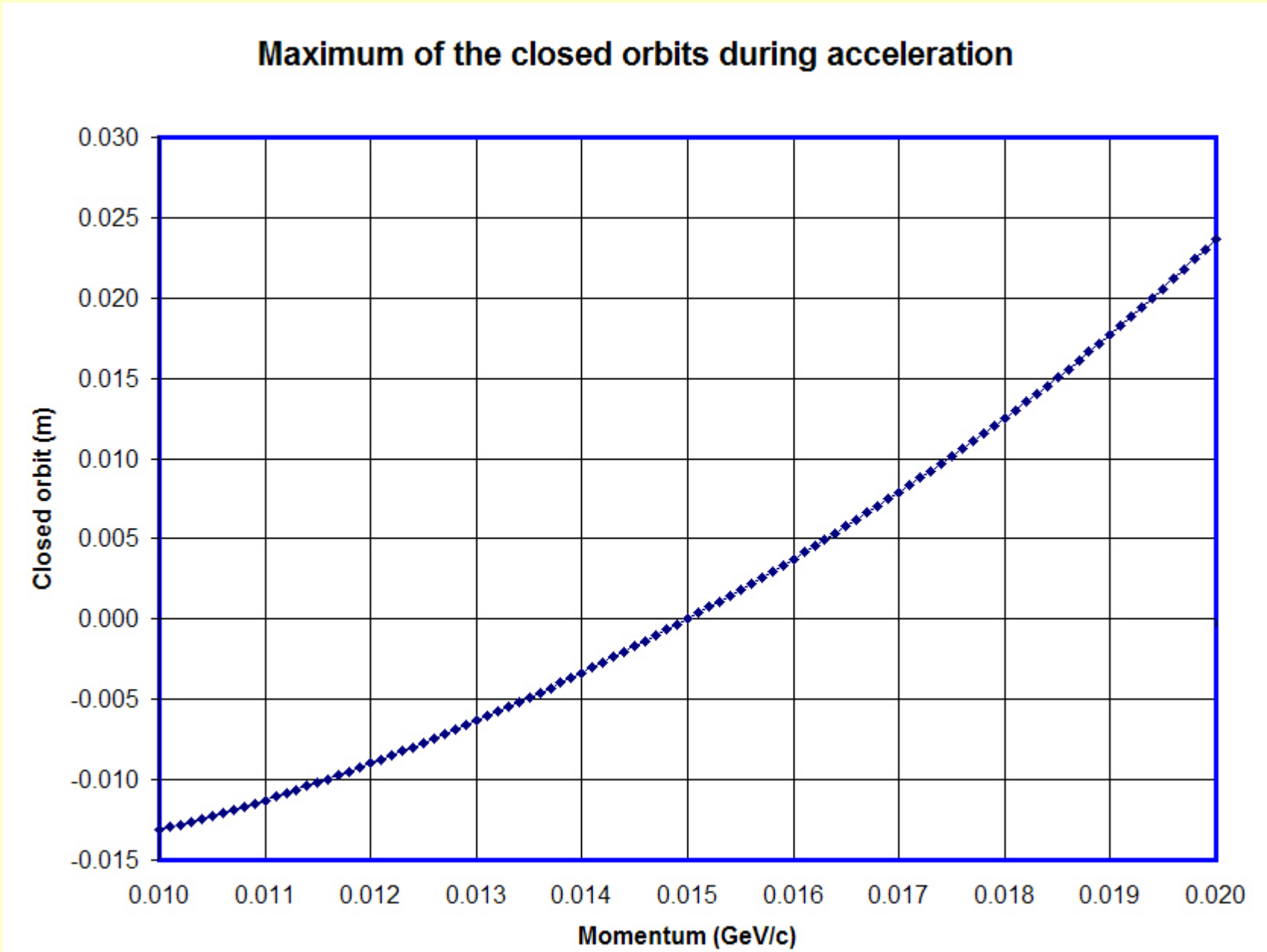


Circumference 12.5 m

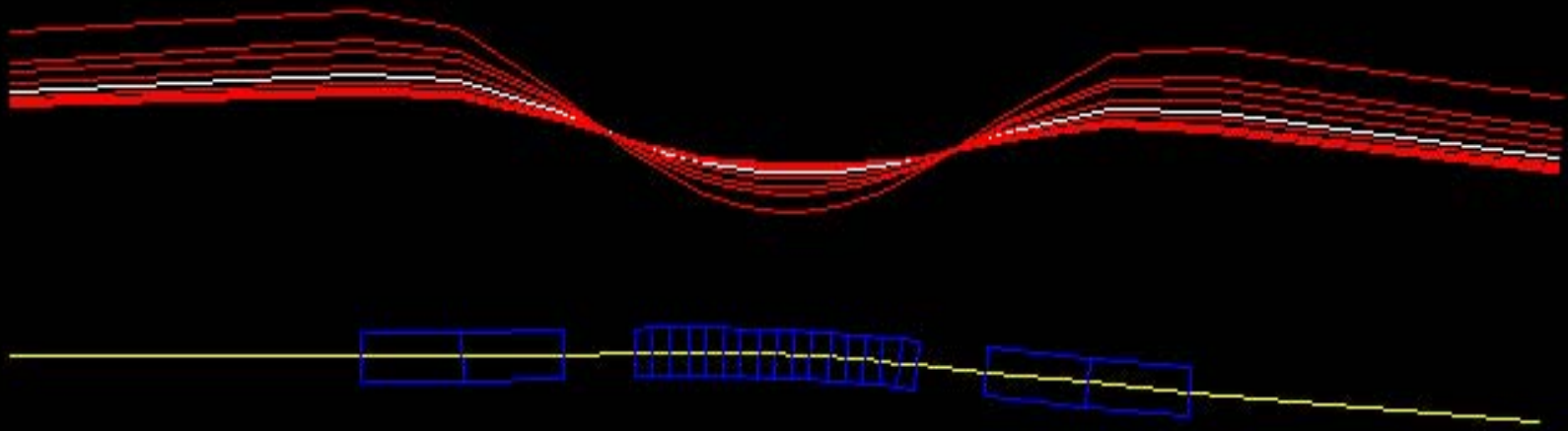


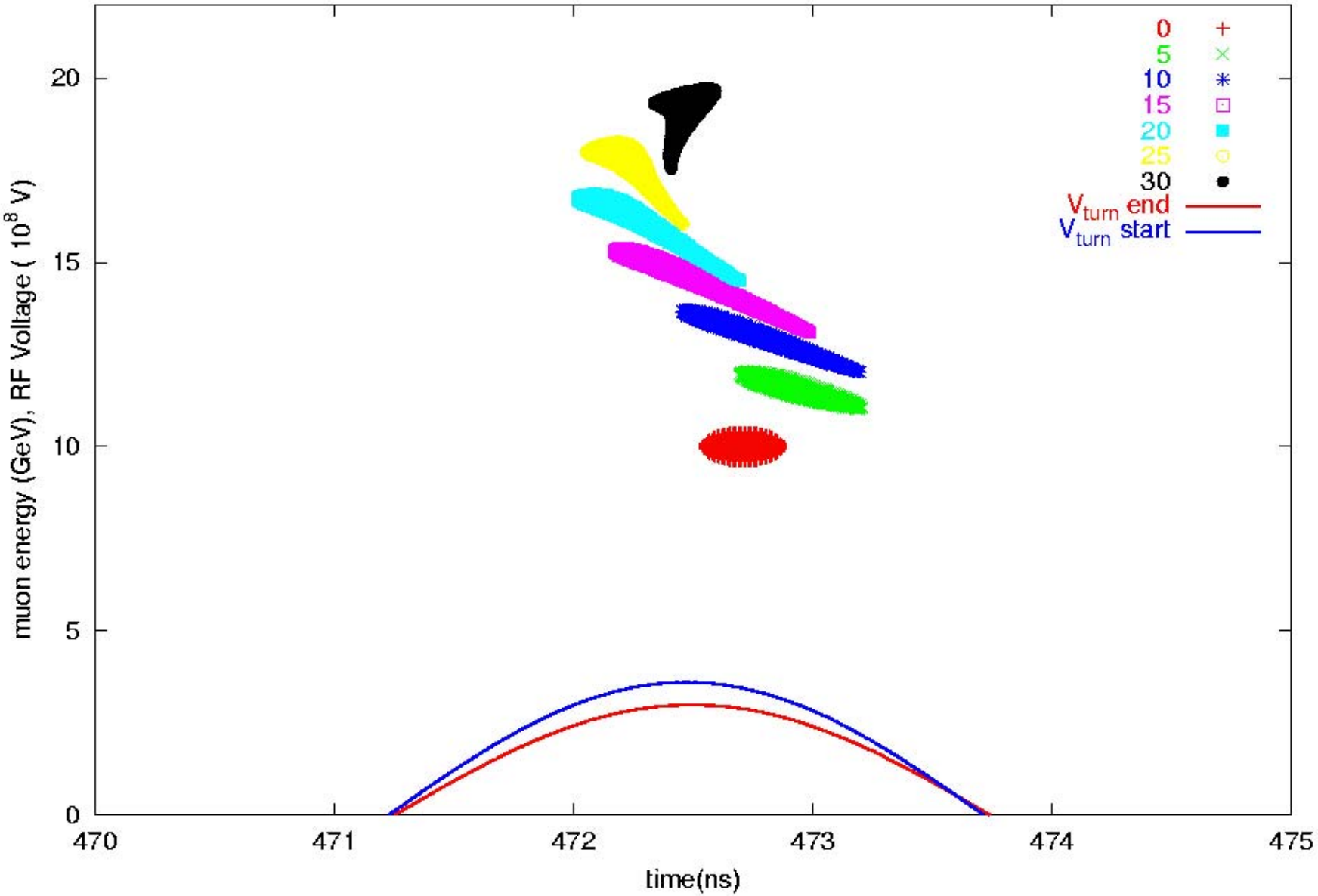
Electron Demonstration Ring non-scaling FFAG



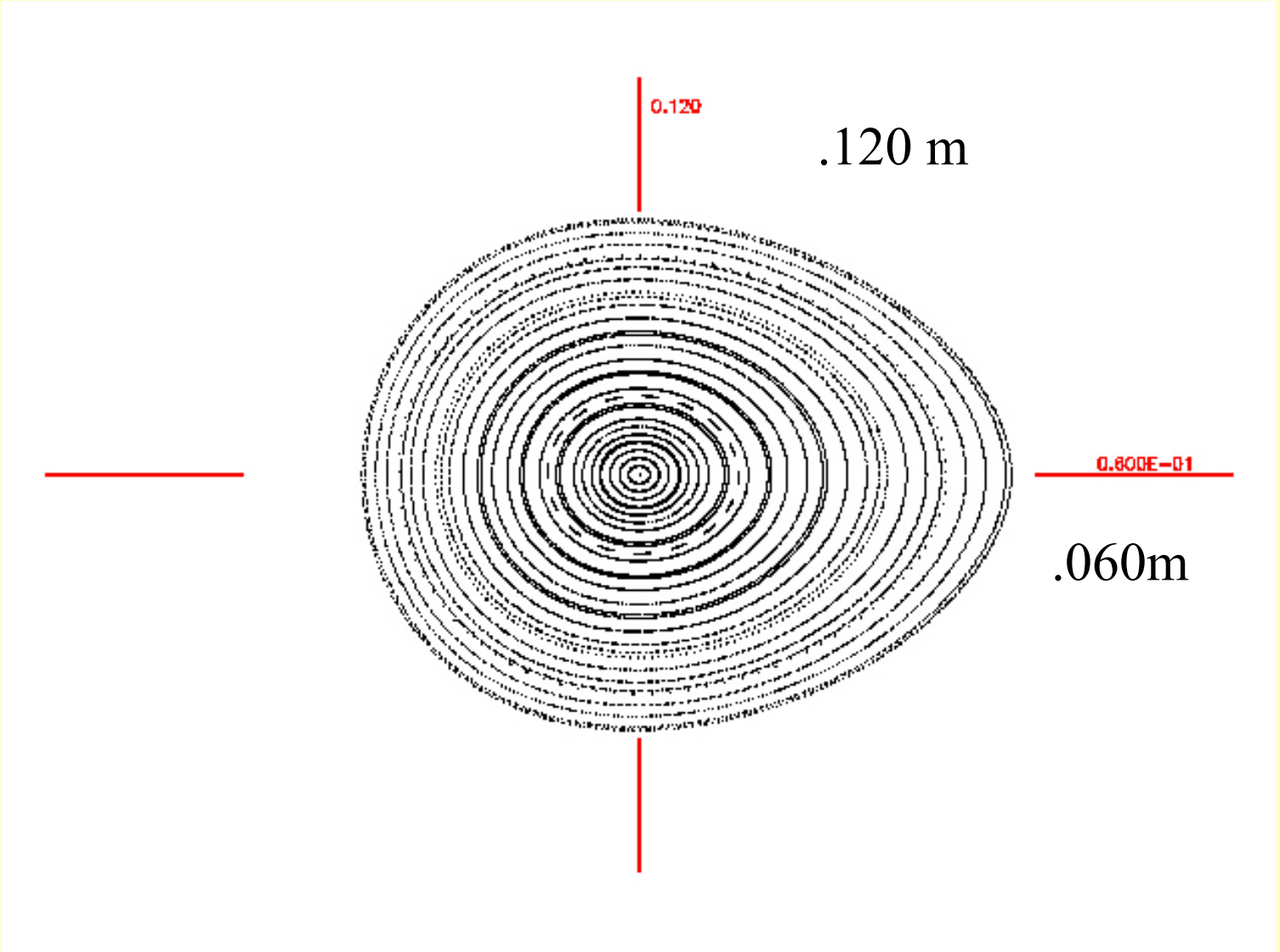

Maximum of the closed orbits during acceleration
Closed orbit (m)
0.030
0.025
0.020
0.015
0.010
0.005
0.000
-0.005
-0.010
-0.015
0.010
0.011
0.012
0.013
0.014
0.015
0.016
0.017
0.018
0.019
0.020
Momentum (GeV/c)

