The standard approach to measure beam emittance in a transfer line

Massimo Giovannozzi - CERN

February 23-24, 2001

Workshop on Instrumentation for Muon Cooling Studies, Imperial College

Summary

- Definitions/Properties
- Measurement in Rings
- Measurement in Transfer Lines
- Emittance reconstruction
- Error sources
- Conclusion

Acknowledgements: R. Cappi, M. Martini

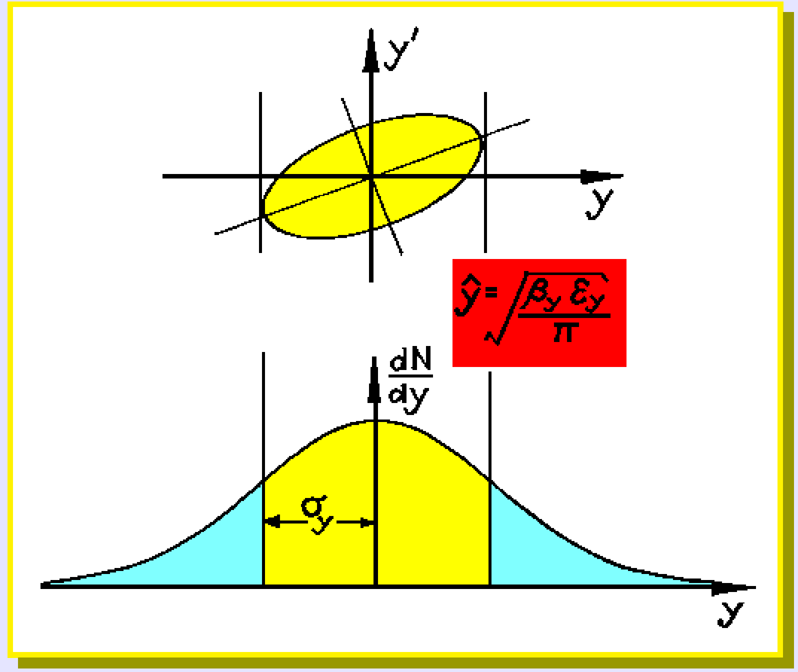
Definitions

- the orbit in phase space of a particle in a linear (i.e. the Hamiltonian is quadratic in the phase space co-ordinates) magnetic transport channel is an ellipse:

$$\gamma y^2 + 2\alpha y y' + \beta {y'}^2 = A^2$$

- The surface (A^2/π) of such an ellipse is an invariant of motion.
- The parameters $\beta, \alpha, \gamma = (1+\alpha^2)/\beta$ are the so-called Twiss-parameters.
- Accelerator beams comprise some $10^{10} - 10^{13}$ particles. the beam distribution is near-Gaussian, hence the beam emittance is defined as

$$\epsilon_y = \frac{\sigma_y^2}{\beta_y}$$



From: P. J. Bryant and K. Johnsen "Circular Accelerators and Storage Rings"

Properties - I

- The beam emittance is an invariant of motion for constant energy.
- It is possible to define an invariant quantity even in presence of acceleration:

$$\epsilon_y^* = \beta\,\gamma\,\frac{\sigma_y^2}{\beta_y} = \beta\,\gamma\,\epsilon_y$$

- Remarks:
  - The emittance is a property of the beam.
  - The Twiss-parameters belong to the transport system.
  - The beam envelope (σ_y) is the convolution of beam and transport system.

Properties - II

So far only particles with the same nominal momentum p_0 are considered. when momentum spread is present, the solution of the equation of motion can be written:

$$y(s) = y_\beta(s) + y_D(s)$$

and

$$y_D(s) = D(s)\,\frac{\Delta \mathrm{p}}{\mathrm{p}_0} \qquad \text{where} \qquad \frac{\Delta \mathrm{p}}{\mathrm{p}_0} = \frac{\mathrm{p} - \mathrm{p}_0}{\mathrm{p}_0}$$

