Scaling Lattices of Gas Filled Rings for Muon Cooling

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Emittance Exchange Workshop, Riverside CA

January 21-26, 2004

Introduction

We have been investigating the possibility of muon cooling in dipole rings with ***scaling*** properties. We have been studying two types of scaling lattices, those with so-called weak or strong focusing.

Scaled magnetic lattices, as well as the closed orbits for different momenta in these lattices, are geometrically invariant under scale transformations relative to the ring center.

The magnets of scaled lattices must have a dipole field component; however they may have certain kinds of radial and azimuthal variations.

Advantages of Scaling for gas-filled Rings

Large dynamic apertures, especially longitudinal

Small fluctuation of beta functions and dispersion with azimuth

Weak focusing Rings with Zero-Gradient Sector Magnets

These are automatically scaling machines. There are a small number of sector magnets – typically 4 or 6 - separated by drifts.

Advantages

Simplicity

Small size > Low cost

Disadvantages

Small size – about 30cm orbit radius – which

1) complicates injection because drift sectors are short

2) limits input beam size in all dimensions

Weak Focusing Rings - Examples

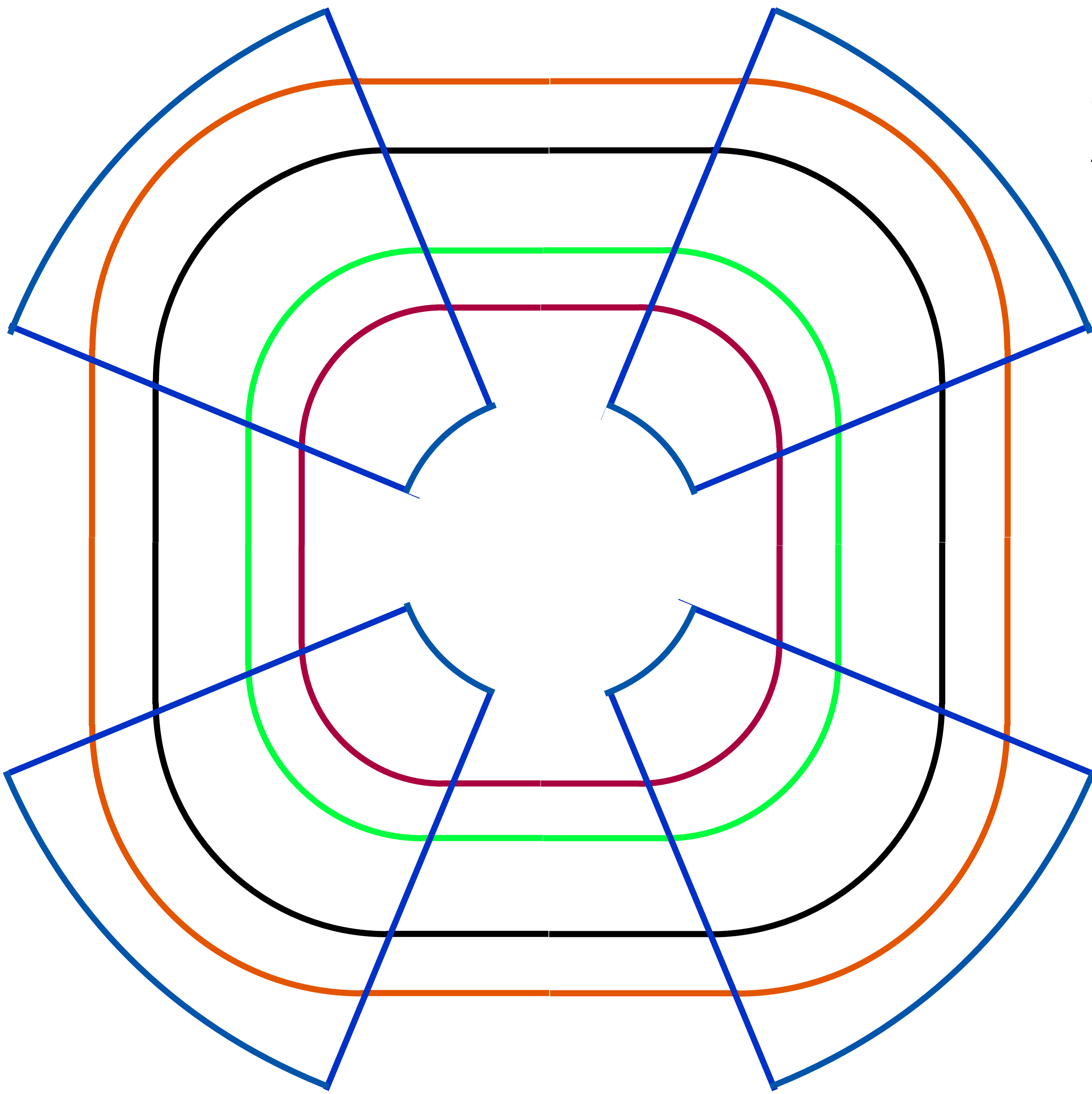
The following example shows a 4-sector, zero-gradient ring. The colored curves are closed orbits for various momentum values

Scaling Weak focusing Rings – Defined

The ring must be composed of one or more identical magnets separated by identical drifts.

The magnet edges must be radial or spiral.

