Do I understand the Concept of Emittance

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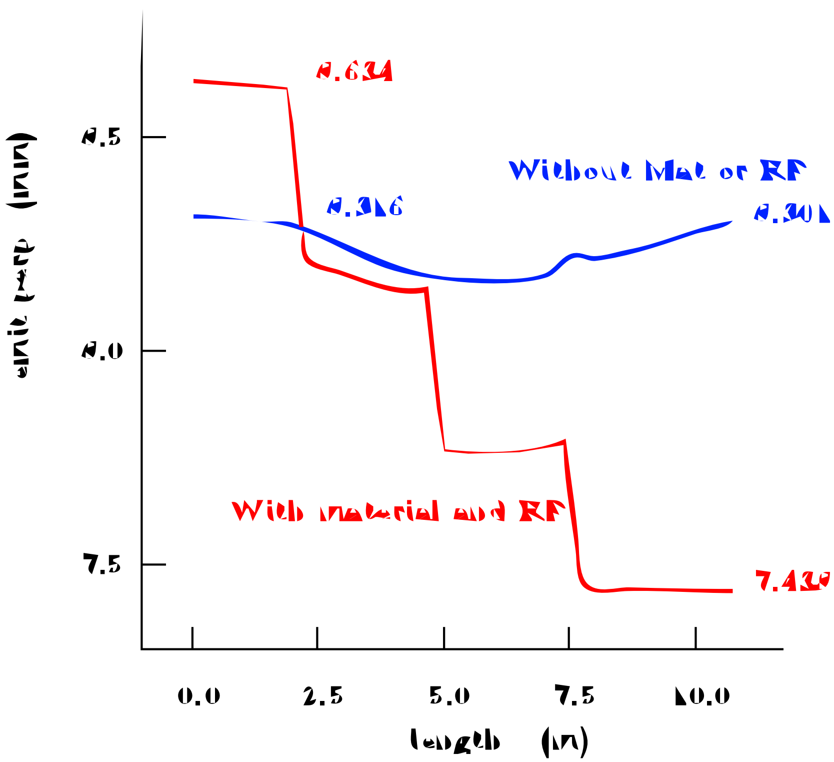
The problem

All those running the code ICOOL have noticed *peculiar* behavior of the transverse emittance (ϵ_T) in several problems. Most recently in a study of the MICE cooling channel, Bob Palmer pointed out that the emittance, in the region of no-absorbers, should have been constant; however, the ICOOL runs show a decrease between the first and second absorber and that the same behavior is shown in an identical channel without absorber and rf.

The problem-cont.

Systematic Problem in MICE

Look at emittance vs Length with and without material and RF (no errors)



- With cooling: Emittance falling too fast between 1st and 2nd absorbers
- Similar drop observed with No Material or RF

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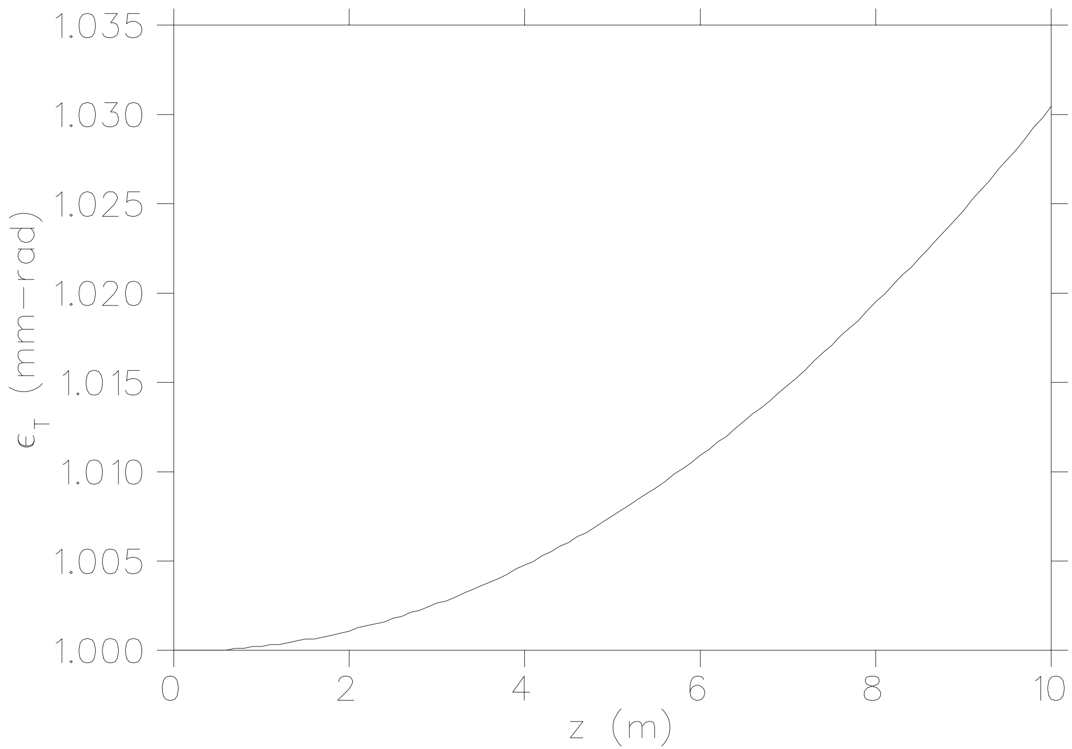
The problem-cont.