Under the hypothesis of statistical independence, one obtains

$$\sigma^2{}_y = \sigma^2{}_\beta + \sigma^2{}_D = \sigma^2{}_\beta + \left(\frac{\sigma_\mathrm{p}}{\mathrm{p}_0}\,D\right)^2$$

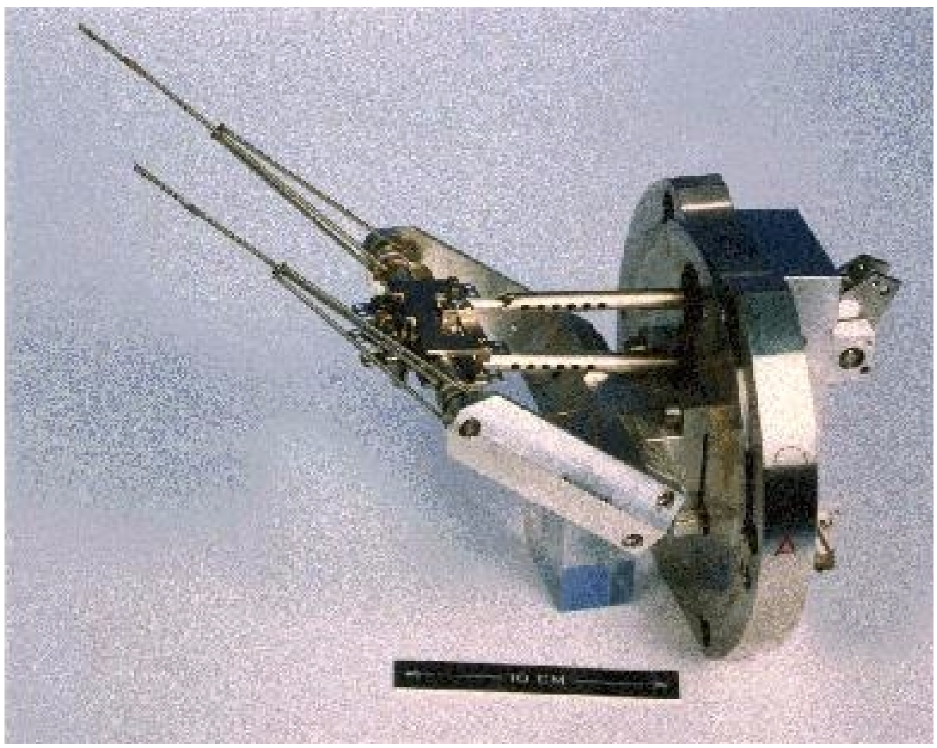
hence

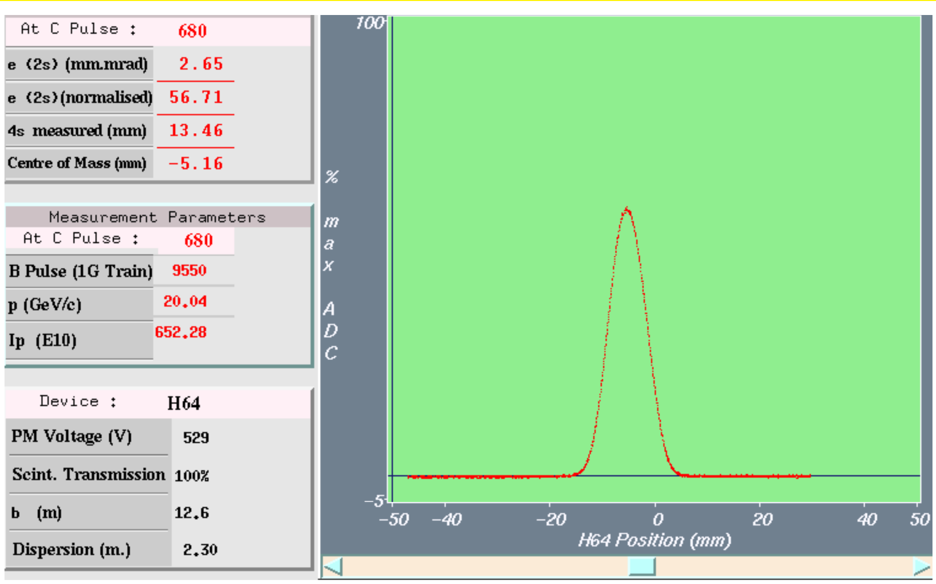
$$\sigma^2{}_\beta = \sigma^2{}_y - \left(\frac{\sigma_\mathrm{p}}{\mathrm{p}_0}\,D\right)^2$$

Measurements in Rings - I

The optics is usually well-known (hence β_y). the beam emittance is then measure by direct application of the very definition.

