FFAG06 slide

Quantum Aspects of charged particle beam transport

Hiroshi Takahashi
Brookhaven National Laboratory

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Motivation

ADS 10MW Proton Beam Mori, A. Ruggiero,
Medical Application, Proton beam for treating cancers, IPP Program Budker Nuclear Physics Institute, Van de Graaf Proton, Li Resonance, Neutron production. BNCT. Neutron Flux Radiation fields,
Space-charge Problem, Radiation effects in hospital.
Minimize radiation and beam leaks.

High-energy Accelerator; primarily, the beam's dynamics have been studied, but rather little attention paid to the radiation field.

Beam dynamics in magnetic fields.

I derived the quantum aspect of the free –electron laser using Matsubara's Two Times Green Function formalism developed in solid-state physics studies. .

Beam and radiation; Quantum effects play an important role in the condensation of beams.

Recently,
Spohn developed the concept of dynamic charged particle radiation using the Pauli-Fierz Hamiltonian formalism that properly takes into account the magnetic fields. Then, the spin of the beam's particles can be described without difficulty. This formalism is very suitable for assessing radioactive particle acceleration, which recently has attracted interest.

In a slow varying external field, the time-adiabatic theorem offers a good tool that can be used to analyze the particle's dynamics. The stability of the particle beam's dynamics is important, and the small gap between the two- fold degenerated ground state ensures this stability. Spohn's article describes this in detail using Fourier transport for momentum space analysis.

Quantum Field Theory (QFT)

Quantum Field Theory was developed for analyses in many branches of physics. Thus, quantum physics play an essential role in elucidating the condensation of particle and fields in condensed-matter physics, and also in high-energy physics, and cosmology. It provides the basic role, and further extends the more basic theory as afforded by the development of string theory and super-string theory. In this paper, we discuss the possibility of applying these theories to our analysis of particle-beam dynamics.

In determining the particle-beam's dynamics, particles are treated as segmented particles in the enveloping RF beam packets; their longitudinal- and transverse-momentums are solved via Hamiltonian formalism, wherein Fourier transformation is extensively used, expressing the segmented particle beam as modes using the Matrix-valued symbol, and mode coupling by linear-, and nonlinear-theory. For analyzing non- linear theory, many tools are used, such as the Simon theory. Furthermore, group theory has a major role in this analysis because not only do the particles have a an effect on the interaction field, but also the fields between these particles; hence, the whole structure of the particles is analyzed using the group theory.

As we know that the group theory offers details of the structural relation between many subgroups and the many transformations between them , we can study such transformations as unitality and anti-unitality processes. Abelian and non-abelian group theory provides a good foundation for these mathematics, and also the soliton, antisoliton instant theory are used for such analyses.

Our particle-beam dynamics under electro-magnetic fields are dictated in the SU(2) group, and its duality plays an essential role. Recently, many approaches to acceleration have been proposed using the ultra-high-intensity laser, but in using the laminated many-modes laser, the SU(3) group structure intervenes this process, and the possibility of establishing another hierarchy of hadron particles, such as muons, and k mesons is the outcome; here, quark-gluon physics will be involved as well as electron and photon physics based on the SU(2) group. The mathematical tools that were developed for quantum chromodynamics (QCD) calculations can be utilized for these analyses.

As mentioned, quantum physics play an important role in charged-particle transportation, and the radiation field associated with it. The condensation of the particle and the associated radiation fields occur through the quantum effect; my theory of free electrons was analyzed some 20 years ago using the second quantification applied as a classical field created by relativist electrons wiggling through the magnetic wiggler using the two times Green function method; however, the relativistic electron beam created at that time was not as refined as present-day ones, and the necessity of using quantum theory for analysis was not appreciated. The above theory, using the two times Green function method, is not recognized in the physics community. However, during the last few decades, the technology of the relativistic electron beam has developed at a rapid pace, and in analyses of the interaction of the electron beam with an extremely high-intensity laser, the quantum effect has a very crucial role. Quantum physics has entailed very controversial issues from its beginning; many discussions on its measurement theory developed for observing physical realities are described in the Einstein , Podowsky- Rosen (EPR) theory and the discussion between Einstein and Bohr's ideas still continues today.