Afterward R. Fernow look at an even simpler problem: a plain **DRIFT** and to our surprise $\epsilon_{\mathbf{T}}, \epsilon_{\mathbf{L}}, \epsilon_{\mathbf{6}}$ changed although one expected that at least ϵ_6 should remain constant. To be precise, *the emittance as calculated by* **ECALC9** *is not*



constant.

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Solution?

Subsequently, we learned of the paper: K. Floettmann, *Some basic Features of the Beam Emittance*, Phys. Rev. ST Accel. Beams **6**, 034202 (2003), where in the abstract, the blasphemous statement is made: *the [transverse] emittance of a beam is not necessarily constant in a drift space*. The author argues that energy spread is responsible for this behavior. The proof given is not complicated but laboriuos.

Solution?

The normalized emittance is:
$\epsilon_x^{norm} = \frac{1}{m_o c}\sqrt{< x^2 >< p_x >^2 - < x p_x >^2}$ In a drift $x(s) = x(0) + \frac{p_x}{p_s} s$ and $p_x(s) = p_x(0)$; hence

$$\frac{1}{N^2}\sum x(0)\frac{p_x}{p_s}\sum p_x^2 \text{ must be } = \frac{1}{N^2}\sum x(0)p_x\sum\frac{p_x}{p_s}p$$

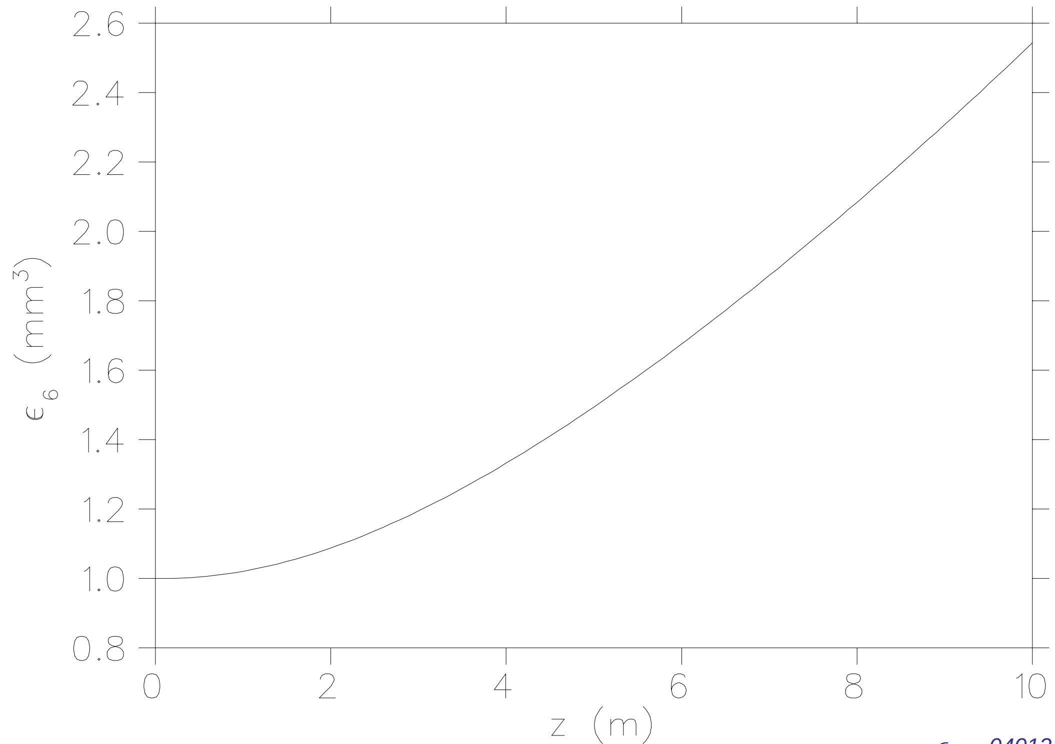
and

$$\frac{1}{N^2}\sum\left(\frac{p_x}{p_s}\right)^2\sum p_x^2 \text{ must be } = \frac{1}{N^2}\left(\sum\frac{p_x}{p_s}p_x\right)^2.$$

which is true only if the energy spread is very small.

