Harmonic Jumping

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Consider Model of Harmonic Jumping FFAG for AGS Booster

$$E_2(proton) \;=\; 1.5 \;\; \mathrm{GeV}$$

so

$$E_2(electron) \;=\; 1500\,\frac{.51}{930} \;=\; 0.82 \;\; \mathrm{MeV}$$

For $E_2/E_1) = 6$ then

$$E_1(electron) \;=\; 140 \;\; \mathrm{MeV}$$

Simple Non-Relativistic Theory for electron ring

L is circumference

f is frequency

$$h = \frac{f\ L}{v} \approx \frac{f\ L}{c}\sqrt{\frac{m\ c^2}{2\ E}}$$

Differentiate and set $dh = 1$ gives $\qquad \Delta E = \frac{2\ E}{h}$

$$\Delta E(max) = \frac{2\ E_2}{h_2}$$

so for $\Delta E/E$=100 i.e. gain over linac of 100, then

$$h_2 = 200$$

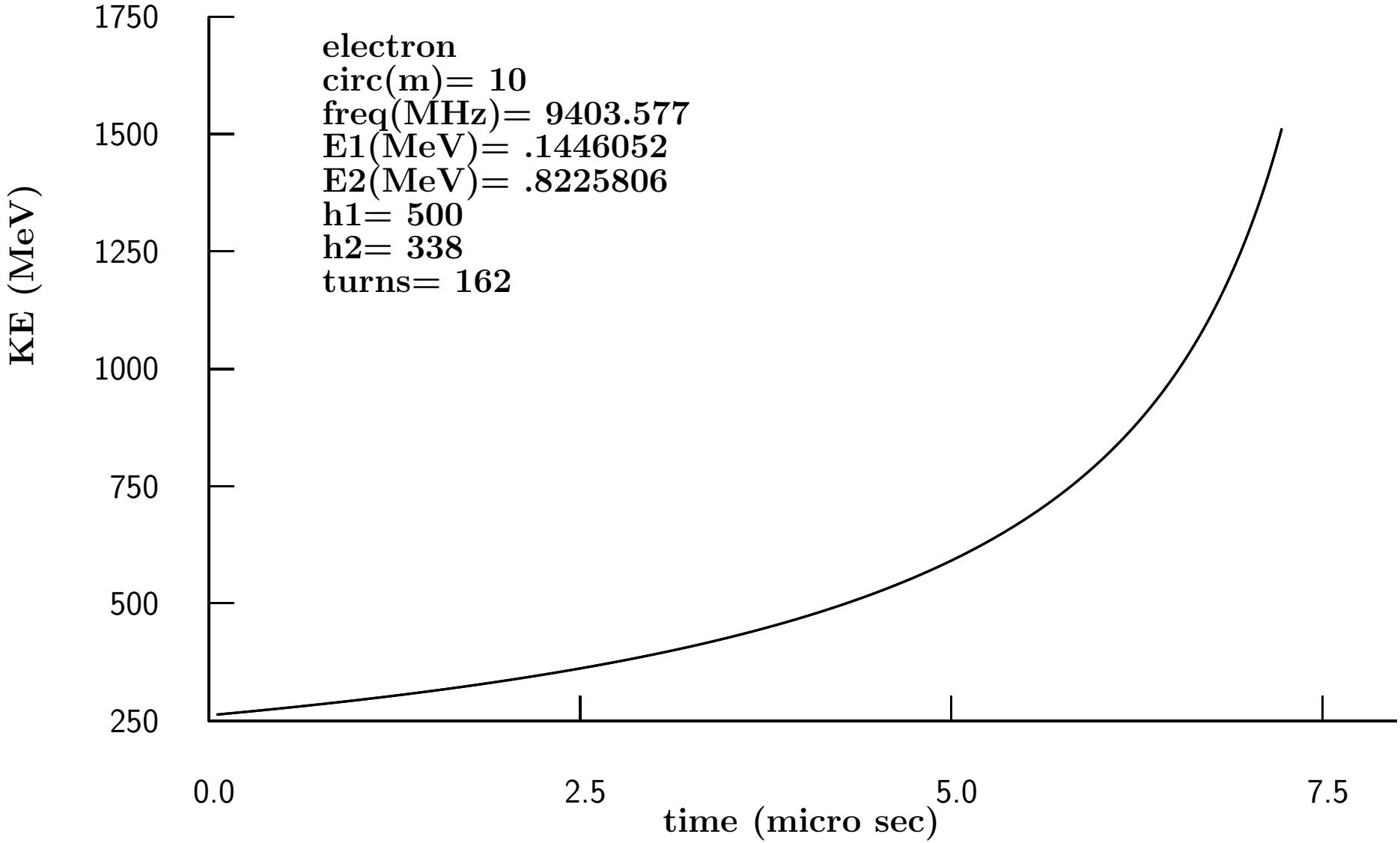
and for $E_2/E_1 = 6$

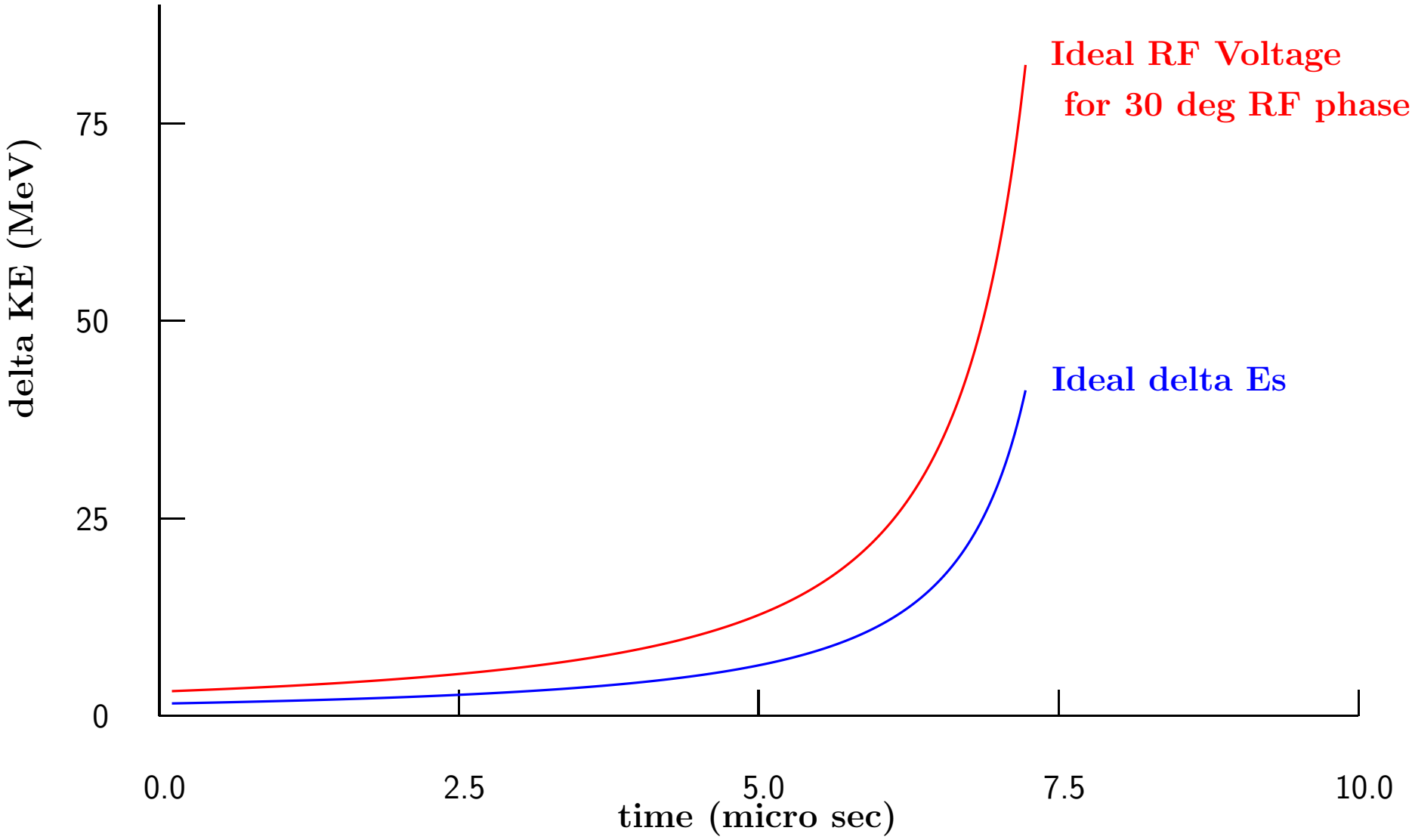
$$h_1 = h_2\sqrt{\frac{E_2}{E_1}} \approx 500$$