The magnetic field must be uniform or may have a constant field index $k = - n = (R / B)\,(dB / dR)$



4-DIPOLE RING

$\lambda = \rho / Rc = 1$

CELL OF 4–CELL RING

$$\lambda = \rho / Rc$$

$$\sigma = (1+\lambda^2+2\lambda\cos\theta)^{1/2}$$

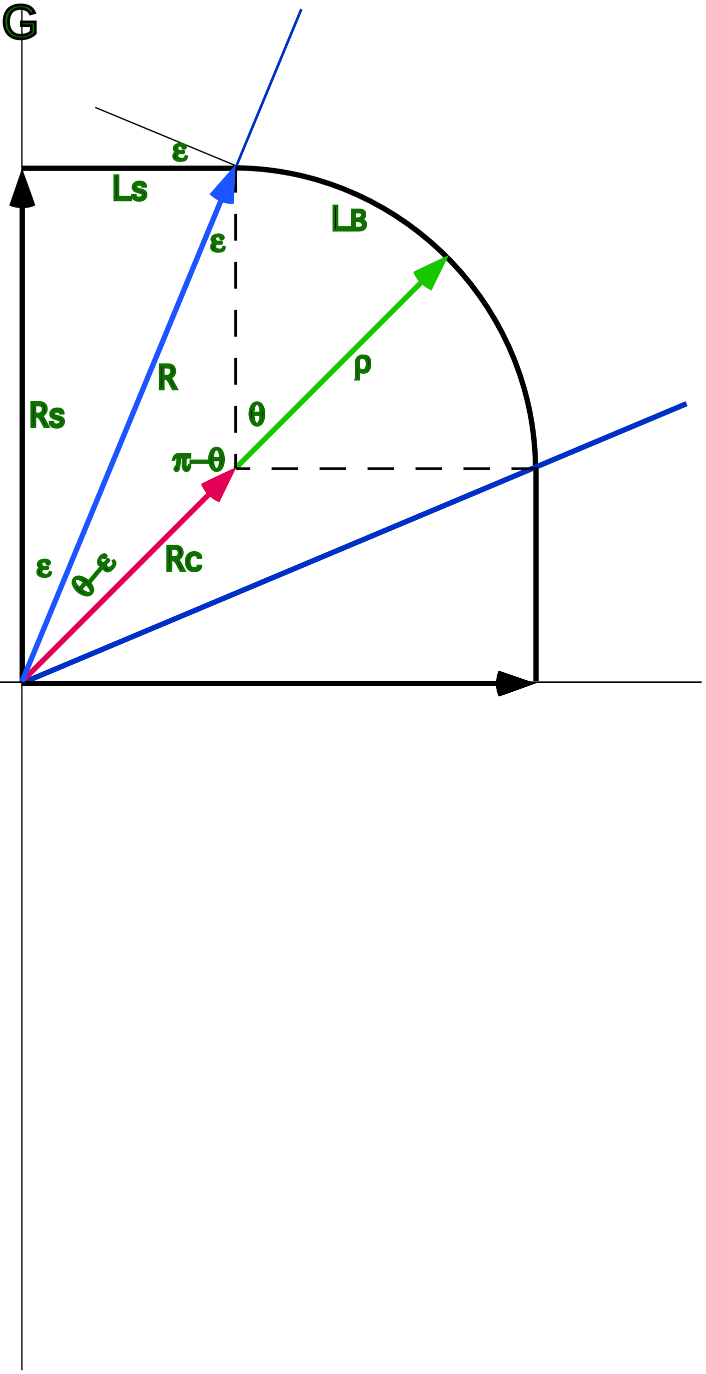
$$R = Rc\ \sigma$$

$$\varepsilon = \sin^{-1}(\sin\theta / \sigma)$$

$$Rs = R\cos\varepsilon$$

$$Ls = R\sin\varepsilon$$

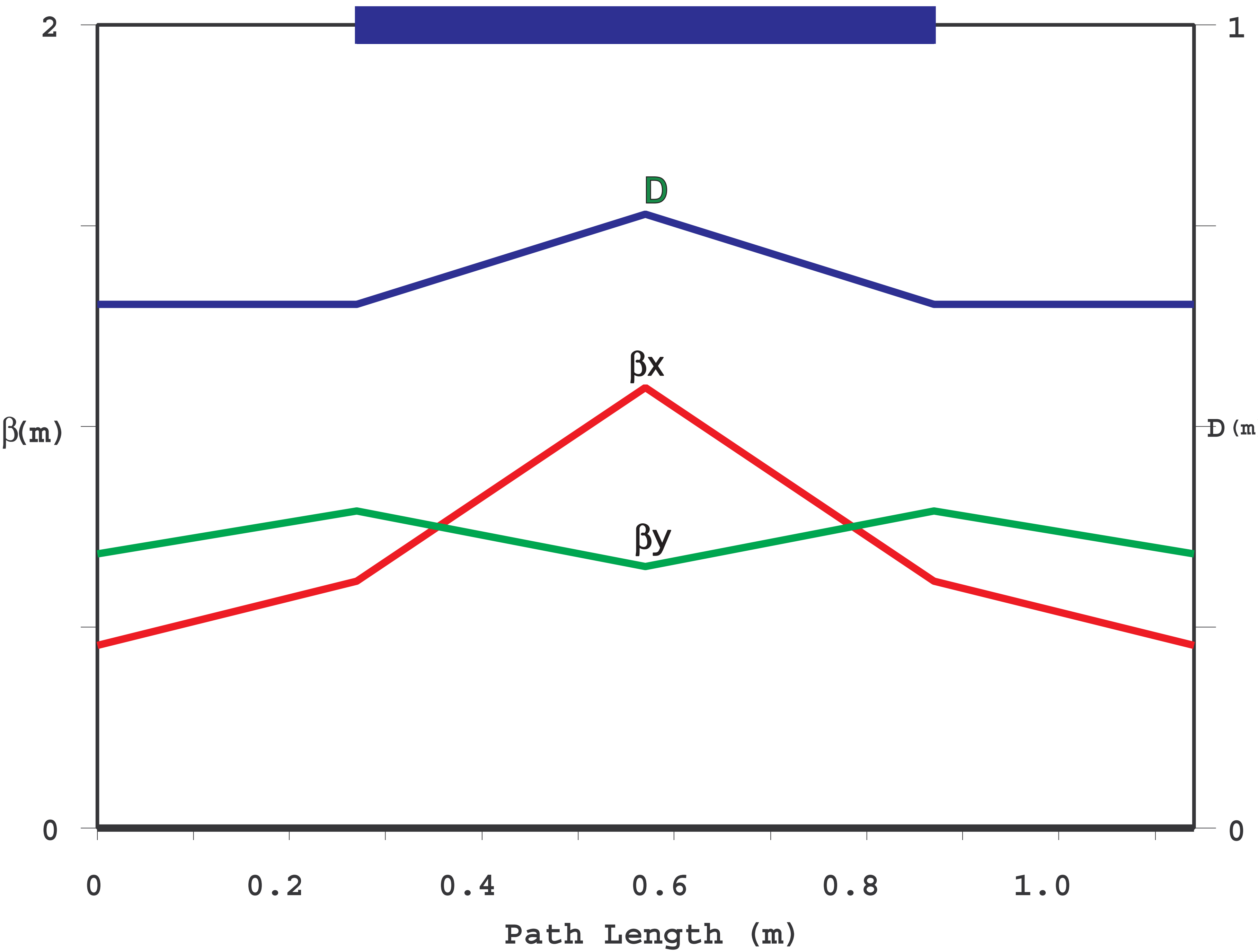
$$LB = \rho\,\theta$$



4 DIPOLE RING