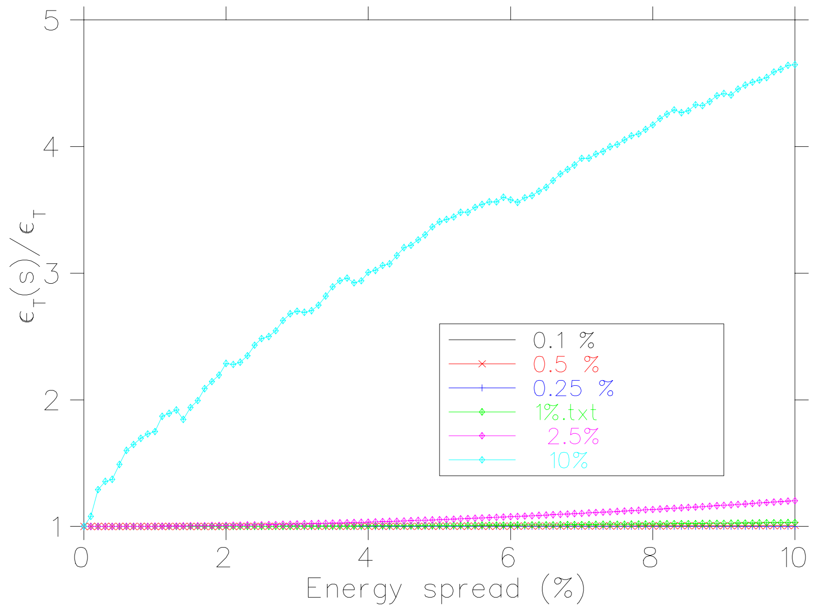
Solution?

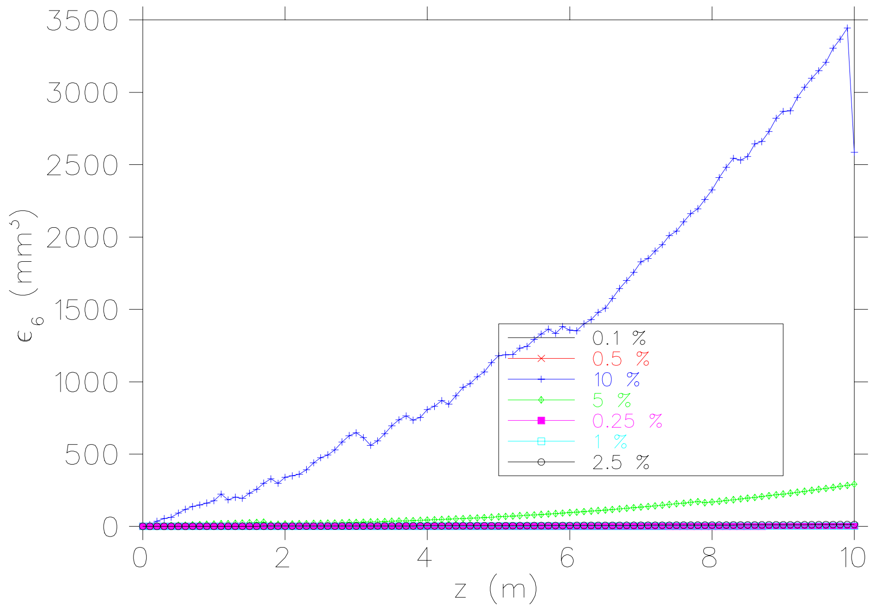
This we may accept because one argues that ϵ_6 is the real constant of motion and ϵ_T may change as long as ϵ_L changes too to keep ϵ_6 constant. So let see what is ECALC9 is saying about ϵ_6 in the drift example:



Solution?

As a function of the energy spread





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The problem

We know that simple drift problem is

- linear
- the integration is symplectic
- Liouville's theorem must not be violated!

ANSWER: I do not know how to calculate emittances.