Betatron Function During Acceleration

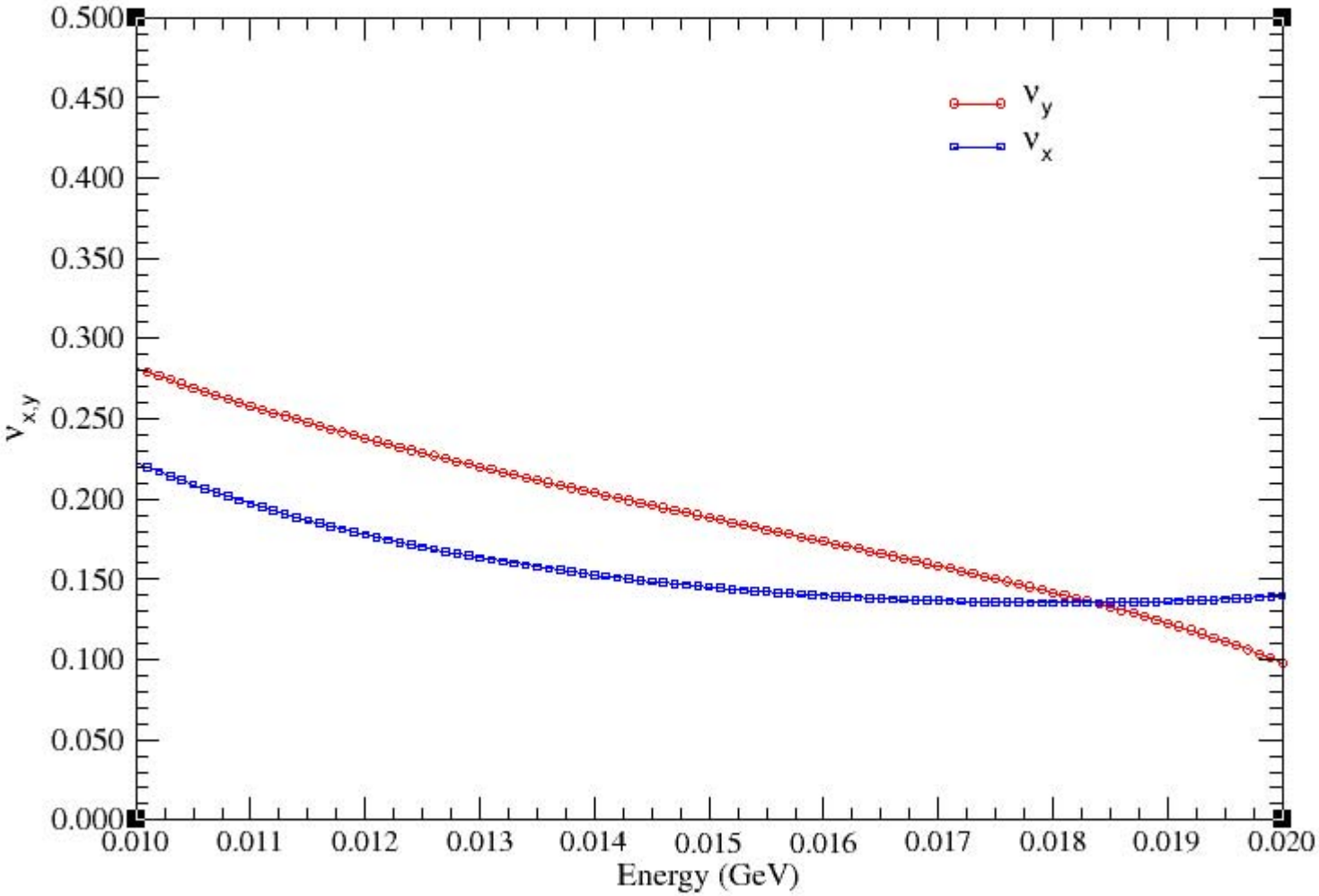


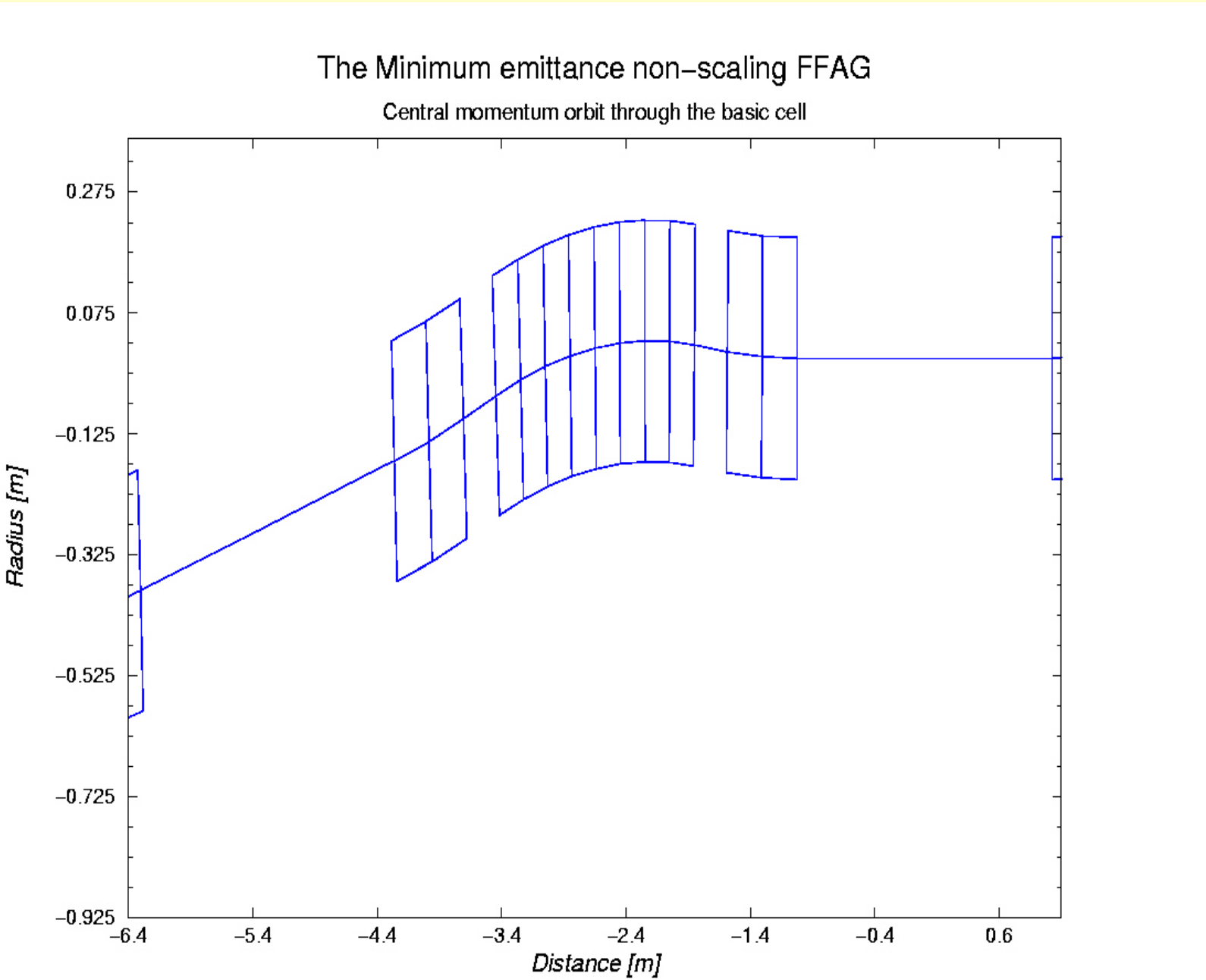

0
5
10
15
20
25
30
V_{turn} end
V_{turn} start
muon energy (GeV), RF Voltage (10^8 V)
time(ns)
0
5
10
15
20
470
471
472
473
474
475

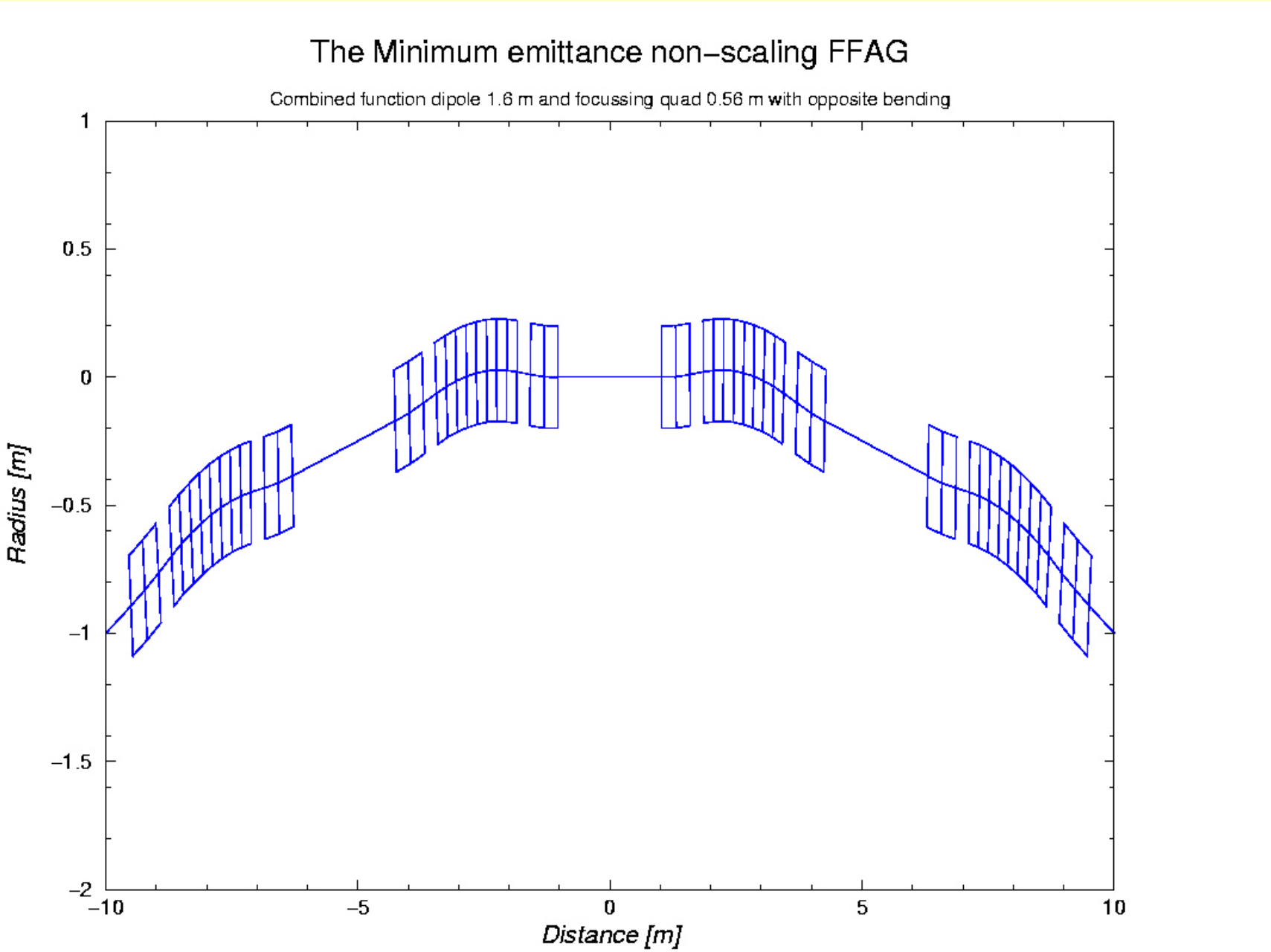
Tracking at the central energy
With COSY