Many methods of quantization have been developed, such as the original Heisenberg's chanonical method, the second quantization method using the creation operator annihilation operator, geometrical quantization, stochastic quantization have been developed; they have many facets and the analyses are carried out with them. .

Among the non-linear theories, the Simon theory is used, and the quark and gluon physics can be treated within this formalism; the classical string theory and super-string theory are easily implemented. Thus, particle acceleration by using the strong laser field is formulated, and manipulation of the quark can be carried out. In this paper, we discuss two methods emerging in recent new mathematical tools, one close to the canonical method, and the other close to the Yang-Mill Gauge theory. They are developed for the foundation of QCD calculations, the Lattice QCD, and Relativistic Heavy Ion collider analysis (RHIC) and Large Hadron Collider (LHC) experiments; the possibility of generating muon particles is studied within this formalism.

Quantum field theory provides the foundation of these physics.

Beam particles are treated as segmented particles; Fourier transformation is extensively used. In the FFAG beam,

acceleration is accomplished using the magnetic field of many poles, and the RF created
in the acceleration cavity along with the beam's path in the accelerator,
Classical beam dynamics are analyzed by methods developed in accelerator physics, and the space charge effect on the beam's stability is analyzed . When the quantum effect describes the condensation of particle beam and the associated field , it plays an important role in analysis, and for this many quantization methods can be applied . I discussed the method using the projective Fourier Duality and Weyl unitarization . This methodology is very closely related to the Yang Mill Gauge theory. The familiar Weyl-Wigner prescription is used extensively in Fourier duality , and its analysis can be performed via Kac algebra developed as a general background for non commutative Fourier analysis. The standard Kac and symmetric Kac algebra describe the new dual framework and well define the projective Fourier duality for the group of translation along the plane. The non-plane geometry can be extended by taking into account the nonlinearity in a similar way as the Simon's analysis.

The details of this are presented in the paper in R. Aldrovandiand L.A. Saeger in the Funct- an /9608004.

The paper starts with a discussion of classical and quantum mechanics in phase space. The Heisenberg group in three – dimensional H_3 is extended to the two dimensional Abelian group of translation on the plane by the torus T. A fourier duality for the Heisenber Group is discussed through the Kac algebra of H_3, $\mathbb{K}^s(H_3)$, Projective Kac algebras and its dual algebra of $\mathbb{K}^s(H_3)$, and the Weyl quantization and duality are described. By using these functional forms the relation between the subgroups are clearly defined and calculations rendered more clearly. Recently, object- oriented programming was extensively developed and a detail analysis can be done through Monte Carlo calculation with a more flexible geometry including algebra manifolds, not restricting the QCD Lattice theory based on the rectangular lattice geometry , but having a hexagonal lattice and Kagome lattice that are fashionable in high Tc super conductor study.

Summarizing the paper, new mathematics have been developed at a rapid pace and the associated computational capability extended, so that the charged particle dynamics can be analyzed from a more fundamental basis than before. We can design not only a high-energy accelerator but also a practical accelerator for medical use, along withmany apparatus in many fields of science and technologies including the implantation of the ions beam.