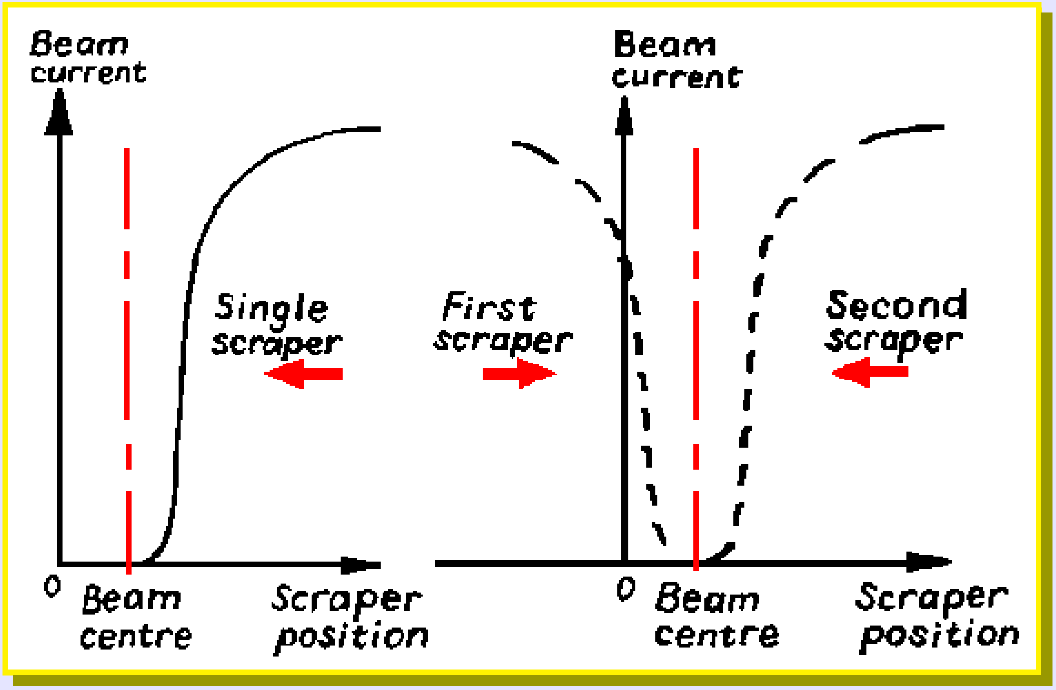
Non destructive devices are: Residual Gas Profile Monitors, Wire Scanners.