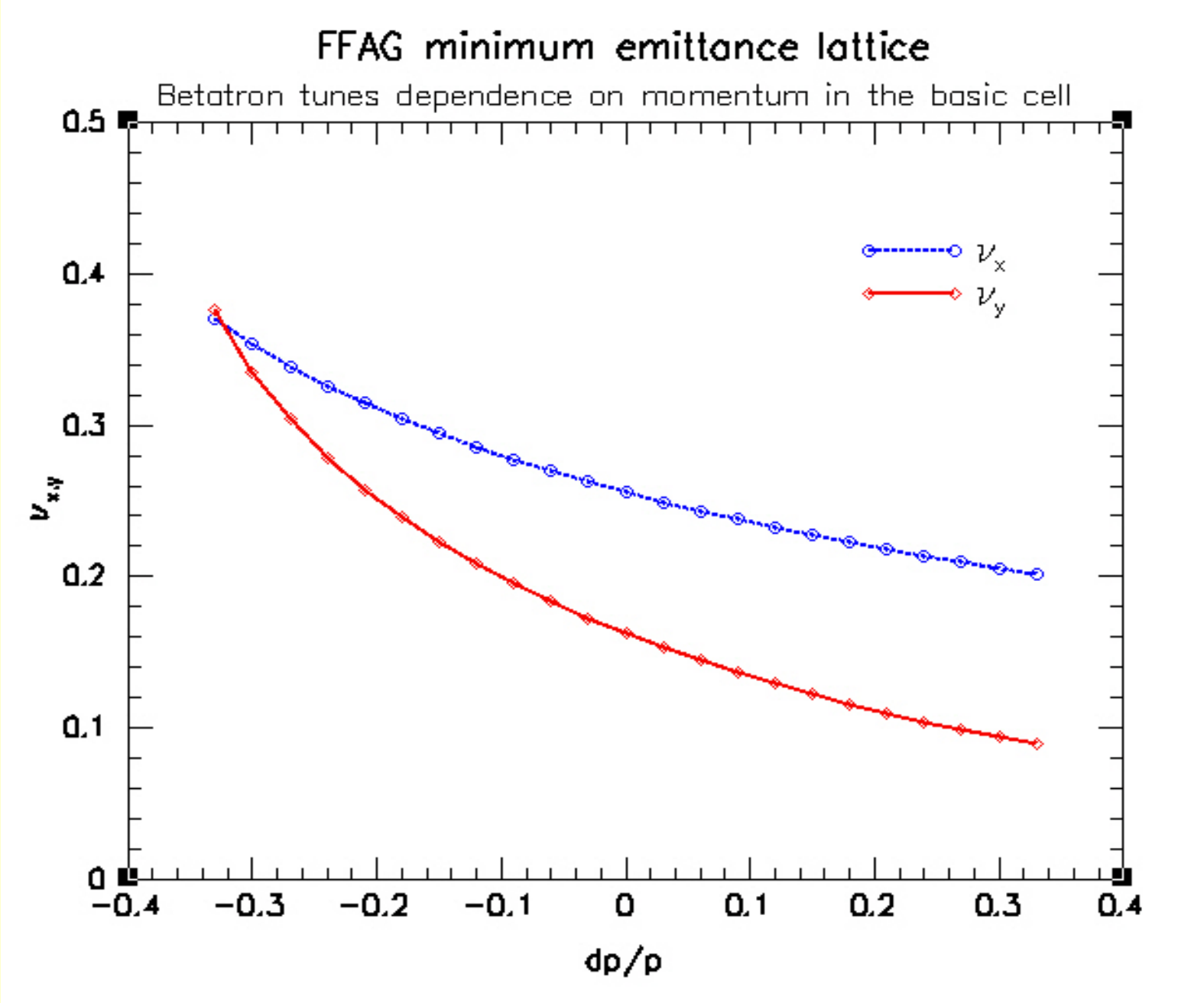


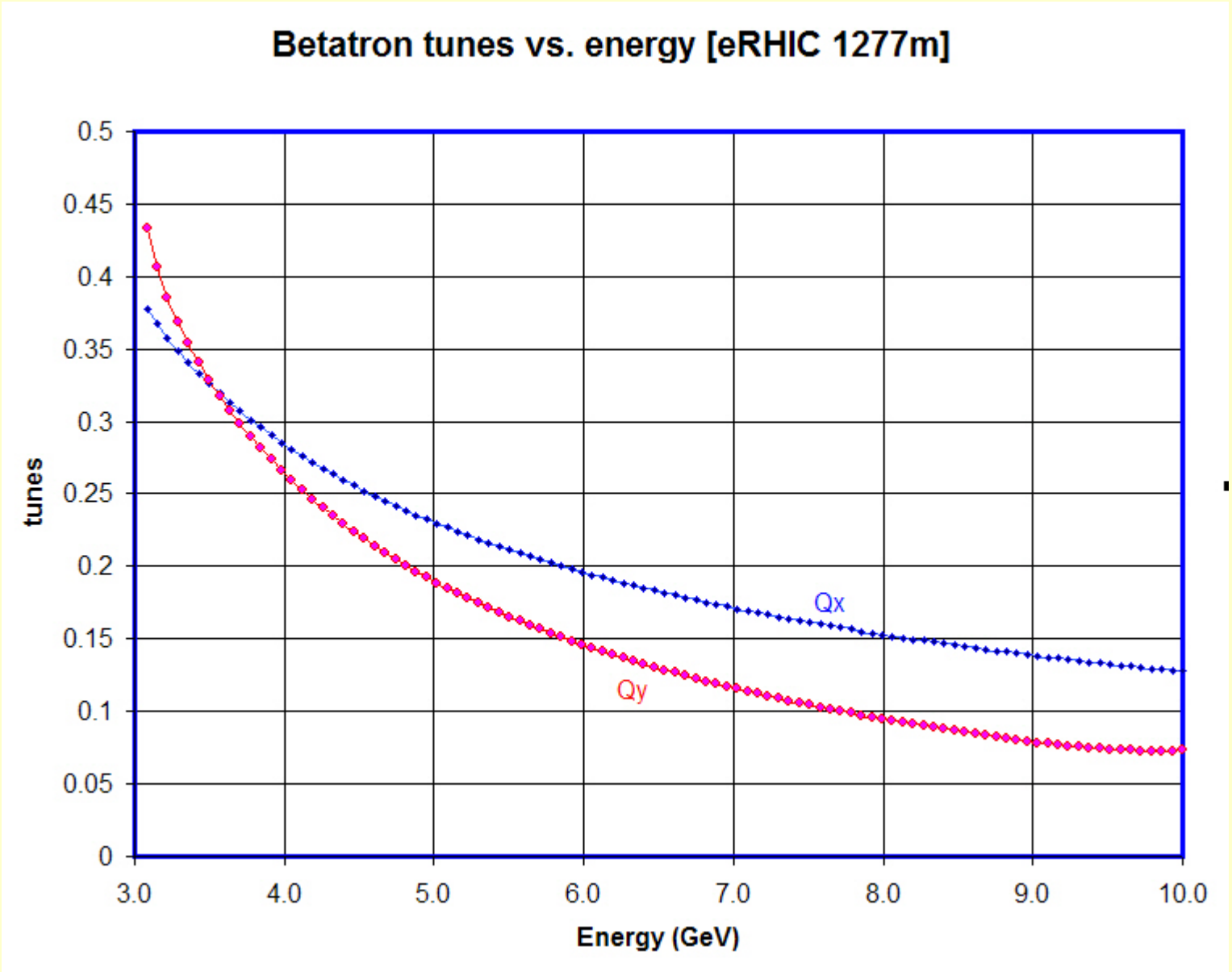
Tune Dependence on Momentum (COSY results)