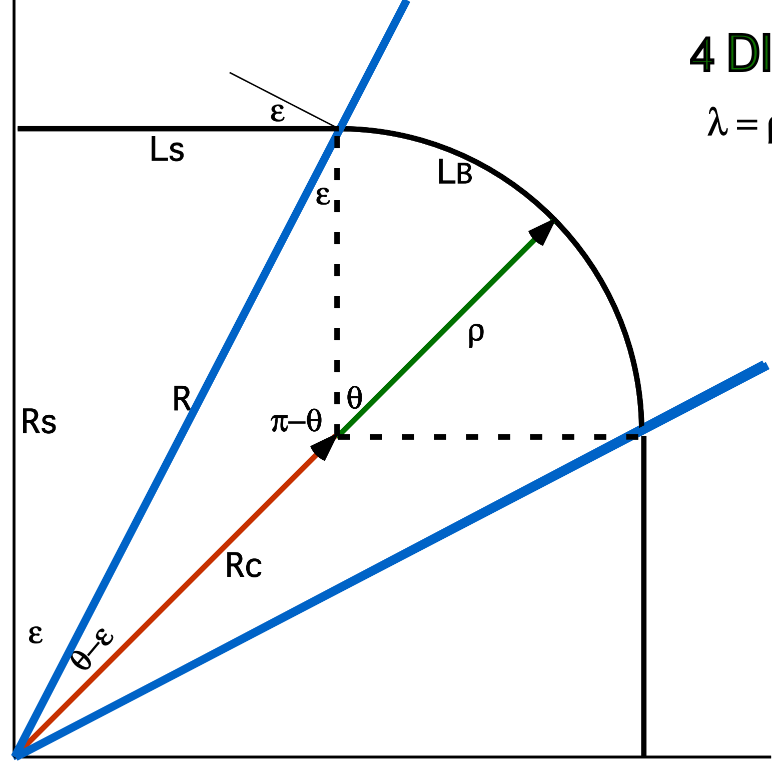
$\lambda = \rho / Rc = 1$

$\rho = Rc = 0.382$, LB = 0.25



4 DIPOLE RING

$\lambda = \rho \,/\, Rc = 2/3$

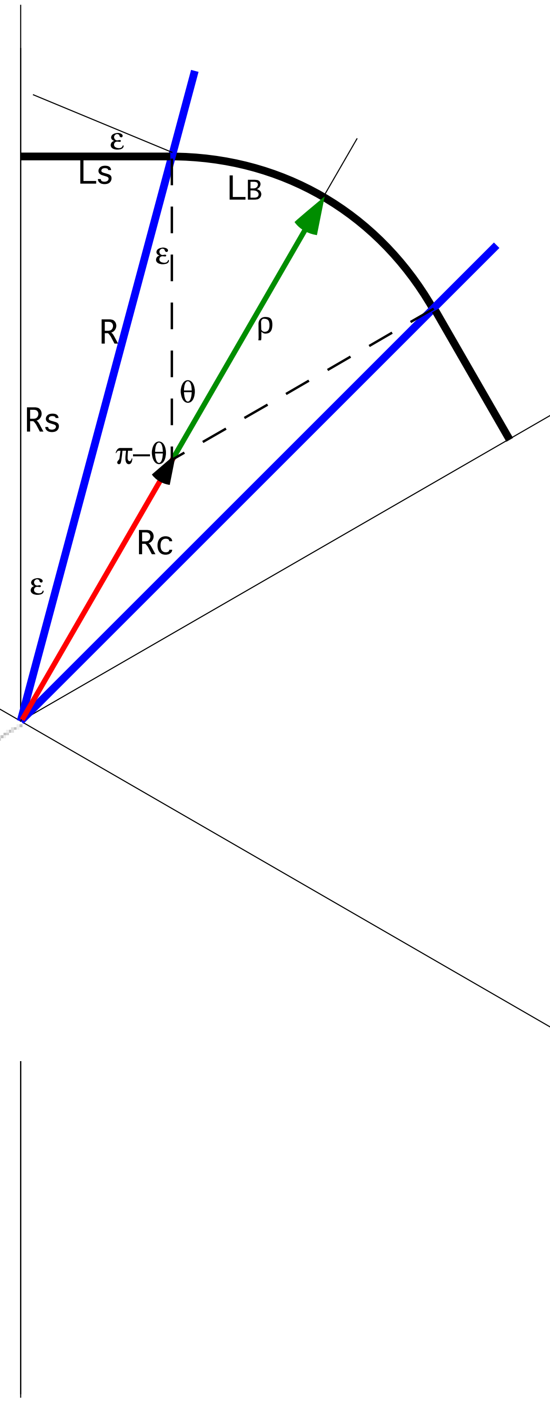


6 DIPOLE RING

$\lambda = \rho / Rc = 1$

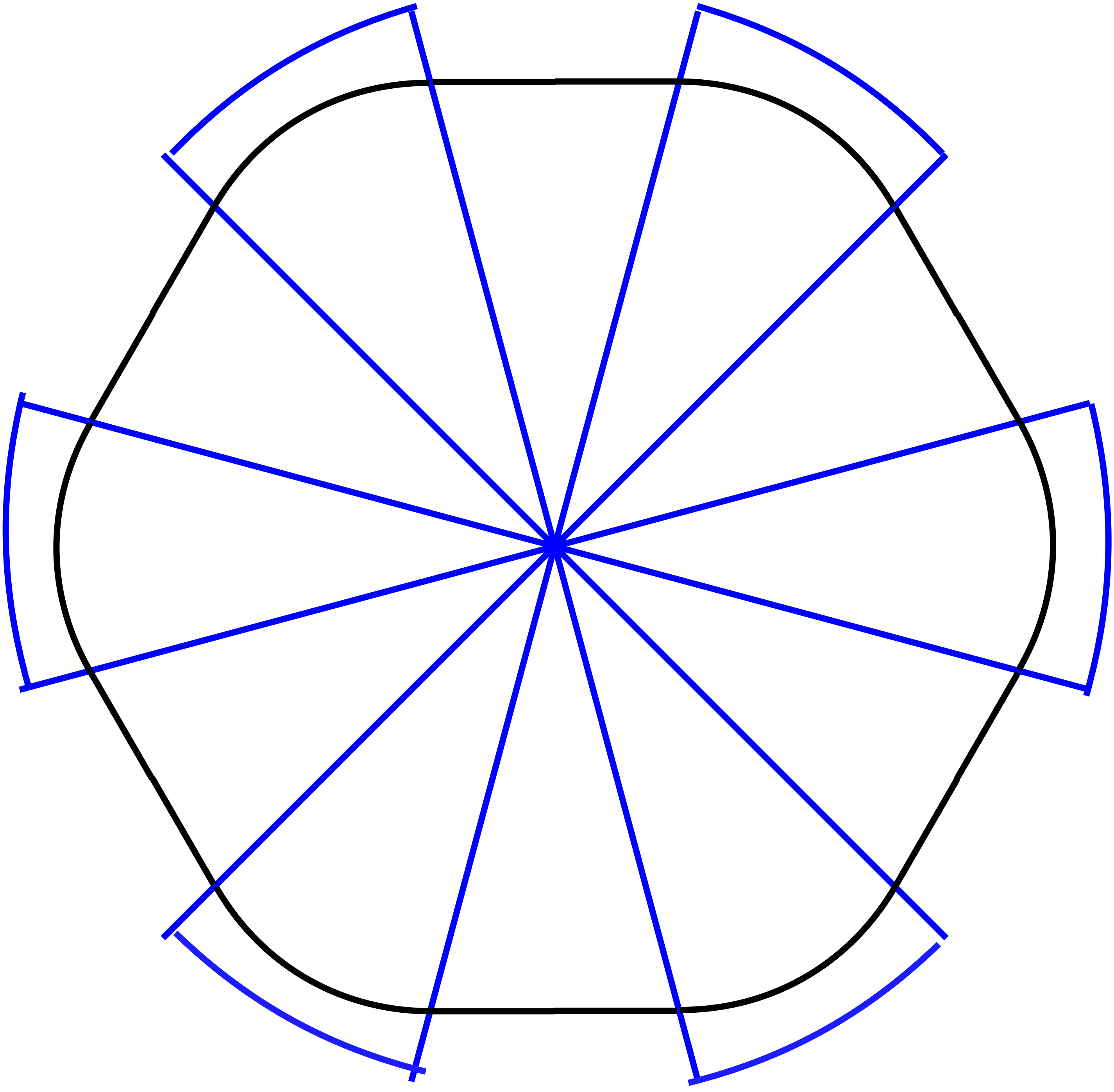
Parameters of 6-Sector Rings

rho=Rc, theta=30deg, eps=15deg



Momentum, GeV/c	0.25
Magnetic field, T	2.62
Magnet half length LB	0.167
Gap half length LS	0.159
Rho	0.318
Rc	0.318
Cell length	0.637
Circumference	3.82
RS	0.594
R	0.615
Bx, max	0.719
By, max	0.645
D, max	0.637

6 DIPOLE RING



Parameters of 6-Sector Ring

Momentum	0.25	GeV/c
Magnetic field	2.62	T
Magnet half length LB	0.167	m
Gap half length LS	0.159	m
Rho	0.318	m
Rc	0.318	m
Cell length	0.637	m
Circumference	3.82	m
RS	0.594	m
R	0.615	m
Bx, max	0.719	m
By, max	0.645	m
D, max	0.637	m
mux	0.173	
muy	0.169	
Bend angle/half cell	30	deg
Edge angle	15	deg

Strong Focusing Scaling Rings

Following are two examples, both with 12 FFAG cells, one with and one without drifts.