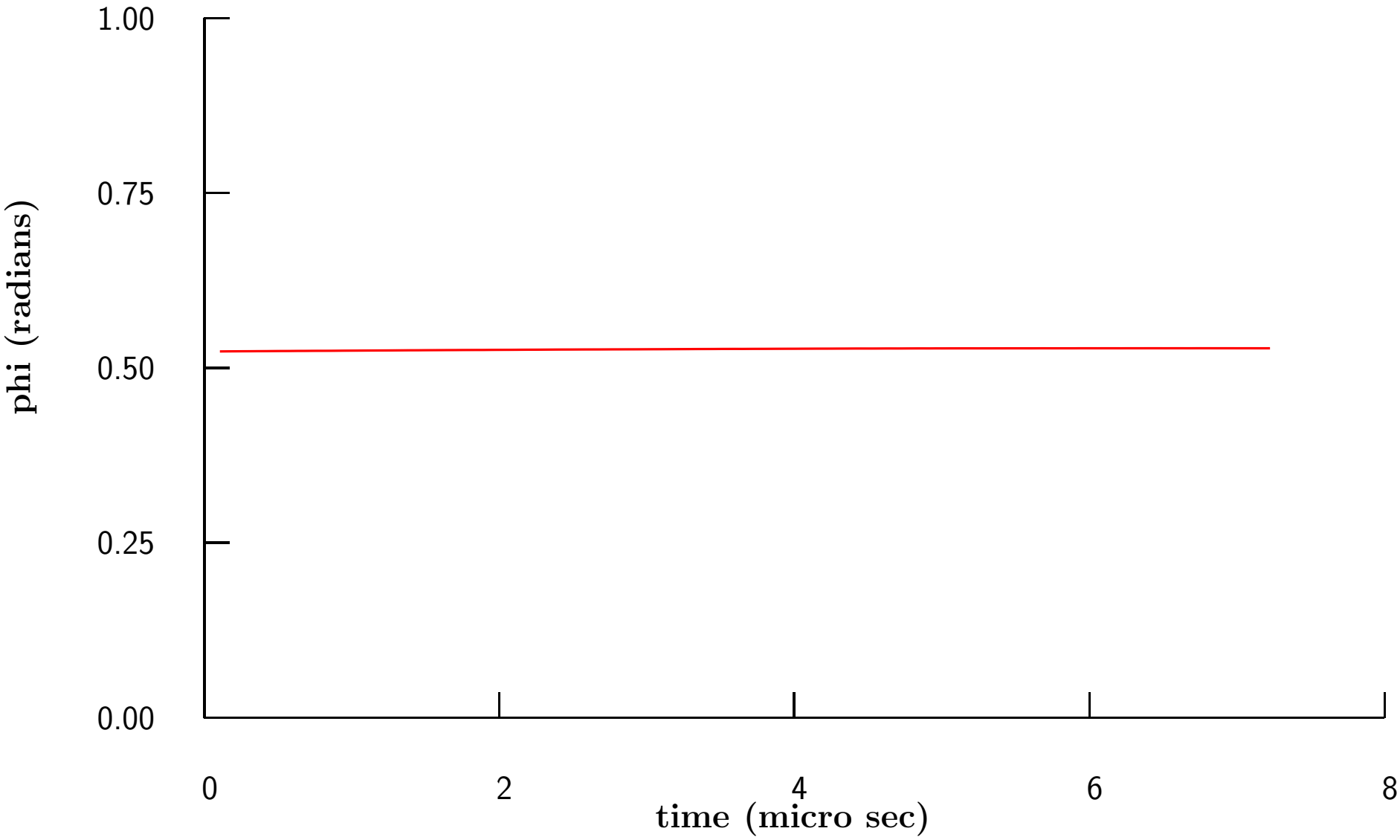
For L=10 m, m=.51 MeV, E_2 = 0.1 MeV/c:

$$f = \frac{h_1 v}{L} \approx \frac{h_1\ c}{L}\sqrt{\frac{2\ E_1}{mc^2}} \approx 10GHz$$