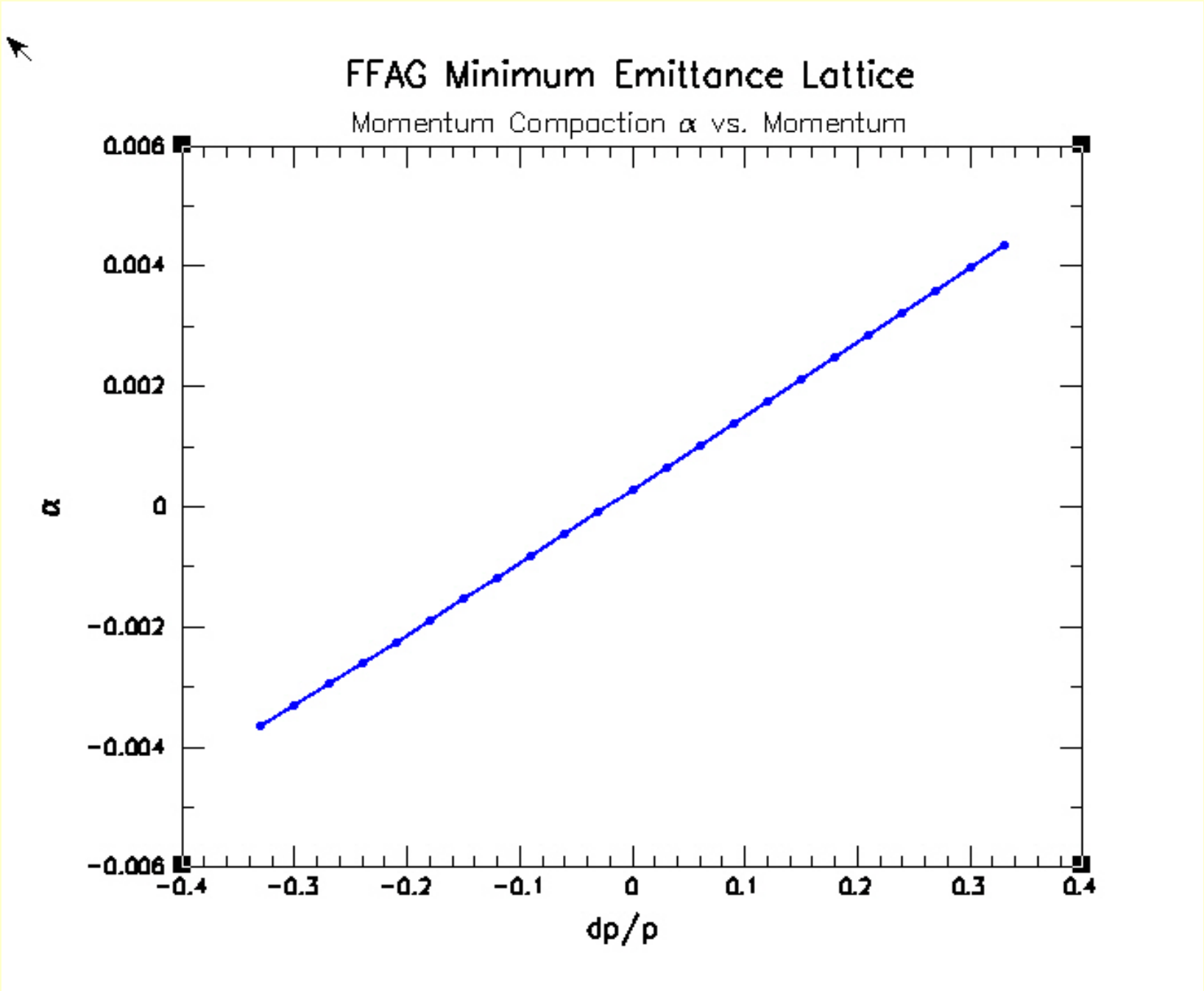


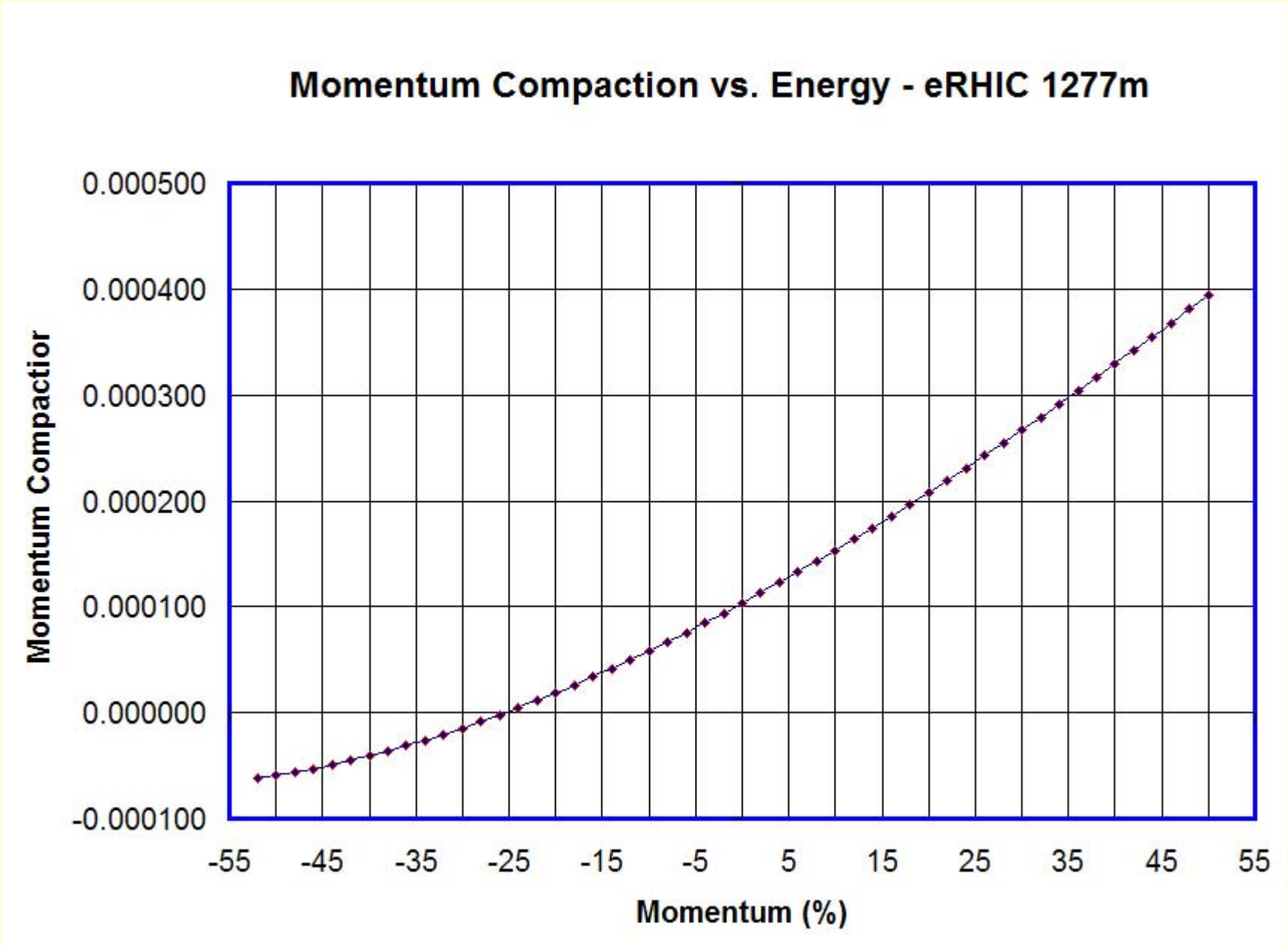

The Minimum emittance non−scaling FFAG
Central momentum orbit through the basic cell
0.275
0.075
−0.125
−0.325
−0.525
−0.725
−0.925
Radius [m]
−6.4
−5.4
−4.4
−3.4
−2.4
−1.4
−0.4
0.6
Distance [m]


The Minimum emittance non−scaling FFAG
Combined function dipole 1.6 m and focussing quad 0.56 m with opposite bending
Radius [m]
Distance [m]
1
0.5
0
−0.5
−1
−1.5
−2
−10
−5
0
5
10


FFAG minimum emittance lattice
Betatron tunes dependence on momentum in the basic cell
ν_x
ν_y
$\nu_{x,y}$
dp/p
0.5
0.4
0.3
0.2
0.1
0
−0.4
−0.3
−0.2
−0.1
0
0.1
0.2
0.3
0.4


Betatron tunes vs. energy [eRHIC 1277m]
tunes
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
Energy (GeV)
Qx
Qy


FFAG Minimum Emittance Lattice
Momentum Compaction α vs. Momentum
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
α
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
dp/p


Momentum Compaction vs. Energy - eRHIC 1277m
Momentum Compactior
0.000500
0.000400
0.000300
0.000200
0.000100
0.000000
-0.000100
-55
-45
-35
-25
-15
-5
5
15
25
35
45
55
Momentum (%)

Energetics of the RF system

For 6.25×10^{12} muons the total charge is 1μC.
Assuming a factor of 2 voltage drop the initial stored energy in the RF cavities is