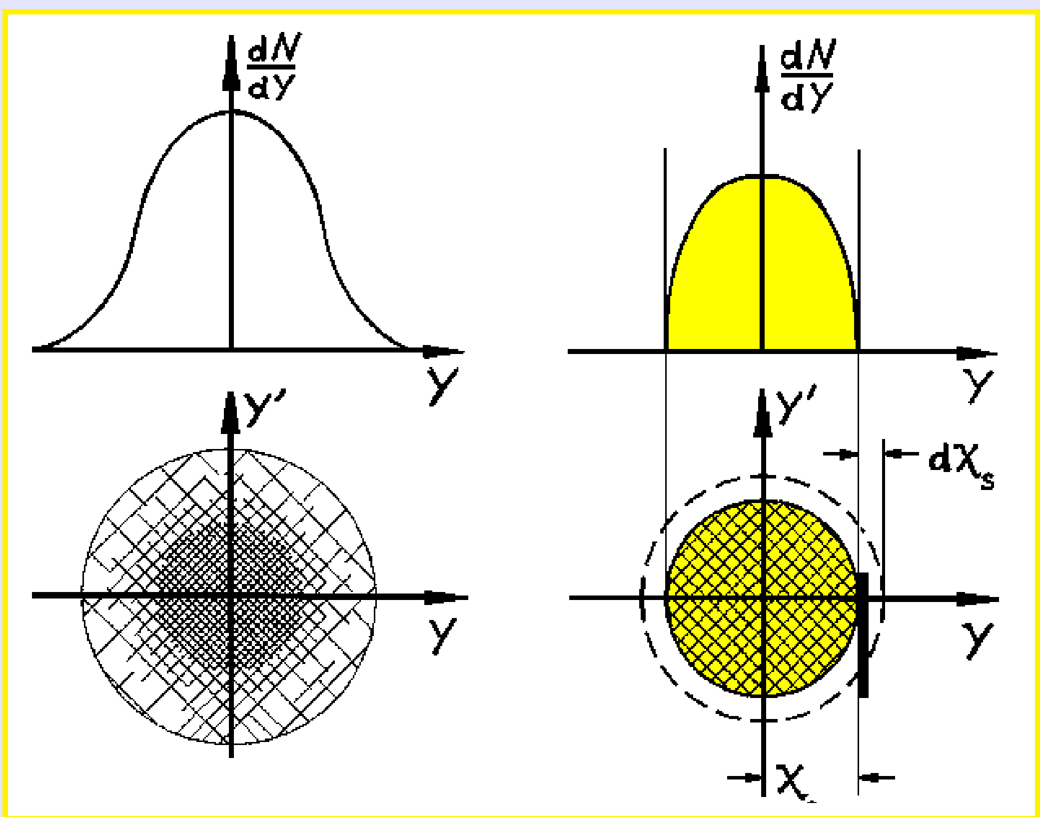




Measurements in Rings - II

Destructive devices are: Beamscope, Fast Scrapers.





From: P. J. Bryant and K. Johnsen "Circular Accelerators and Storage Rings"

Measurements in Transfer Lines - I

- The beam profile should be measured at different locations along the transport channel (usually <u>three</u> monitors are used).

- Some examples of beam profiles monitors used in transfer lines are:

  - Secondary Emission Monitors (wires).
  - Optical Transition Radiation Monitors.
  - Secondary Emission monitors (grids).
  - Multi Proportional Wire Chambers.

Increasing effect on beam:

Multiple Coulomb Scattering

Energy loss

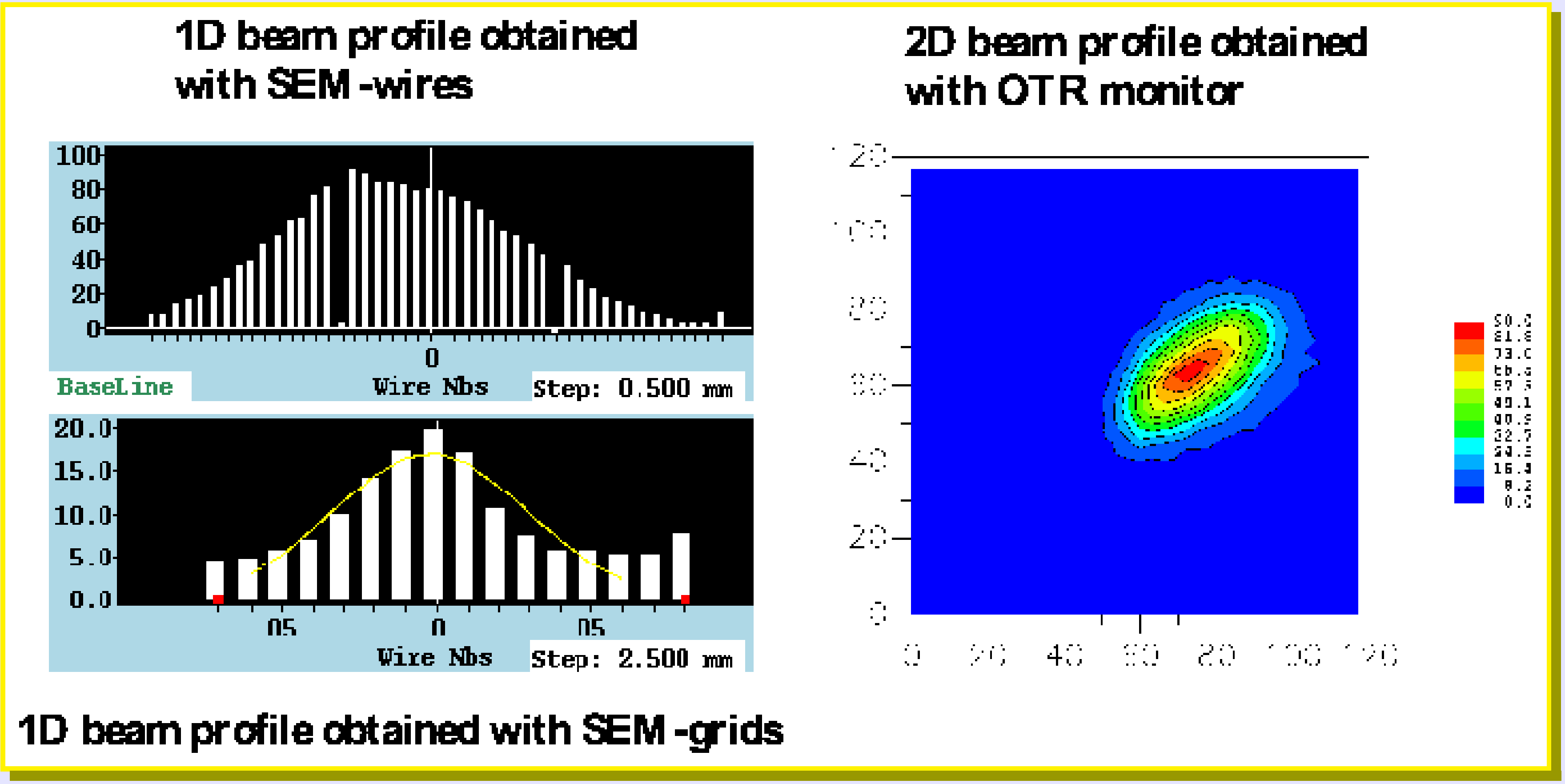
- The transfer matrices between monitors are known.

- The dispersion function at monitors' location is known (it should be measured independently).

- The beam momentum spread $\sigma_{\mathrm{p}}/\mathrm{p}_0$ should be also known.

Measurements in Transfer Lines - II

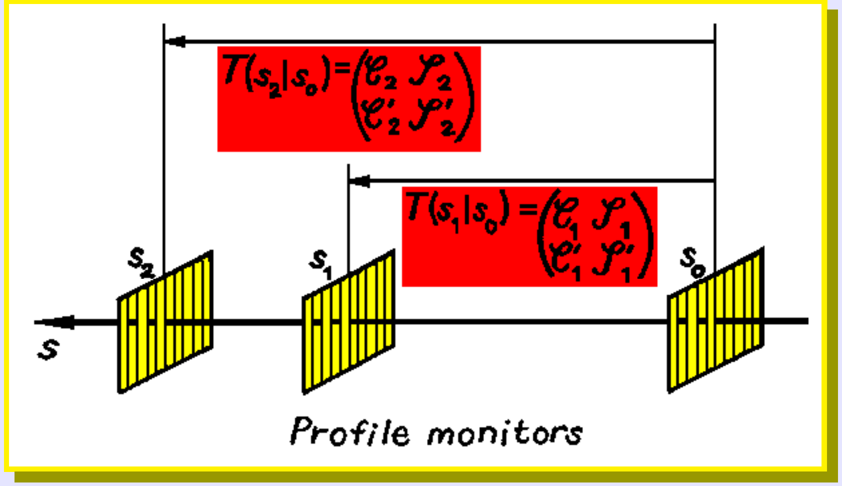
Some examples of beam profiles as measured by SEM-wires, SEM-grids and OTR.