Electron Ideal




Ideal RF Voltage
for 30 deg RF phase
Ideal delta Es
delta KE (MeV)
0
25
50
75
0.0
2.5
5.0
7.5
10.0
time (micro sec)



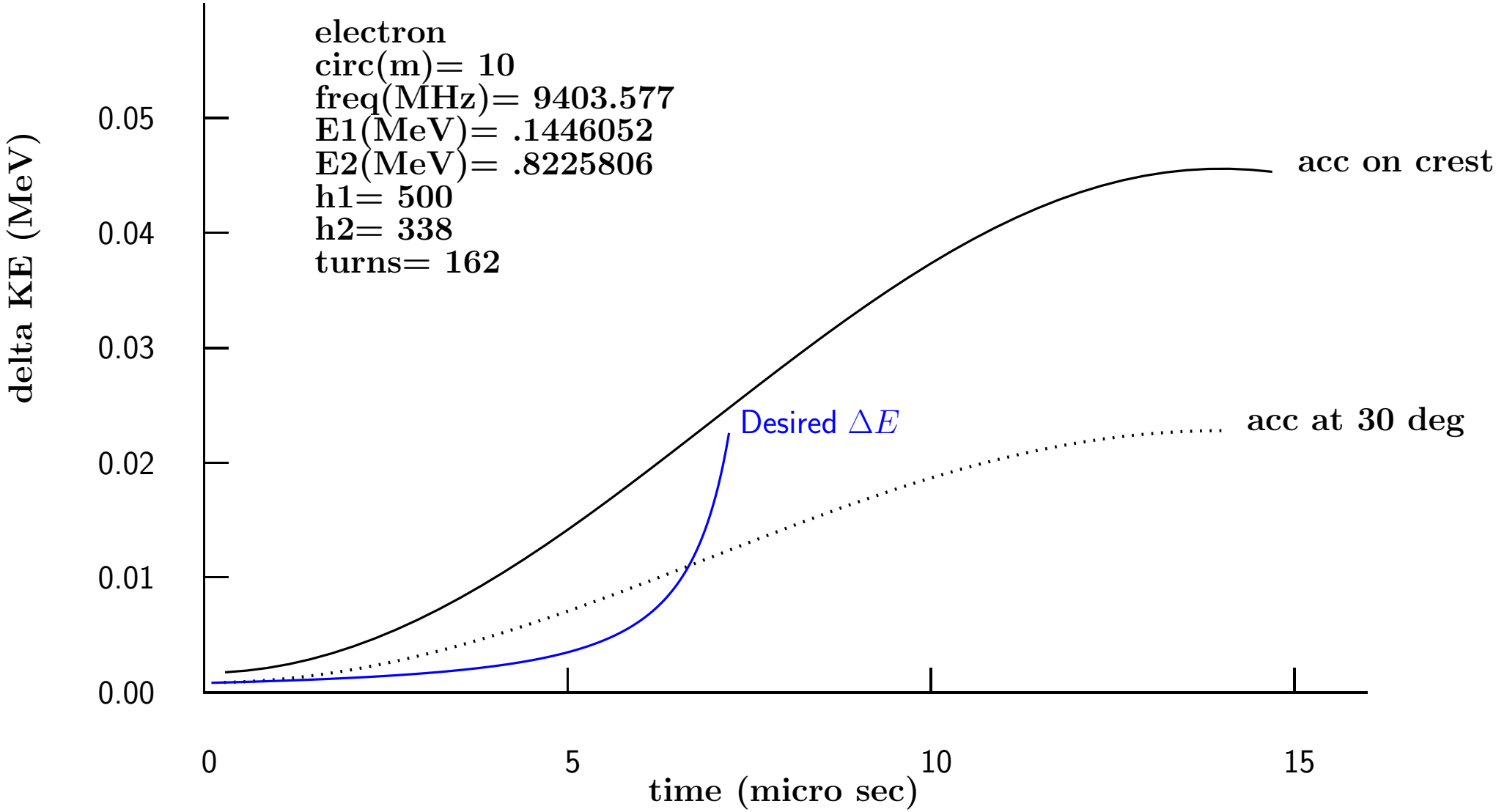
Phase constant because bunches come back at correct time