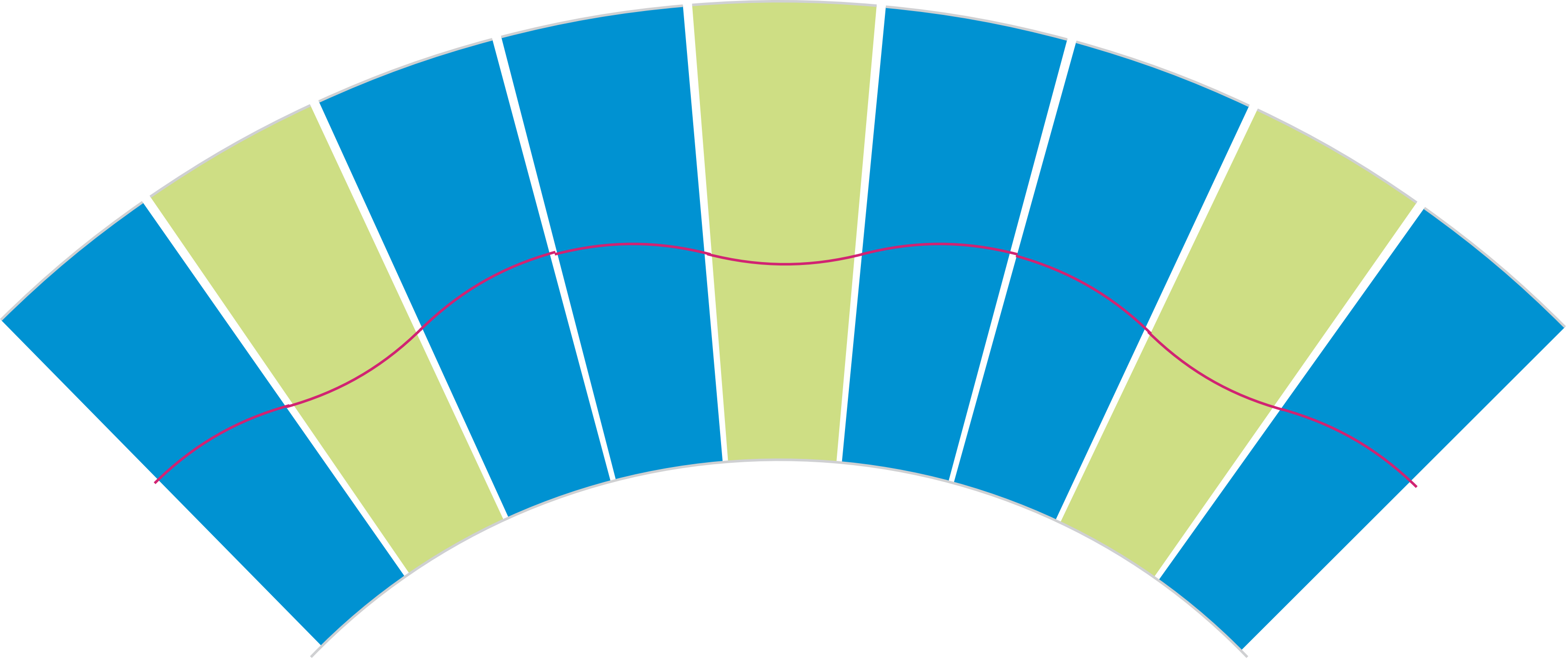
Advantages

Allows lower magnetic fields and rf gradients

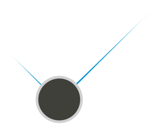
Large size > Accepts larger transverse beam size and longer bunch train