Emittance reconstruction

The deconvolution of beam and optical parameters is based on the following assumptions:

- No linear coupling is present in the transfer line (i.e. no skew quadrupoles or solenoids).
- The beam should fill completely the phase space ellipse.



The following formulas are used

$$\epsilon_y = \sigma^2_{\beta,0}\,\Lambda \qquad \beta_y = \frac{1}{\Lambda} \qquad \alpha_y = \frac{\Gamma}{2\,\Lambda}$$

$$\Gamma = \frac{[(\sigma_{\beta,2}/\sigma_{\beta,0})^2 - C_2^2]/S_2^2 - [(\sigma_{\beta,1}/\sigma_{\beta,0})^2 - C_1^2]/S_1^2}{(C_1/S_1) - (C_2/S_2)}$$

$$\Lambda^2 = (\sigma_{\beta,1}/\sigma_{\beta,0})^2/S_1^2 - (C_1/S_1)^2 + (C_1/S_1)\Gamma - \Gamma^2/4$$

Remark: Under certain hypothesis, D, D' can be measured using more than three monitors.

Error sources - I

Two main error sources affect the reconstructed beam and optical parameters:

- Transport matrices: Their computation is based on the knowledge of the geometry (very accurate) of the transfer line and of the magnetic properties of its elements.

  In general these errors are very small and they can be neglected. Exceptions are extreme cases, i.e. low or high energy beams. Also a possibility exists of direct measuring the transfer matrices.

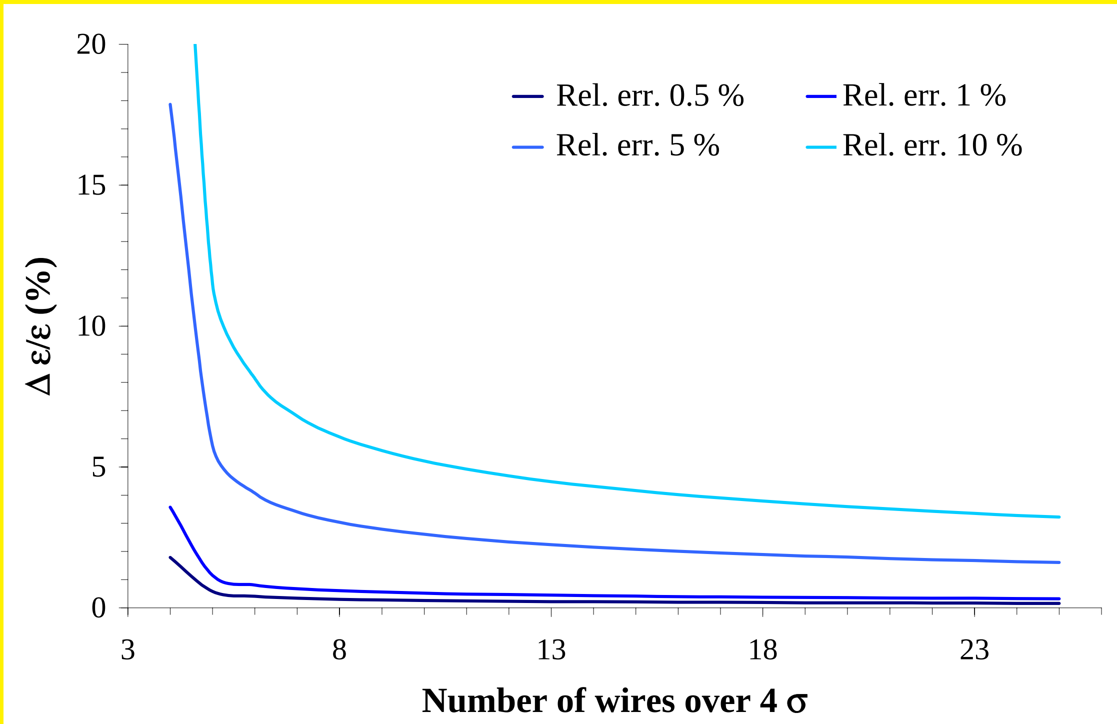
- Beam profiles: Many different effects influence the accuracy of the measured beam profiles.