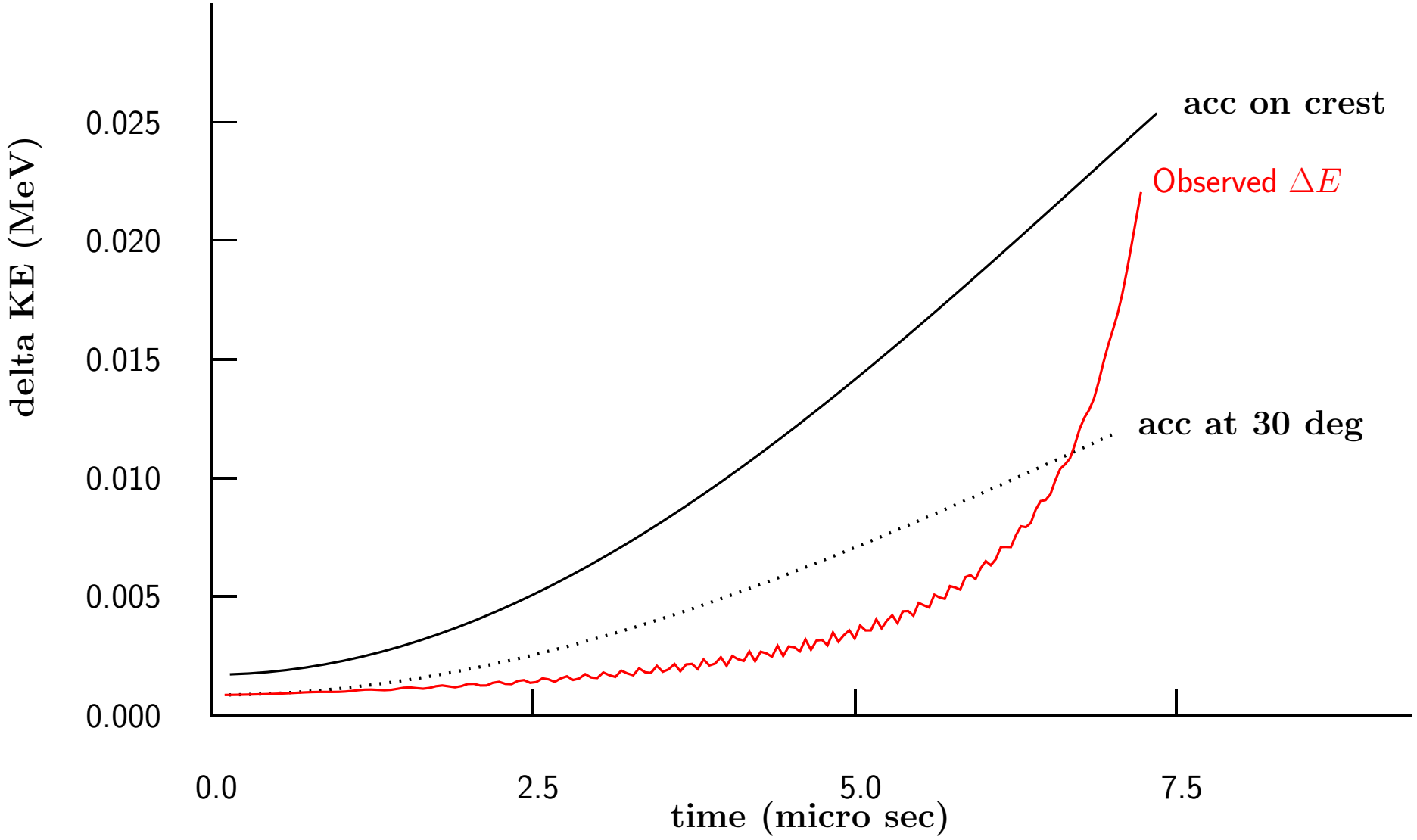
Generation of Rapid change in ΔE

