Proton acceleration with added edge focusing

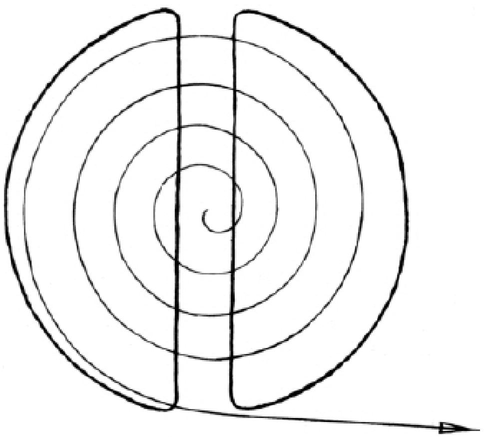
- Introduction:
  - Rick Baartman: “Spiral focusing slides”

- Fixed parameters: QF, QD, θ_{QD}, θ_{QF}
  Variables: edge angles

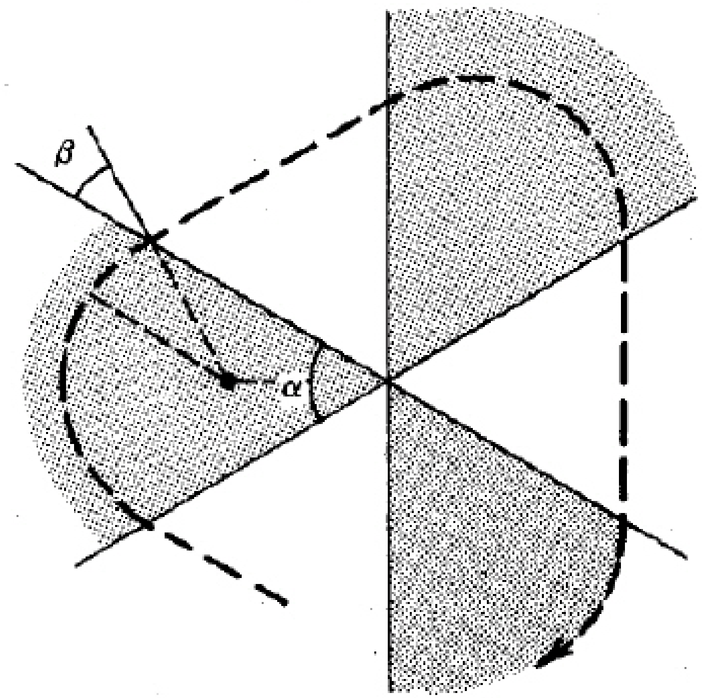
Slides from the Rick Baartman presentation: 'Cyclotrons: Classic to FFAG' - 2002

Invention (Lawrence, 1930)

$mv^2/r = qvB$, so $m\omega_0 = qB$, with $r = v/\omega_0$



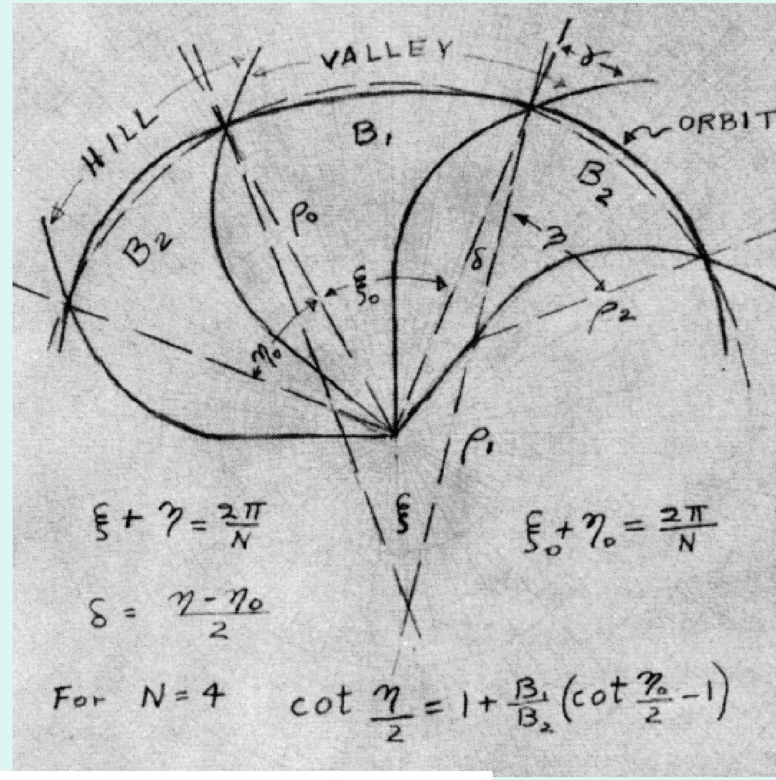
With B constant in time and uniform in space, as particles gain energy from the rf system, they stay in synchronism, but spiral outward in r.