Disadvantage

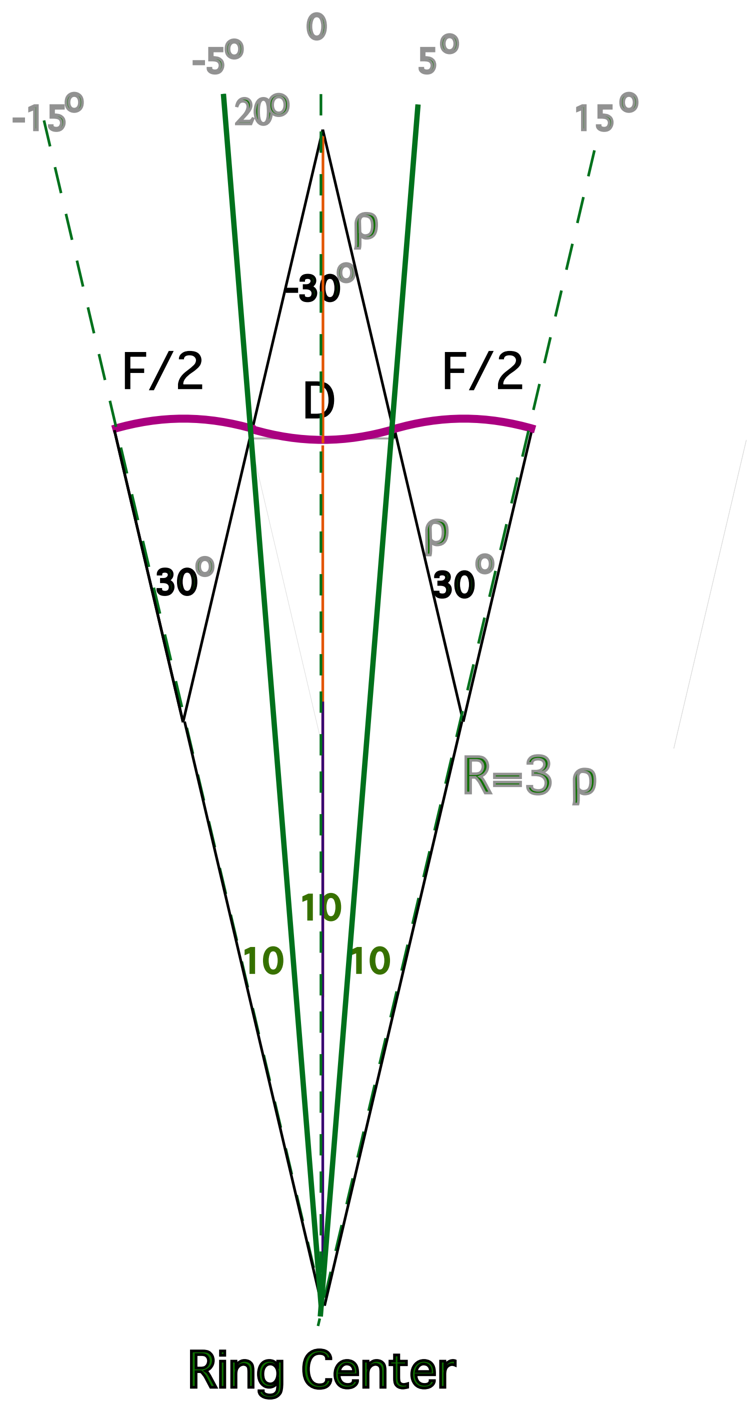
More complicated, higher cost



3 Cells - 90°



12 Cell Ring without Drifts
Layout of 1 Cell



$k = nd = -nf = 0.9$

$B/B0 = (R/R0)^{k} = (\rho/\rho o)^{-n}$

$P/P0 = (R/R0)^{k+1}$

$(R/R0) = (P/P0)^{1/k+1}$

$D = dR/d(p/po) = R/(k+1)$ = .2581m

Lf = .3333m ; Ld = .1667m

Lcell = 0.5m ; Circumference = 6m

ρ0 = .3183m ; R0 = .9549m ; B0 = 2.620T

$B/B0 = (R/R0)^{k} = (\rho/\rho o)^{-n}$

$P/P0 = (R/R0)^{k+1}$

$(R/R0) = (P/P0)^{1/k+1}$