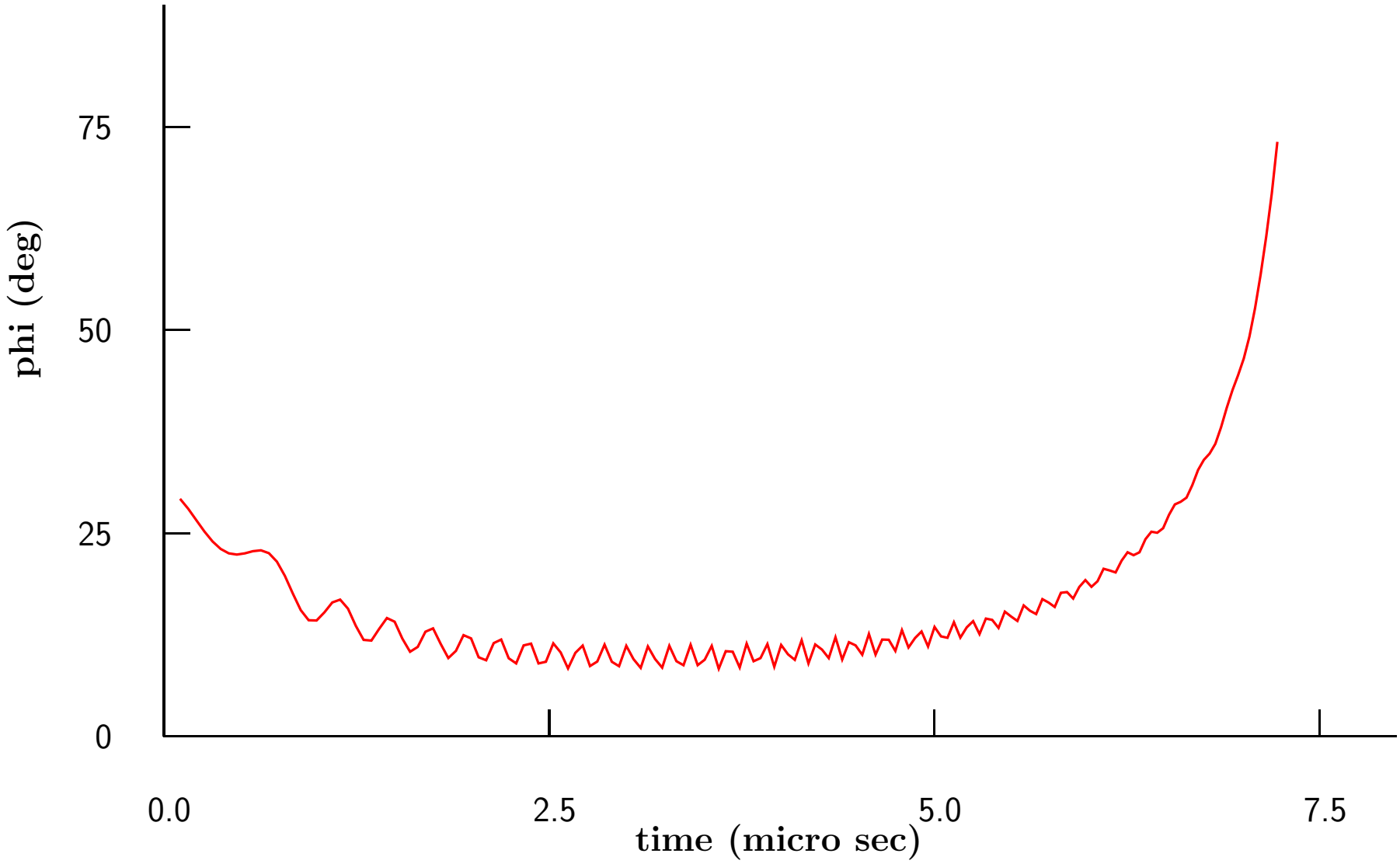
- Final rise is very Rapid
- Can one put energy into cavity this fast?