Thomas focusing and later the Okhawa-Symon-Kolomenski FFAG

Slides from the Rick Baartman presentation:

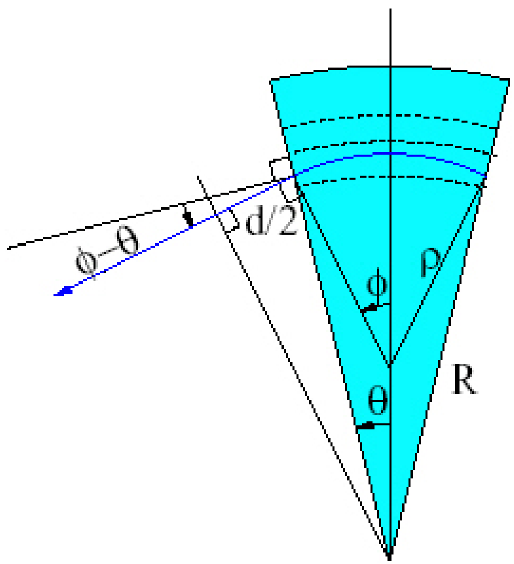
In 1954, Kerst realized that the sectors need not be symmetric.By tilting the edges, the one edge became more focusing and theother edge less. But by the strong focusing principle (larger betatron amplitudes in focusing, smaller in defocusing), one could gain nevertheless. This had the important advantage that reverse bends would not be needed (reverse bends made the machine excessively large). (Figure is from J.R. Richardson notes.) The resulting machines no longer had *alternating gradients*, but Kerst and Symon called them FFAGs anyway.



$$M_z := \left(\begin{pmatrix} \mathrm{Cosh}[k_y\, s] & \mathrm{Sinh}[k_y\, s]/k_y \\ k_y\, \mathrm{Sinh}[k_y\, s] & \mathrm{Cosh}[k_y\, s] \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{pmatrix} \cdot \begin{pmatrix} \mathrm{Cosh}[k_y\, s] & \mathrm{Sinh}[k_y\, s]/k_y \\ k_y\, \mathrm{Sinh}[k_y\, s] & \mathrm{Cosh}[k_y\, s] \end{pmatrix}\right)$$

Slides from the Rick Baartman presentation:

Tunes in an FFAG



$$\frac{\sin(\theta)}{\rho} = \frac{\sin(\phi)}{R}$$

$$d/2 = R\sin(\phi - \theta)$$

To make it transparent, let us consider all identical dipoles and drifts; no reverse bends. We have drifts d, dipoles with index k, radius ρ, bend angle ϕ, and edge angles $\phi - \theta$:

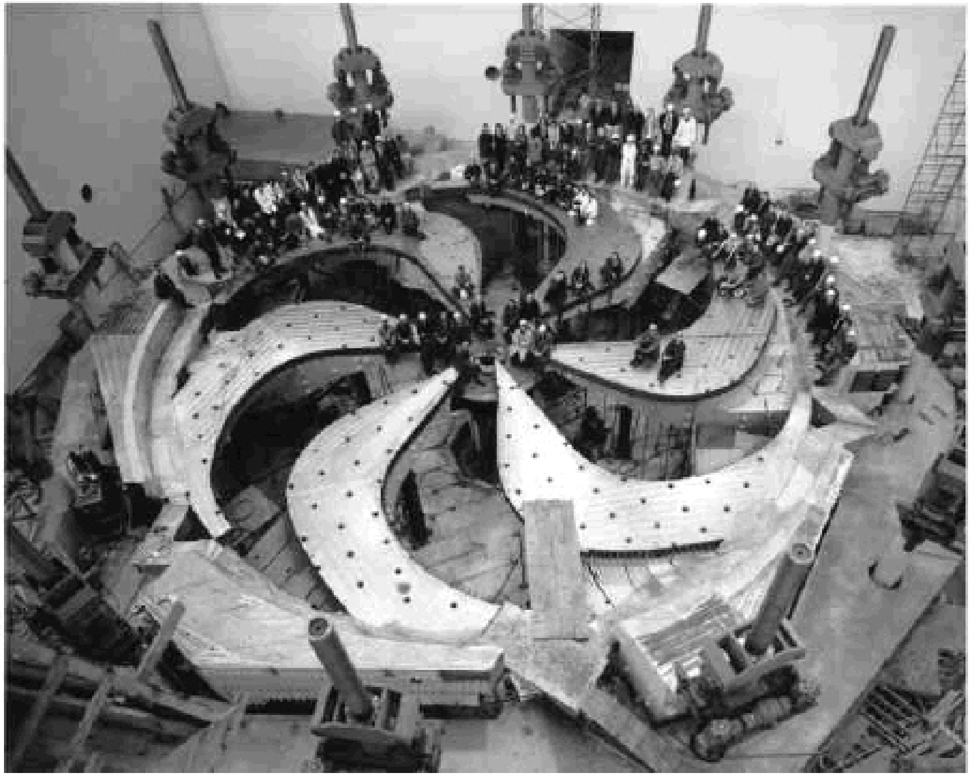
In addition, imagine that the edges are inclined by an extra angle ξ. This is called the “spiral angle” (hard to draw).

In this hard-edged case, the “flutter” $F^2 \equiv \langle (B - \overline{B})^2 \rangle / \overline{B}^2 = R/\rho - 1$.

Aside: Notice that the particle trajectory (blue curve) does not coincide with a contour of constant B (dashed curves). This has large implications for using existing transport codes to describe FFAGs.