$D = dR/d(p/po) = R/(k+1)$ = .2581m

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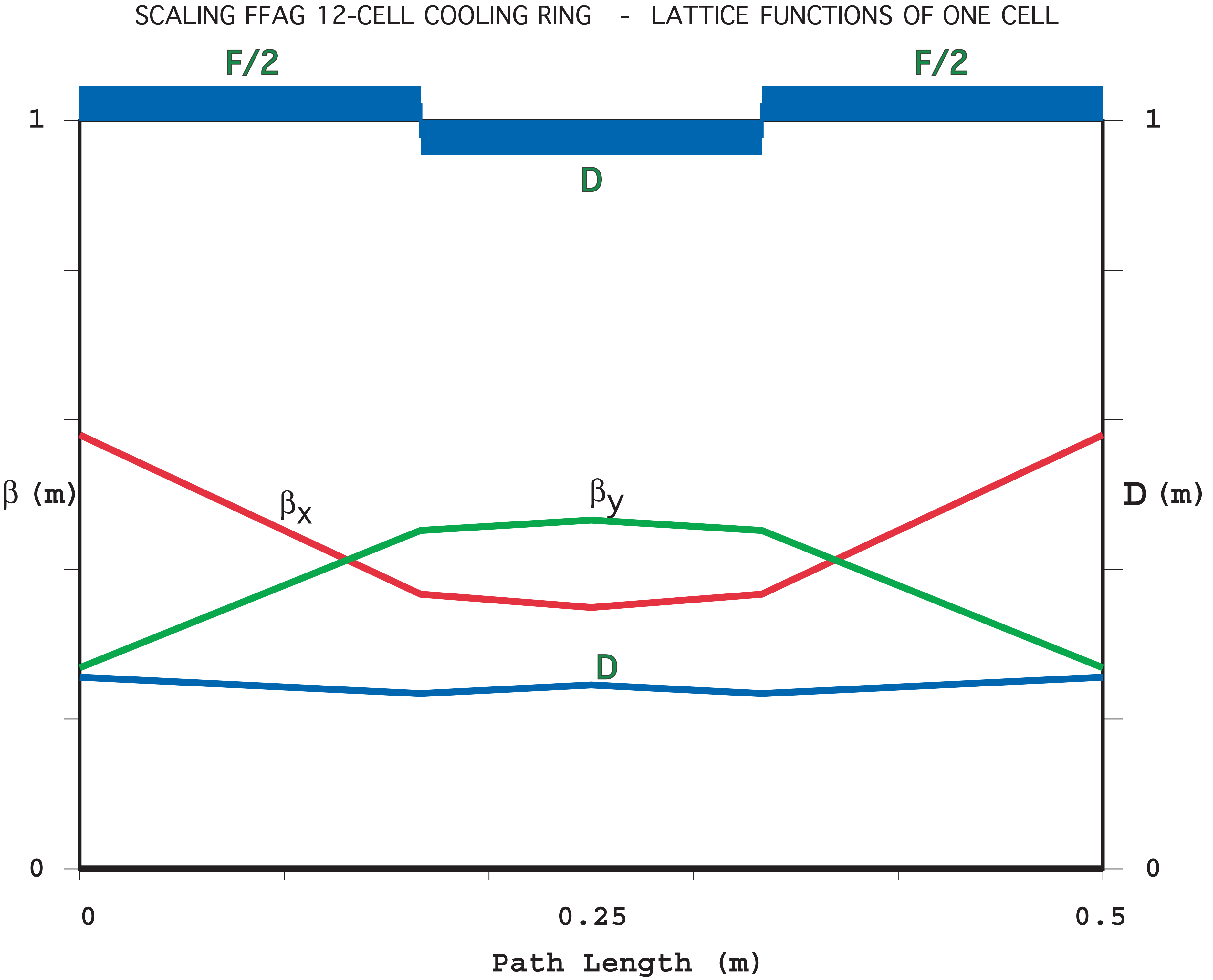
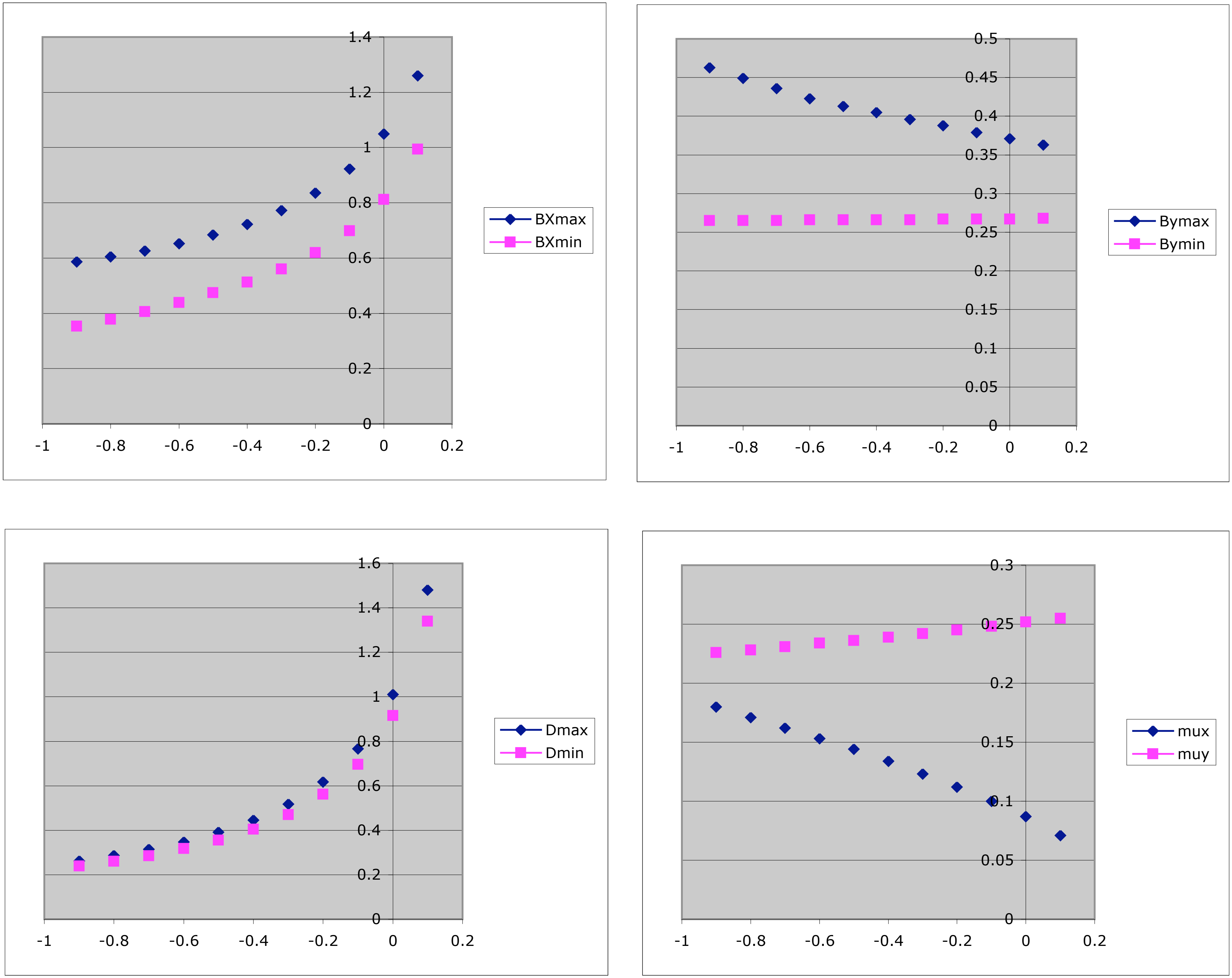
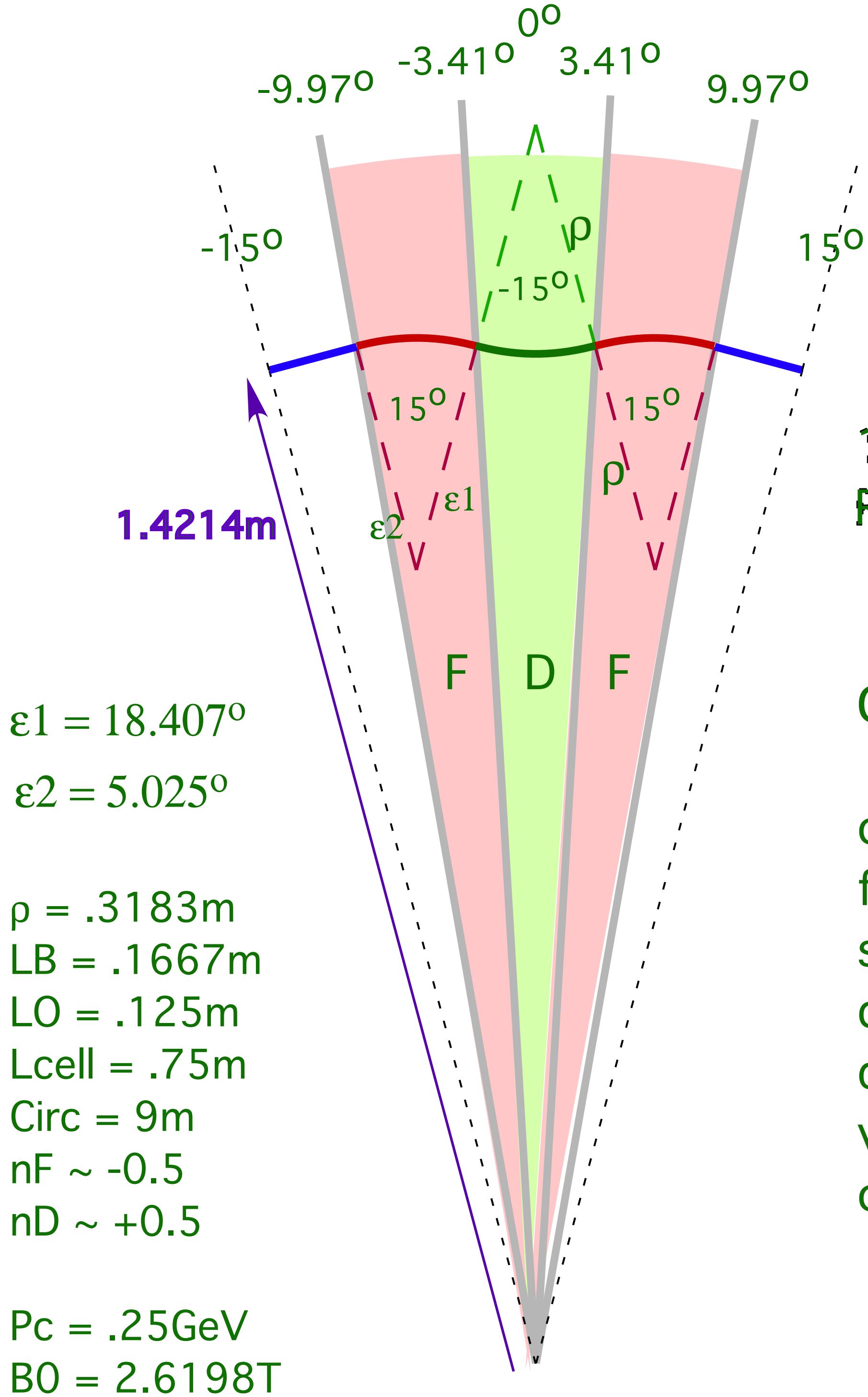
P/PO	P	R/RO	ρ	R	R-RO
	GeV/c		m	m	m
1.3	.325	1.0735	.342	1.025	.0702
1.2	.300	1.0505	.334	1.003	.0482
1.1	.275	1.0261	.327	0.980	.0249
1.0	.250	1.0000	.318	0.955	.0000
0.9	.225	.9719	.309	0.928	-.0268
0.8	.200	.9415	.300	0.899	-.0559
0.7	.175	.9081	.289	0.867	-.0878