The equations used this formalism are the following

$$i_{X_H}\omega = -dH.$$

$$\{f, g\} = -\omega(X_f, X_g).$$

$$[\mathcal{F}f](x,y) = \tilde{f}(x,y) = \frac{1}{2\pi}\int_{\mathbb{R}^2} dqdp\, f(p,q)\, e^{-i(yq+xp)},$$

$$\int_{\mathbb{R}^2} dxdy\, \chi_{(x,y)}(q,p)\overline{\chi_{(x,y)}(q',p')} = (2\pi)^2\delta(q-q')\delta(p-p'),$$

$$f(p,q) = \frac{1}{2\pi}\int_{\mathbb{R}^2} dxdy\, \tilde{f}(x,y)\, e^{i(yq+xp)}.$$

$$\hat{f}_\hbar = \int_{\mathbb{R}^2} dqdp\, f(q,p)\, e^{-\frac{i}{\hbar}(p\hat{q}+q\hat{p})},$$

the Glauber identity, the operatorial kernel $S'_\hbar(x,y) \equiv e^{-\frac{i}{\hbar}(y\hat{q}+x\hat{p})}$ can be written also as

$$S_\hbar(x,y) = e^{\frac{i}{2\hbar}xy}\, U(y)V(x),$$

in terms of the Weyl operators $U(y) = e^{-\frac{i}{\hbar}y\hat{q}}$ and $V(x) = e^{-\frac{i}{\hbar}x\hat{p}}$.

$$U(y)V(x) = e^{-\frac{i}{\hbar}xy}\, V(x)U(y),$$

while for $S_\hbar$ holds

$$S_\hbar(x,y)S_\hbar(x',y') = e^{\frac{i}{2\hbar}(xy'-yx')} S_\hbar(x+x', y+y').$$

In this work the three-dimensional Heisenberg group H_3 is regarded as the cen the two-dimensional Abelian group of translations on the plane by the torus $\mathbb{T}$.

an *integration* on operator space (taking of the trace):

$$f(q,p) = \mathrm{Tr}[S_\hbar^\dagger(q,p)\hat{f}_\hbar].$$

twisted:

$$\begin{aligned} \hat{f}_\hbar \cdot \hat{g}_\hbar &= \int_{\mathbb{R}^2\times\mathbb{R}^2} dxdydx'dy' f(x,y)g(x',y')\, e^{\frac{i}{2\hbar}(xy'-yx')} S_\hbar(x+x', y+y') \\ &= \int_{\mathbb{R}^2} dx''dy''\, (f \circledast g)(x'',y'')\, S_\hbar(x'',y''), \end{aligned}$$

where

$$(f \circledast g)(x'',y'') = \int_{\mathbb{R}^2} dxdy\, e^{\frac{i}{2\hbar}(xy''-yx'')} f(x,y)g(x''-x, y''-y).$$

give rise to inequivalent central extensions. Thus, for a chosen cocycle $\Omega \in H^2(\mathbb{R}^2, \mathbb{R}/2\pi)$, e.g.

$$\Omega(x, y) = \frac{1}{2}(x_1 y_2 - y_1 x_2), \tag{7}$$

the product on $H_3 = \mathbb{R}^2 \times \mathbb{T}$ is given by

$$(x, \alpha)(y, \beta) = (x + y, \alpha\beta\, e^{i\Omega(x,y)}),$$

where associativity is ensured by the closeness of Ω in H^2, namely,

$$\delta\Omega(x, y, z) = \Omega(y, z) - \Omega(x + y, z) + \Omega(x, y + z) - \Omega(x, y) = 0.$$

restricting to $\mathbb{R}^2$ the irreducible linear representations of H_3 shown in

$$[S_\nu(x)\xi](q) = e^{-i\nu\Theta(q;-x)}\xi(q - x_1), \qquad \nu \in \mathbb{Z} - \{0\}.$$

$$L_\Omega = \sum_{\nu \in \mathbb{Z}-\{0\}}^{\oplus} \mu(\nu)\, S_\nu.$$

$$[S_\nu(x) f_\nu](y) = e^{i\nu\Omega(x,y)}\, f_\nu(y - x),$$

$L^1_\Omega(\mathbb{R}^2)$ through

$$L_\Omega(f) = \sum_{\nu \in \mathbb{Z}-\{0\}} \mu(\nu)\, S_\nu(f),$$

where

$$S_\nu(f) \equiv \hat{f}_\nu = \int_{\mathbb{R}^2} dx\, f(x)\, S_\nu(x).$$

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