Slides from the Rick Baartman presentation:

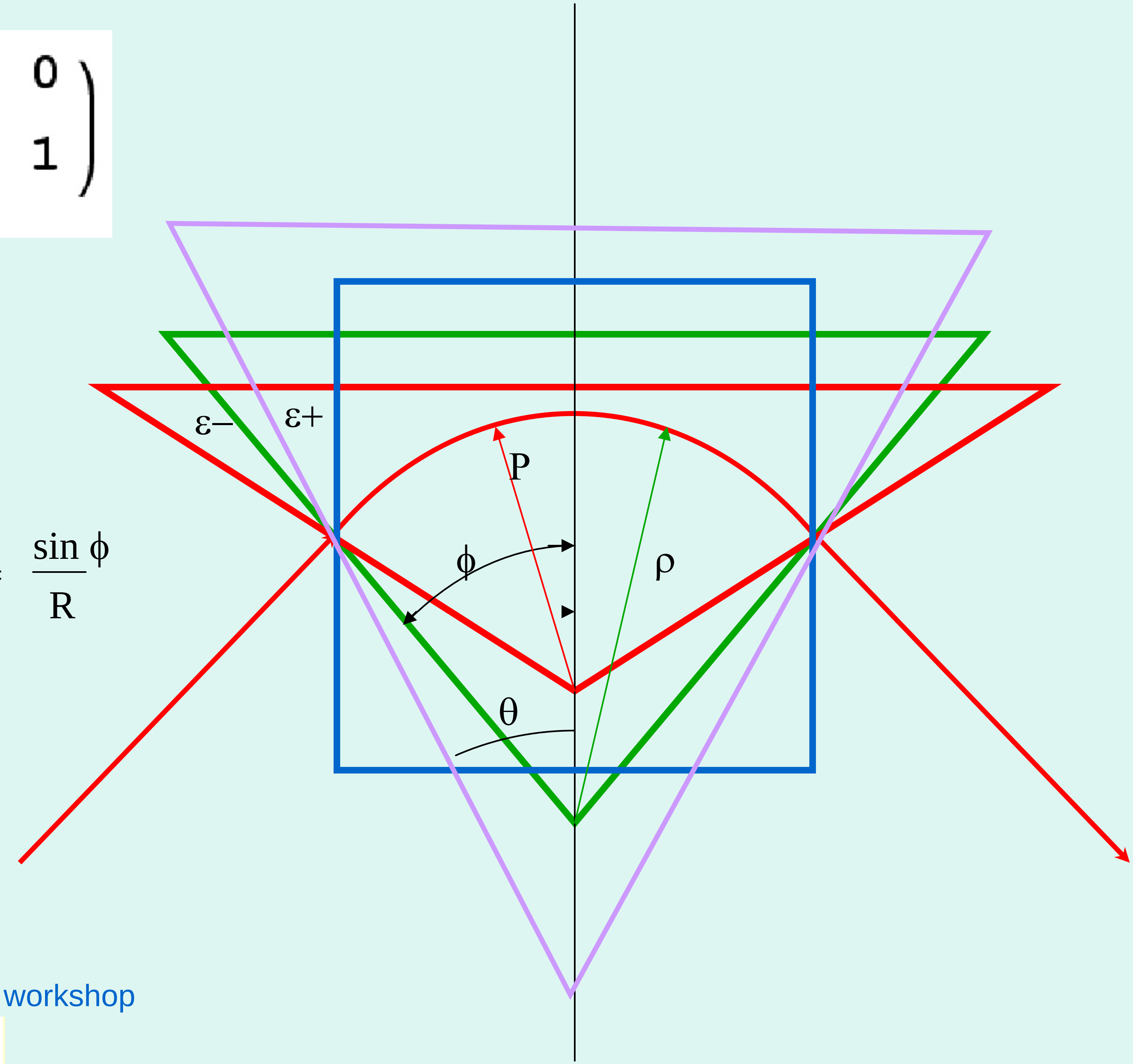
Example: TRIUMF cyclotron



Energy	R	$\beta\gamma$	ξ	$1+2\tan^2\xi$	F^2	ν_z
100 MeV	175 in.	0.47	0°	0.0	0.30	0.28
250 MeV	251 in.	0.78	47°	3.3	0.20	0.24
505 MeV	311 in.	1.17	72°	20.0	0.07	0.24