$$U = 10\text{GV} \times 1\mu\text{C} \times \frac{4}{3} = 13\text{kJ}$$

The stored energy is related to the voltage and impedance by

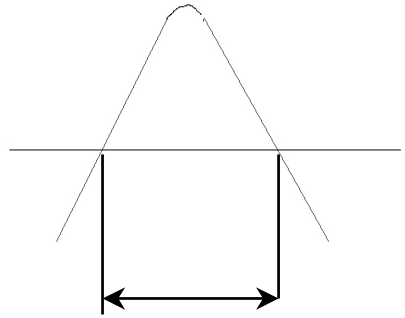
$$U = \frac{V^2}{2\omega_{rf}\left(\frac{R}{Q}\right)}$$

Taking a total voltage of 500 MV and $\omega_{rf} = 2\pi \times 200$MHz one obtains $(R/Q) = 7.6$ kΩ.
The simulations used this impedance and $V = 600$ MV so the voltage dropped to 400 MV at the end of the cycle.
Taking 10 MV per cavity the requisite R/Q per cavity is 126Ω.
The stored energy per cavity is 300 J.
For $E = 10$ MV/m the volume is $0.7m^3$.
With 60 cavities some extra straight sections may be required but, since 10 GeV $\gg$ 106 MeV $= m_\mu c^2$, the straights will have a negligible effect on dT/dE.

References

[1] N. Holtkamp, D. Finley *eds.*, "A feasibility study of a neutrino source based on a muon storage ring", FNAL 2000.

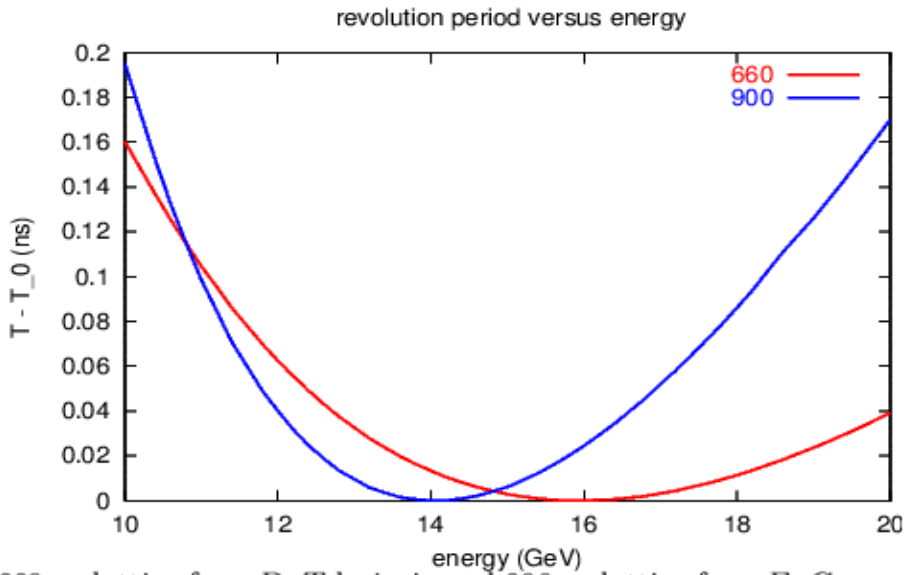
200 MHz RF



~2.5 ns

5 cm = 0.166 ns
10 cm = 0.3335 ns
15 cm = 0.5003 ns
20 cm = 0.667 ns
30 cm = 1.00 ns

RF considerations for FFAG rings
M. Blaskiewicz, BNL



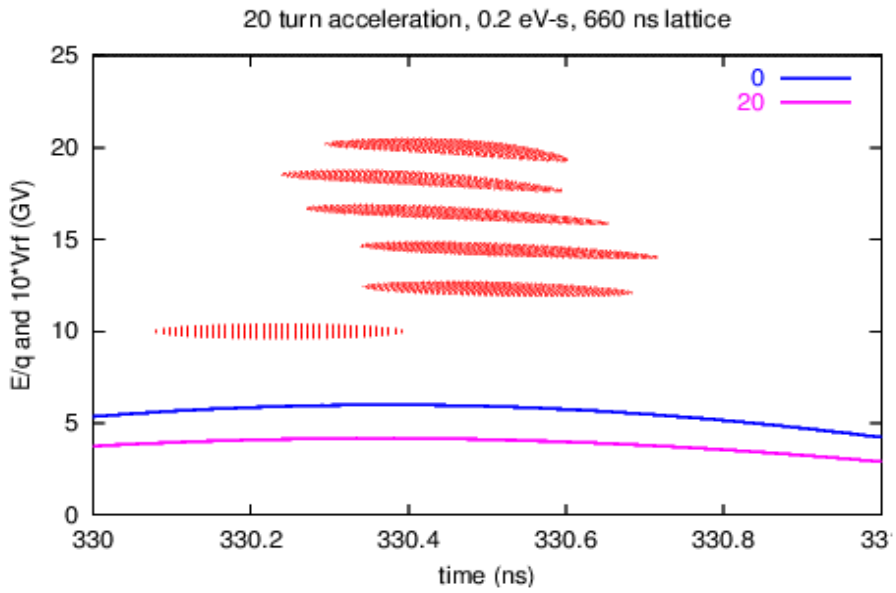
660 ns lattice from D. Trbojevic and 900 ns lattice from E. Courant. Assume negligible energy input to the RF system during acceleration[1] 1-D update equations are