SCALING FFAG 12-CELL COOLING RING - LATTICE FUNCTIONS OF ONE CELL
F/2
F/2
D
1
1
β (m)
D (m)
βx
βy
D
0
0
0
0.25
0.5
Path Length (m)

12-CELL STRONG FOCUSING COOLING RING: ORBIT FUNCTIONS VS. FIELD INDEX n

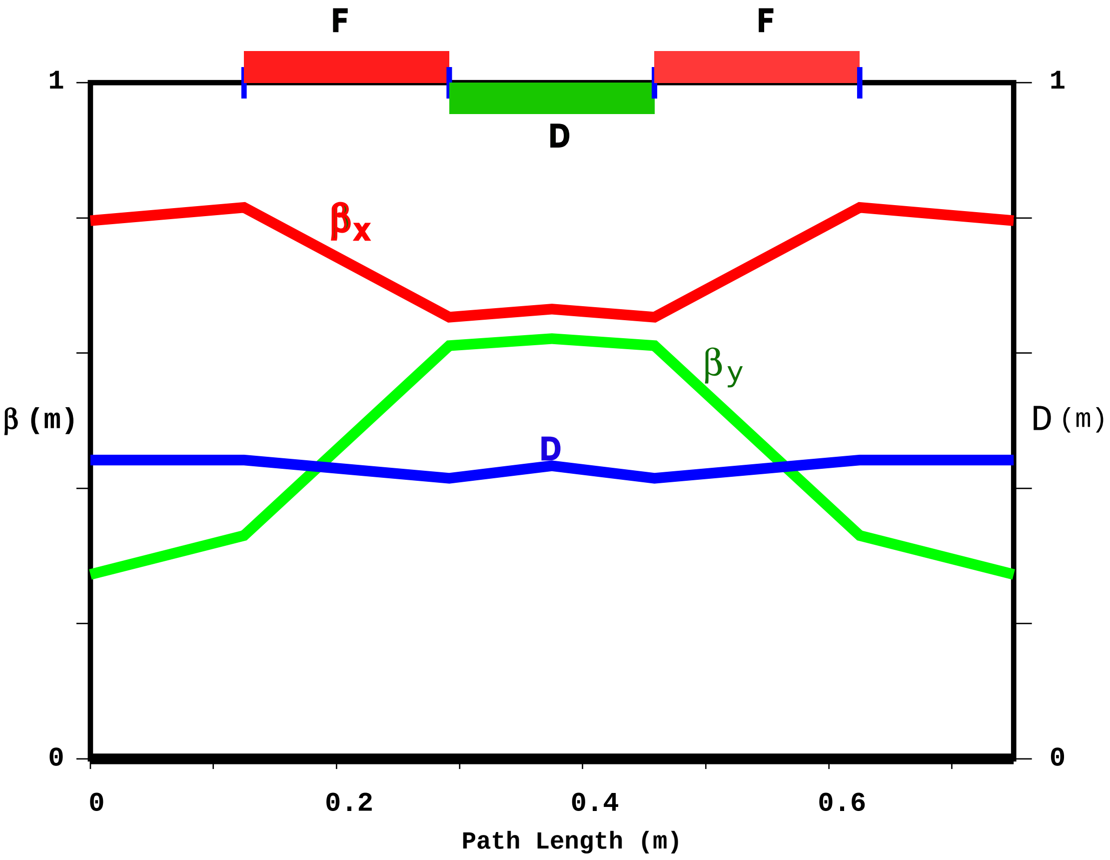




12 CELL STRONG FOCUSING RING WITH DRIFTS

Cell Diagram

The magnets have equal and opposite field strengths and field indices. The diagram shows the central, 0.25GeV closed orbit. Closed orbits of different momenta are scaled versions of the central closed orbit.



Beta functions of one cell of FFAG 12-cell ring with drifts

3 CELLS OF THE 12-CELL RING