$$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{pmatrix}$$

$$\frac{\sin\theta}{\rho} = \frac{\sin\phi}{R}$$



Polymorphic Tracking Code (PTC) edge effect calculation

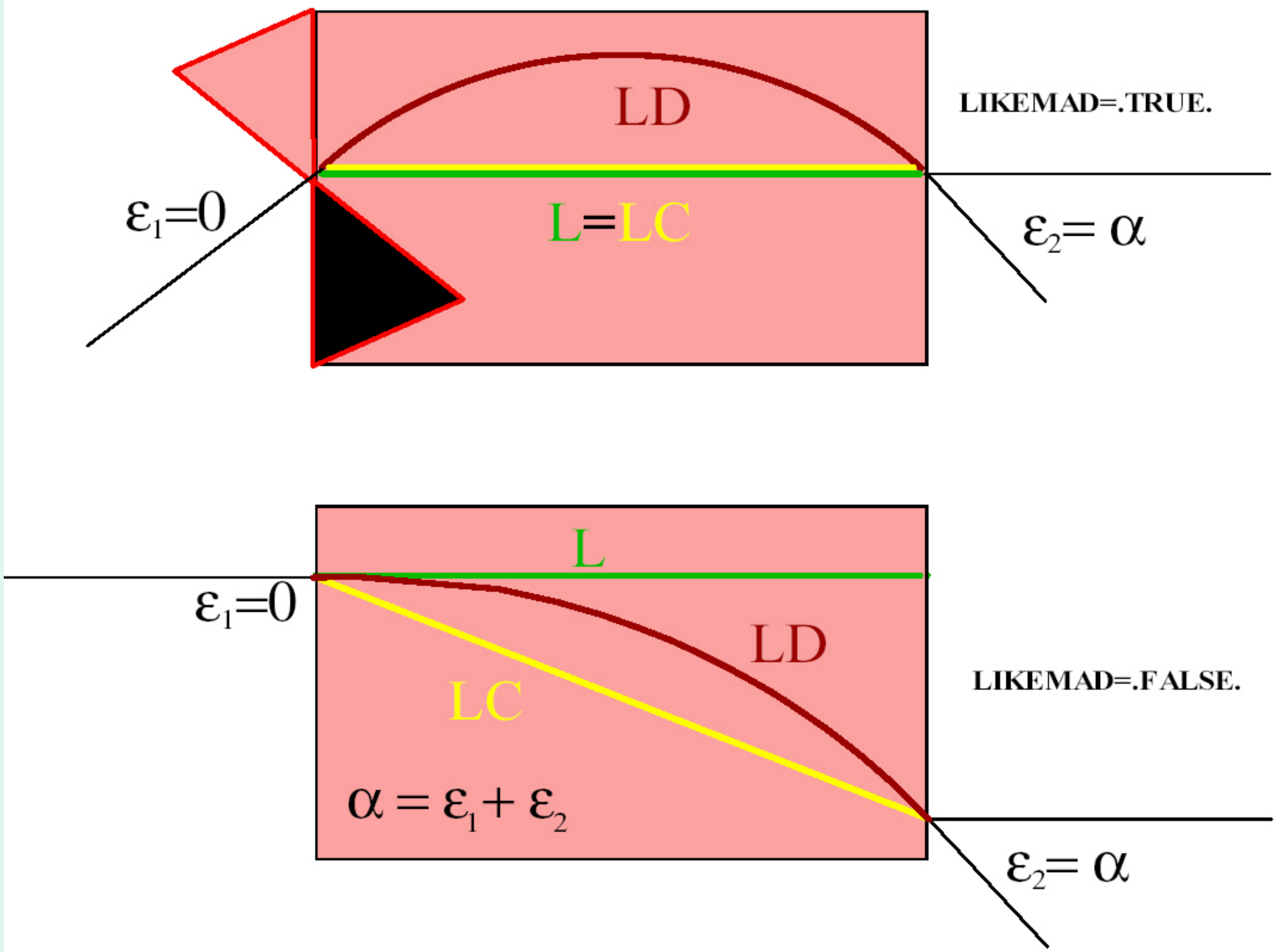


Figure 19: Two Different Bends constructed from the RBEND by PTC

Dejan Trbojevic
December 4, 2005

Q.5.8 The Rectangular Bend

The rectangular bend is created with the command

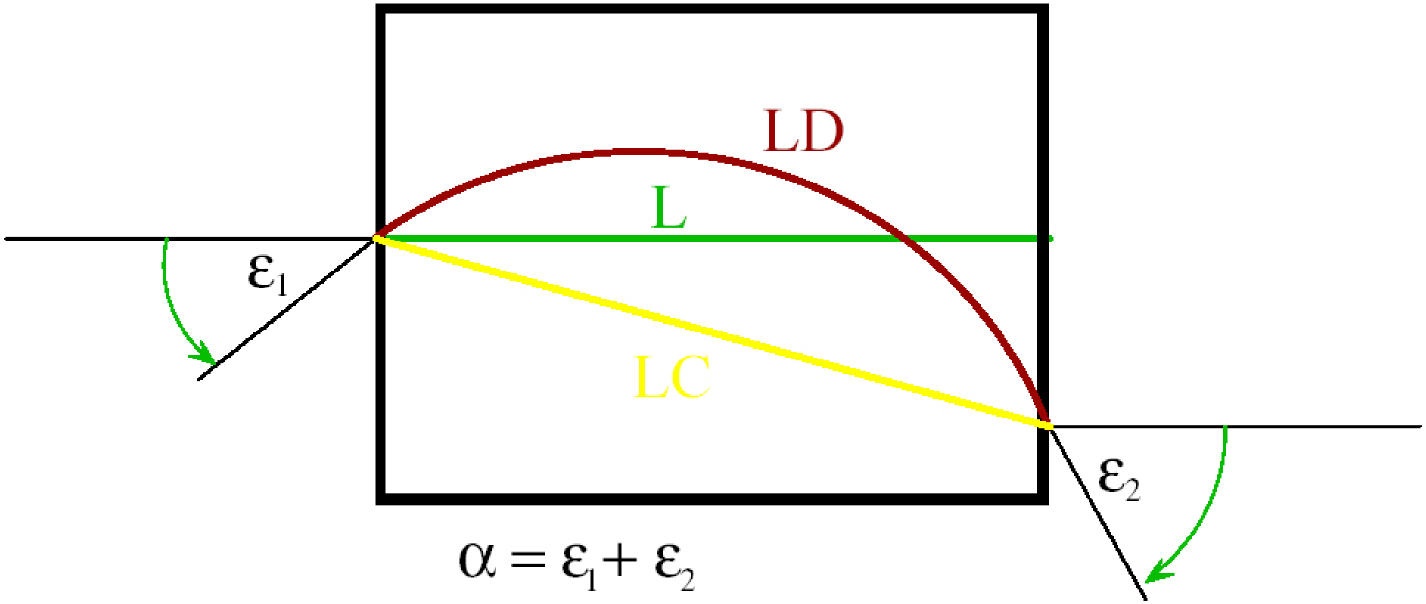


Figure 22: The True RBEND of PTC

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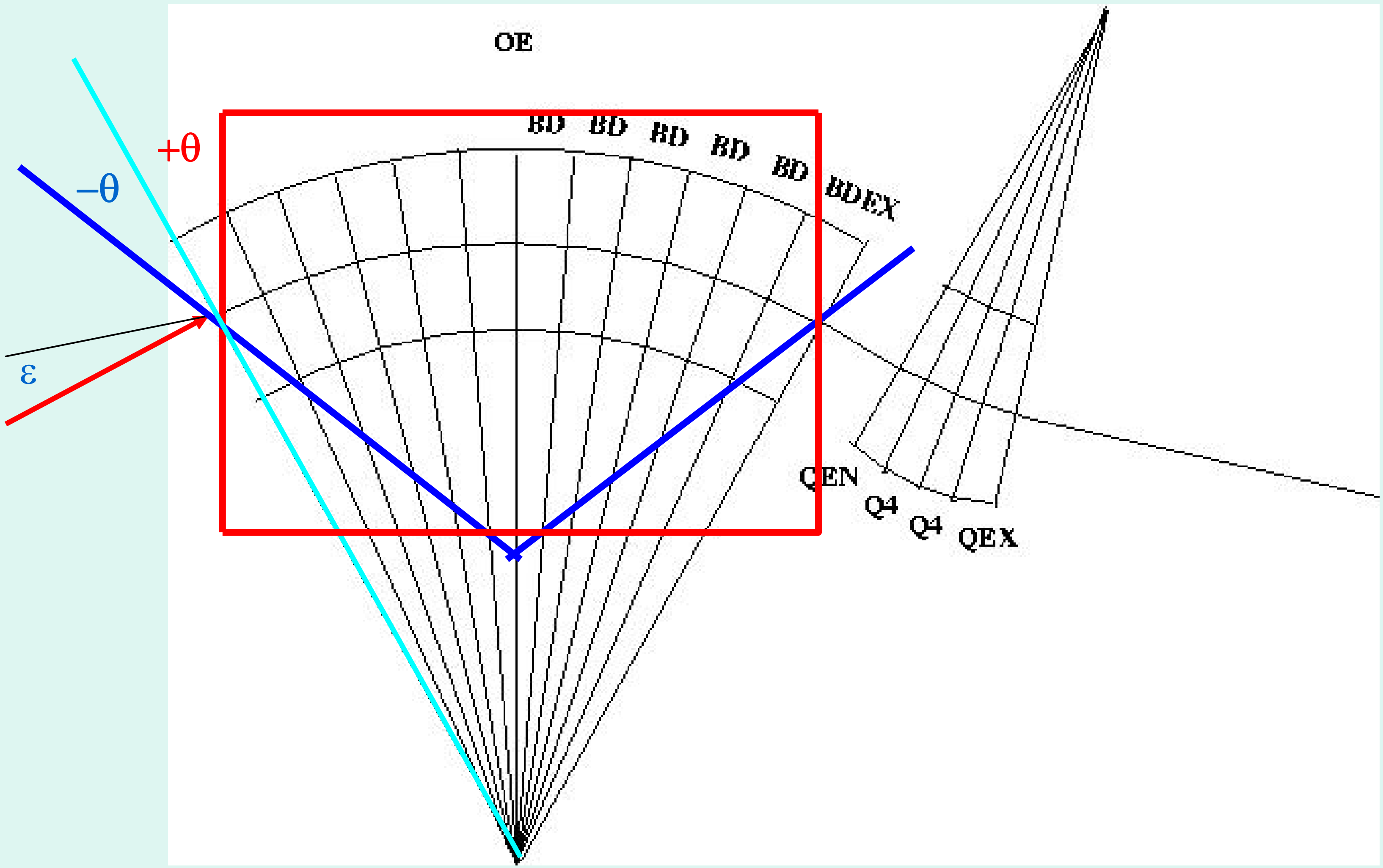
April 3, 2005