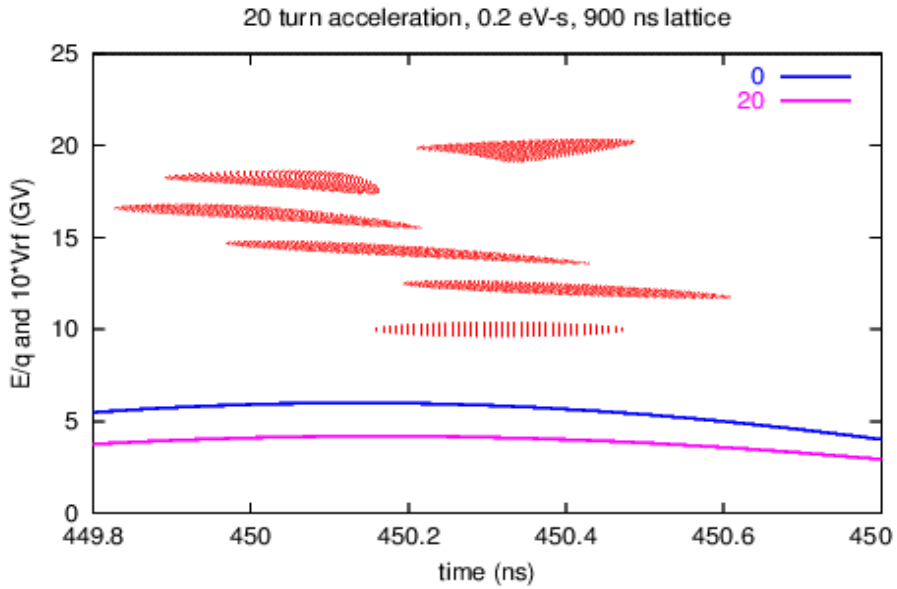
$$\tau_{n+1} = \tau_n + T(E_n) \quad (1)$$

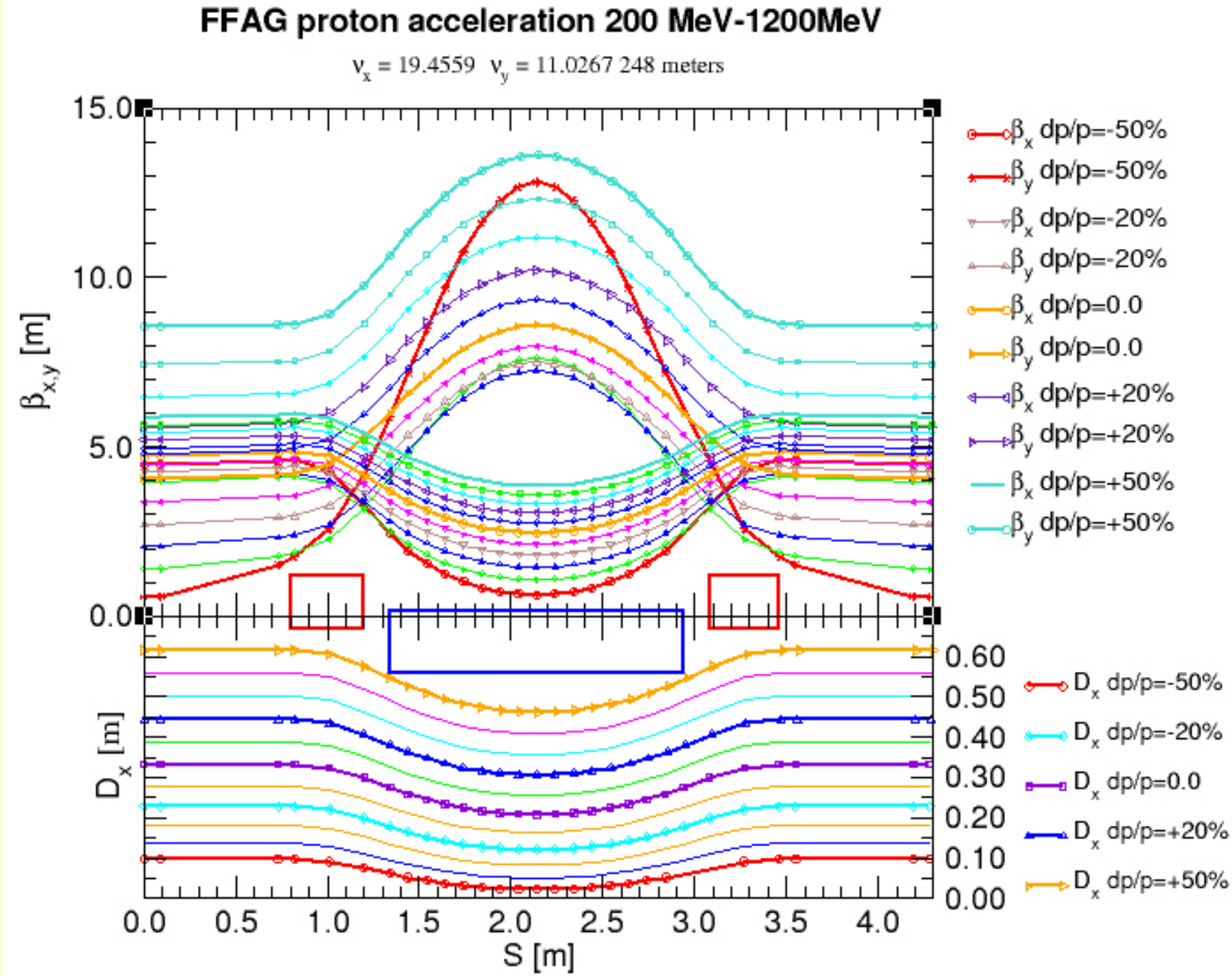
$$\left(\frac{R}{Q}\right) I(t) = \frac{1}{\omega_{rf}} \frac{dV(t)}{dt} + \omega_{rf} \int_0^t dt_1 V(t_1) \quad (2)$$