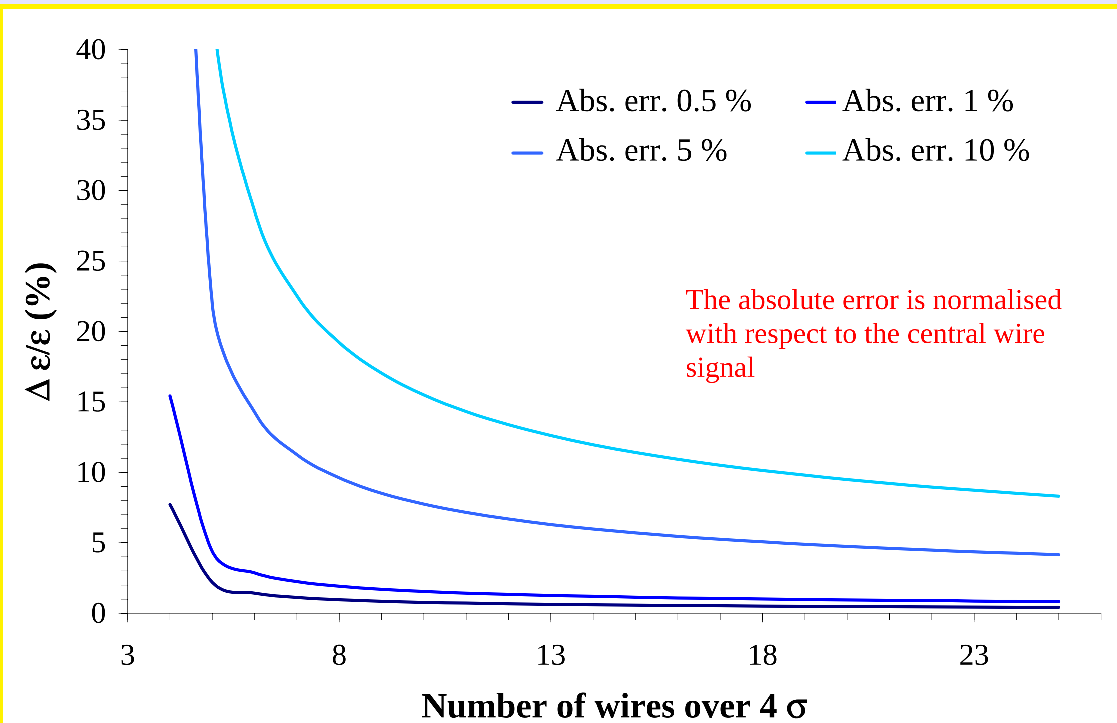
  In the case of SEM-wires (mostly used at CERN) noise on the wires and the number of wires covering the beam width are two important sources of errors.

  The fitting procedure (Gaussian or spline at CERN) can also introduce a bias in the value of the reconstructed emittance.

Error sources - II

The following plots shows the error on the reconstructed beam emittance vs number of wires and relative error per wire (see M. Martini, CERN PS/PA Note 92-03).





Conclusion

It seems interesting to conclude this overview by presenting a paragraph from the conclusions of the *4th ICFA Beam dynamics Mini-Workshop on Transverse Emittance Preservation and Measurement* (CERN PS/DI Note 98-03, by H. Koziol and K. Wittenburg)

> *...The overall conclusion is that no emittance measurement is yet proven to be precise to better than 10 %.*
>
> *Certainly, a number of instruments are basically capable of measuring the beam size quite precisely, but the details of data treatment play an important rôle for the final result.*
>
> *Furthermore, when calculating emittance from beam size, one relies on the knowledge of the beam optical parameters at the place of the instrument and these are often fraught with considerable uncertainties...*