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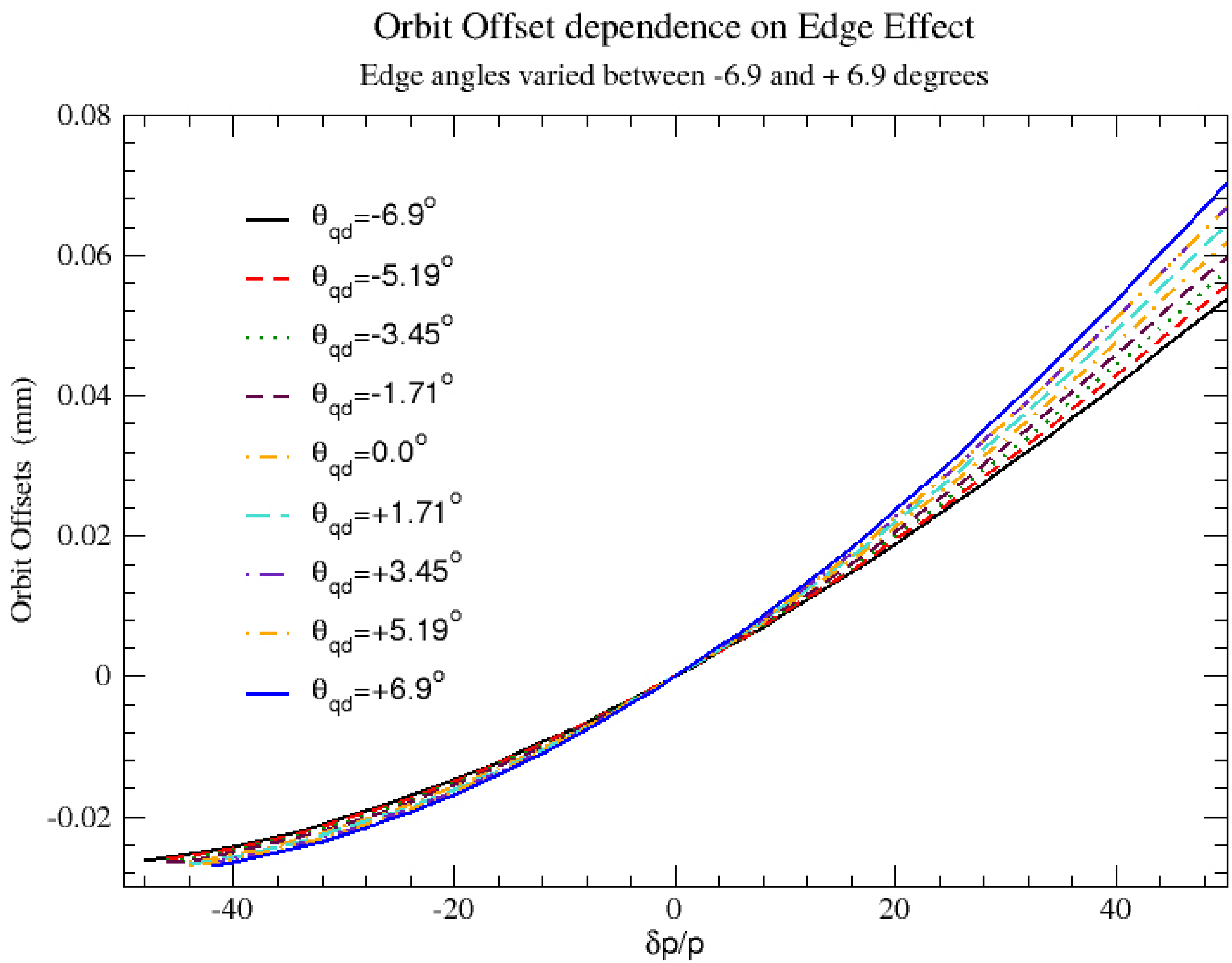