$$E_{n+1} = E_n + qV(\tau_{n+1}) \quad (3)$$

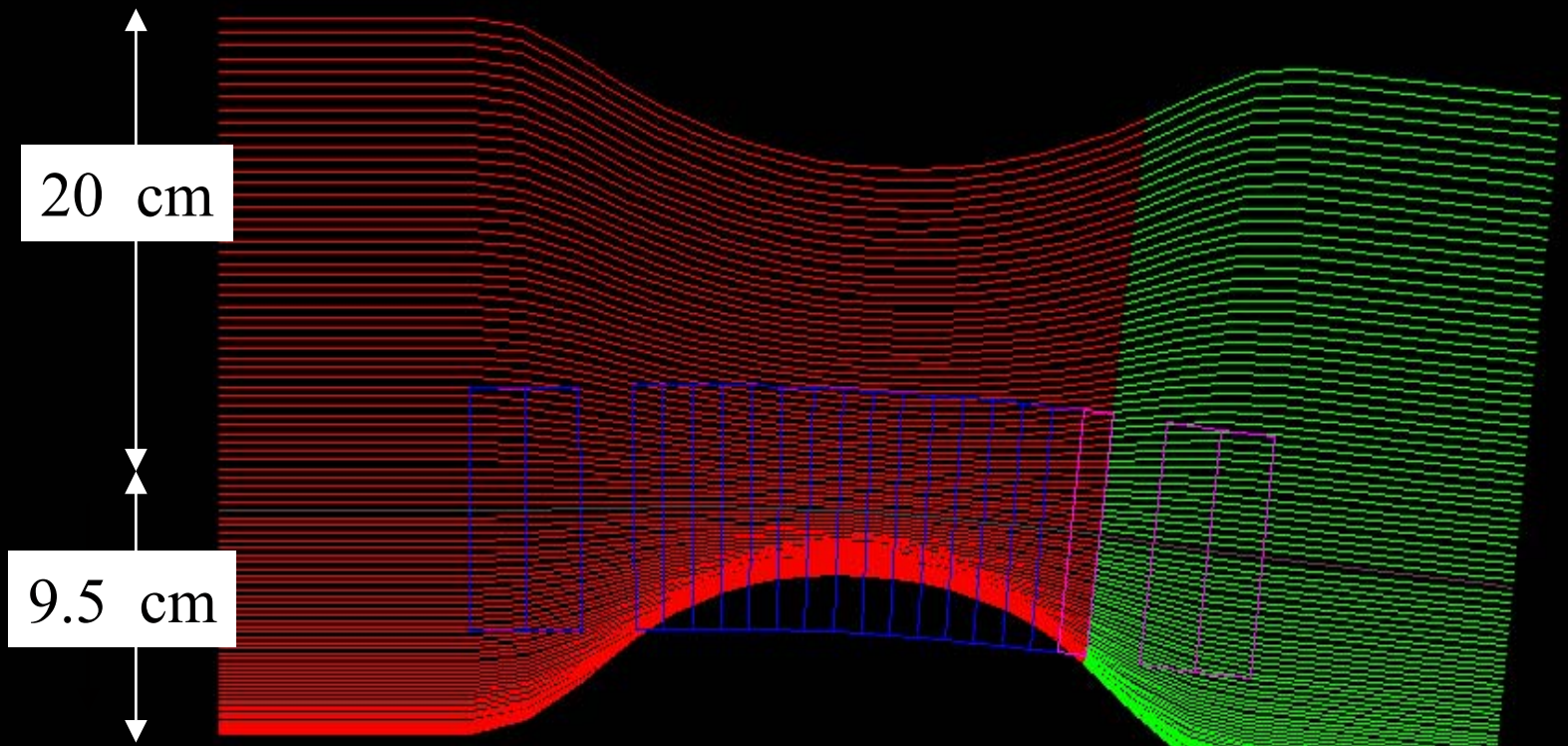
$I(t)$ smoothed by 0.5 ps. V(t) updated with $\Delta t = 0.15$ ps.

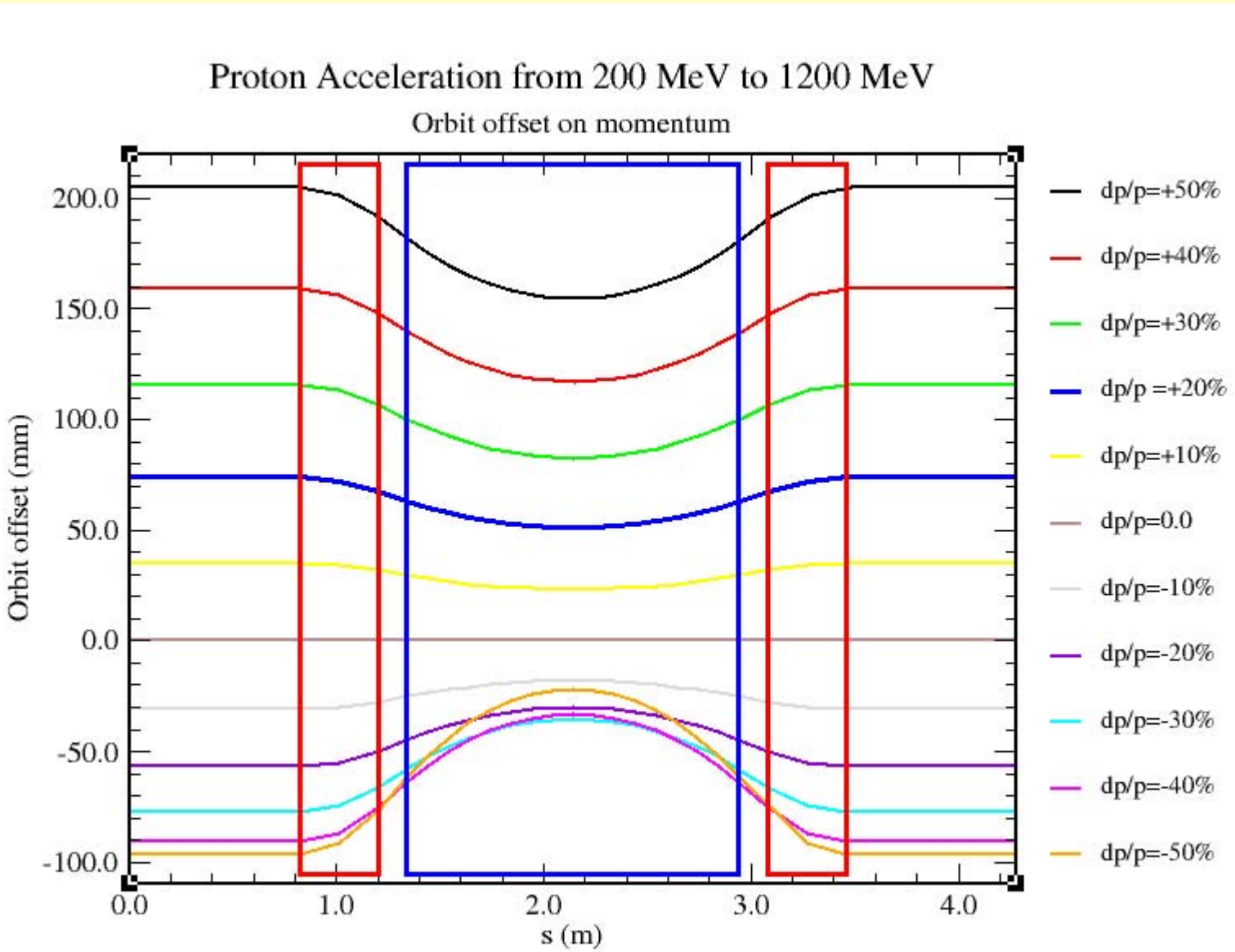

20 turn acceleration, 0.2 eV-s, 660 ns lattice
E/q and 10*Vrf (GV)
time (ns)
0
20
25
20
15
10
5
0
330
330.2
330.4
330.6
330.8


20 turn acceleration, 0.2 eV-s, 900 ns lattice
E/q and 10*Vrf (GV)
time (ns)
0
20
25
20
15
10
5
0
449.8
450
450.2
450.4
450.6
450


FFAG proton acceleration 200 MeV-1200MeV
ν_x = 19.4559 ν_y = 11.0267 248 meters
$\beta_{x,y}$ [m]
D_x [m]
S [m]
β_x dp/p=-50%
β_y dp/p=-50%
β_x dp/p=-20%
β_y dp/p=-20%
β_x dp/p=0.0
β_y dp/p=0.0
β_x dp/p=+20%
β_y dp/p=+20%
β_x dp/p=+50%
β_y dp/p=+50%
D_x dp/p=-50%
D_x dp/p=-20%
D_x dp/p=0.0
D_x dp/p=+20%
D_x dp/p=+50%

Proton Acceleration 0.2 – 1.2 GeV




Proton Acceleration from 200 MeV to 1200 MeV
Orbit offset on momentum
Orbit offset (mm)
200.0
150.0
100.0
50.0
0.0
-50.0
-100.0
0.0
1.0
2.0
3.0
4.0
s (m)
dp/p=+50%
dp/p=+40%
dp/p=+30%
dp/p =+20%
dp/p=+10%
dp/p=0.0
dp/p=-10%
dp/p=-20%
dp/p=-30%
dp/p=-40%
dp/p=-50%

Conclusions:

- **Electron Demonstration ring is becoming a reality.**
- **There are many different studies going at the same time and all reports are very encouraging [J.S. Berg, M. Blaskiewicz, E.D. Courant, M. Craddock, A. Garren, C. Johnstone, E. Keil, S. Koscielniak, A. G. Ruggiero, and A. Sessler, ….].**
- **More detail studies of six dimensional simulations during acceleration are the next step and there are available tools.**

- **We have shown that the non-scaling triplet FFAG's can accelerate muons, protons, heavy ions, electrons.**