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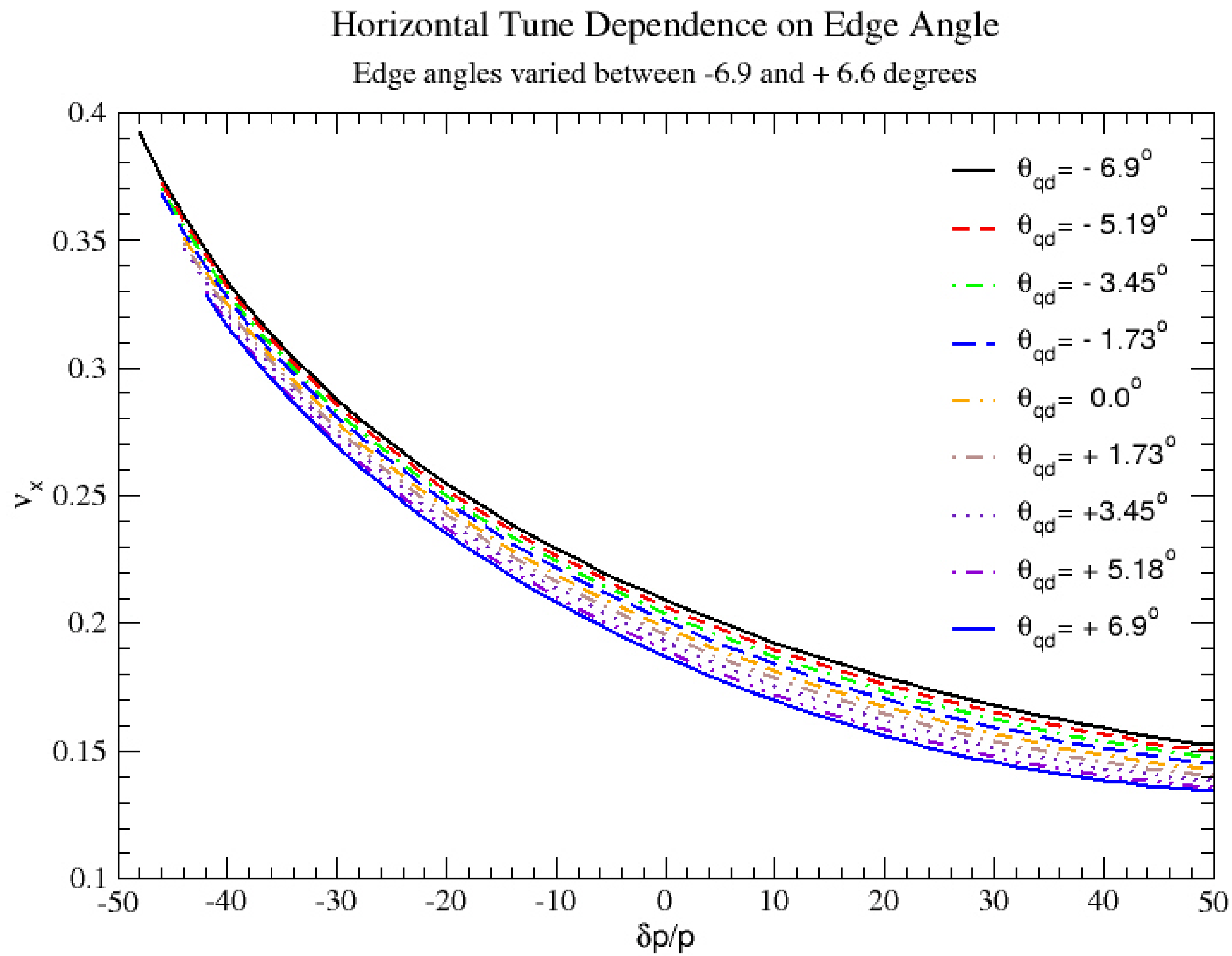
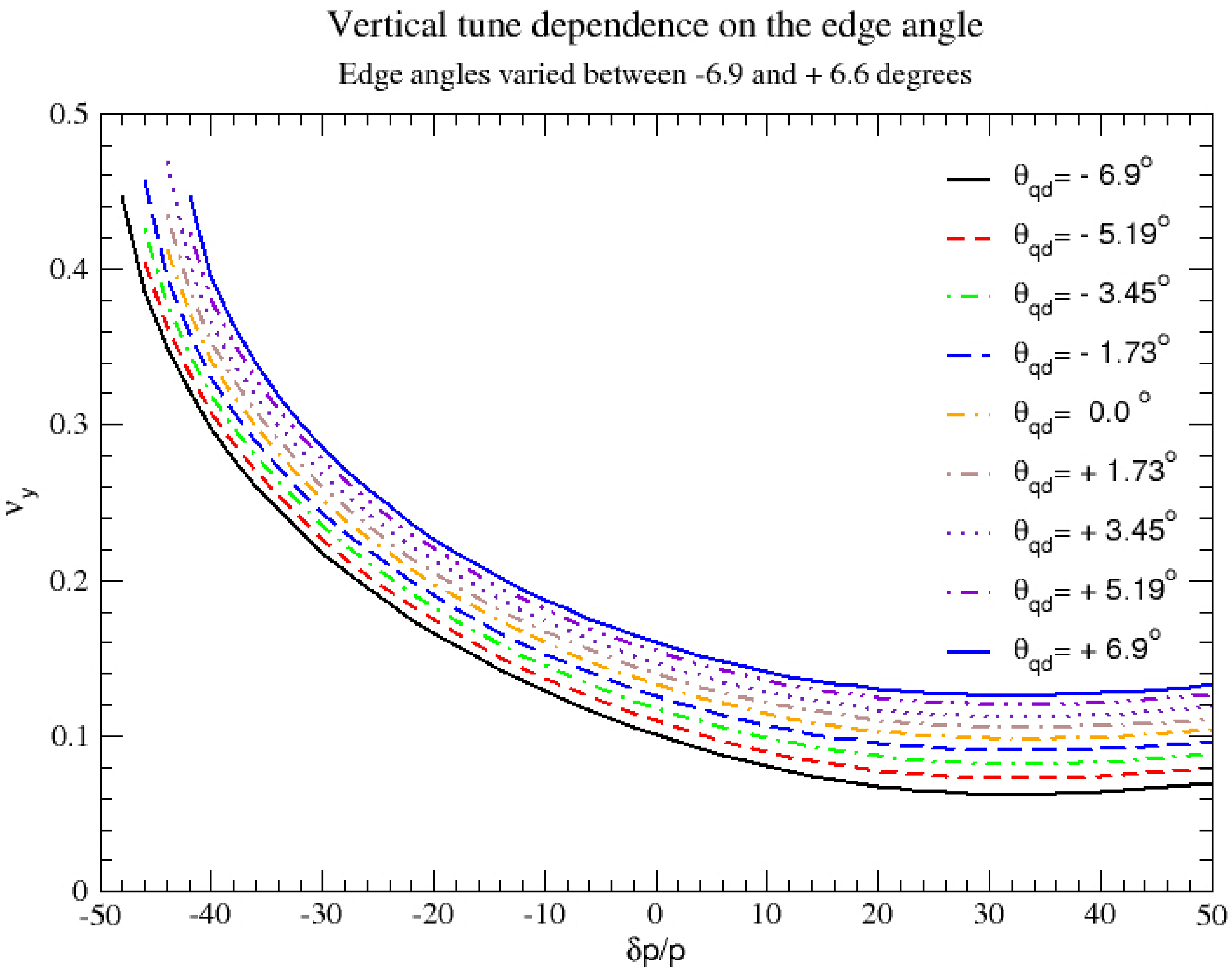
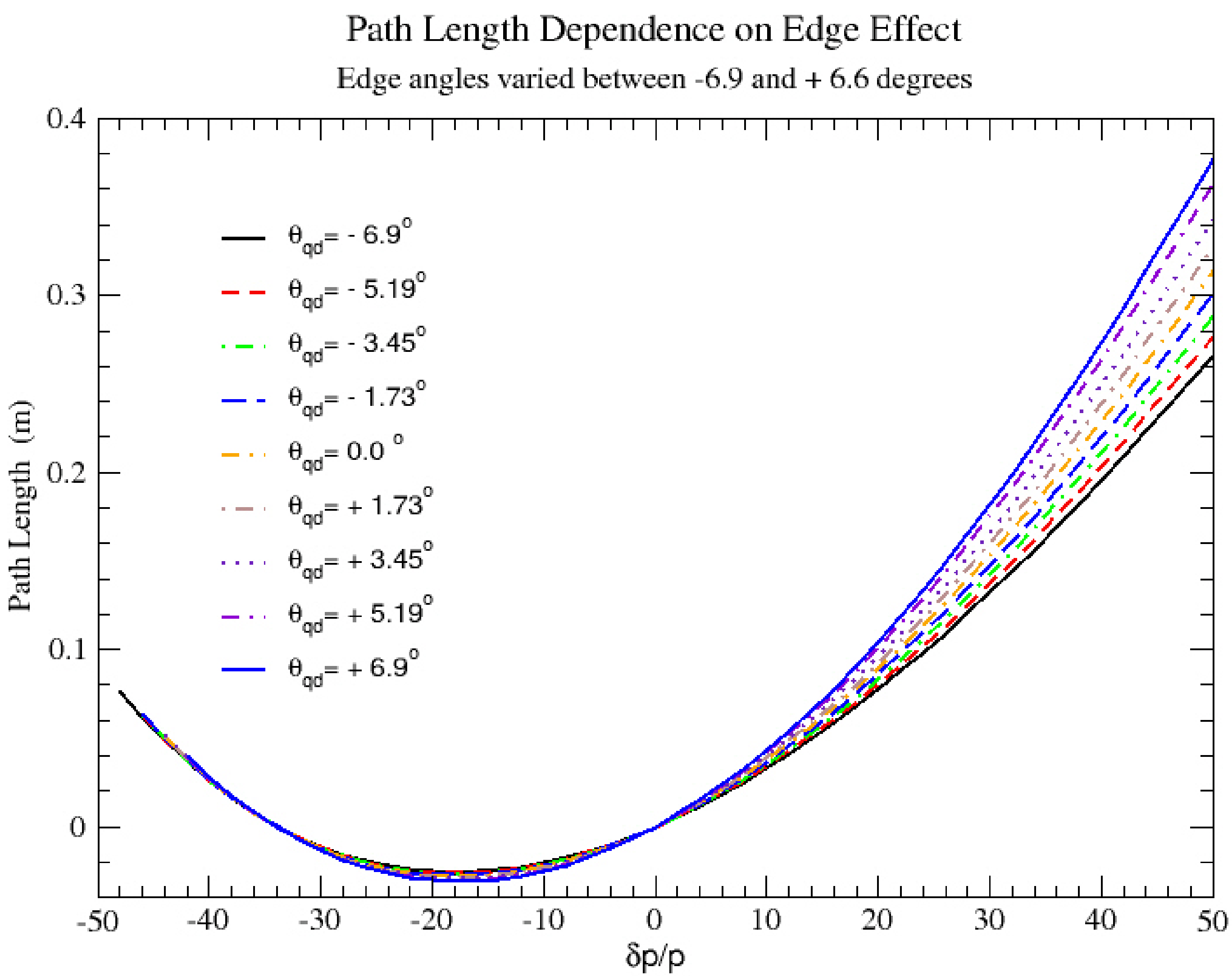

Horizontal Tune Dependence on Edge Angle
Edge angles varied between -6.9 and + 6.6 degrees
ν_x
δp/p
0.4
0.35
0.3
0.25
0.2
0.15
0.1
-50
-40
-30
-20
-10
0
10
20
30
40
50
θ_{qd}= - 6.9°
θ_{qd}= - 5.19°
θ_{qd}= - 3.45°
θ_{qd}= - 1.73°
θ_{qd}= 0.0°
θ_{qd}= + 1.73°
θ_{qd}= +3.45°
θ_{qd}= + 5.18°
θ_{qd}= + 6.9°


Vertical tune dependence on the edge angle
Edge angles varied between -6.9 and + 6.6 degrees
0.5
0.4
0.3
0.2
0.1
0
-50
-40
-30
-20
-10
0
10
20
30
40
50
δp/p
ν_y
θ_qd= - 6.9°
θ_qd= - 5.19°
θ_qd= - 3.45°
θ_qd= - 1.73°
θ_qd= 0.0 °
θ_qd= + 1.73°
θ_qd= + 3.45°
θ_qd= + 5.19°
θ_qd= + 6.9°



Path Length Dependence on Edge Effect
Edge angles varied between -6.9 and + 6.6 degrees
Path Length (m)
δp/p
θqd= - 6.9°
θqd= - 5.19°
θqd= - 3.45°
θqd= - 1.73°
θqd= 0.0 °
θqd= + 1.73°
θqd= + 3.45°
θqd= + 5.19°
θqd= + 6.9°
0.4
0.3
0.2
0.1
0
-50
-40
-30
-20
-10
0
10
20
30
40
50