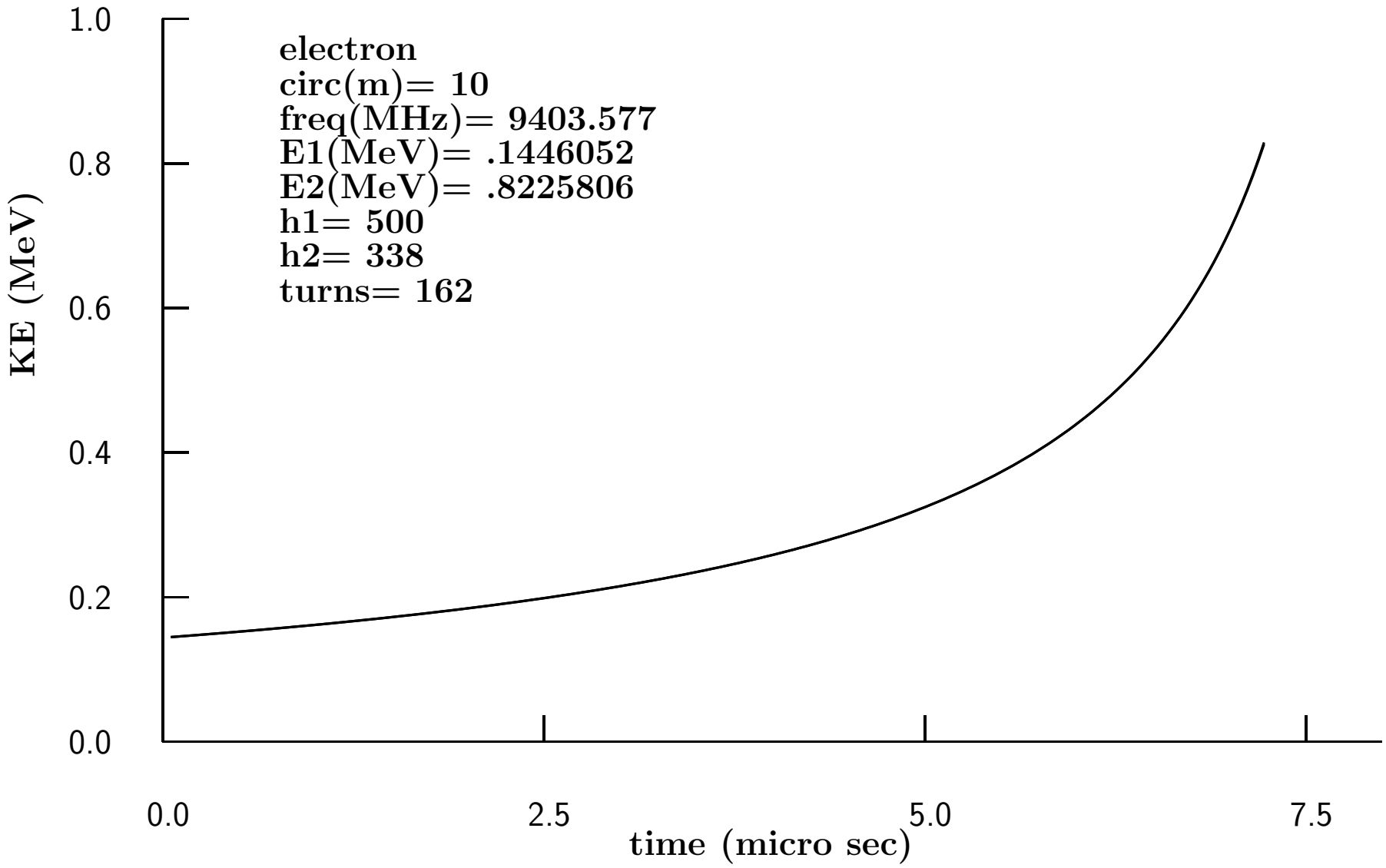
- Try beating two frequencies against one another
  - sum is frequency $(f_1 + f_2)/2$
  - varying as $\sin(2\pi(f_1 - f_2)/2\ t)$
- But this does not match shape of required rise
- Allow phase changes to take up the difference

Electron Simulation with Sin RF




delta KE (MeV)
0.025
0.020
0.015
0.010
0.005
0.000
0.0
2.5
5.0
7.5
time (micro sec)
acc on crest
Observed ΔE
acc at 30 deg


75
50
25
0
phi (deg)
0.0
2.5
5.0
7.5
time (micro sec)

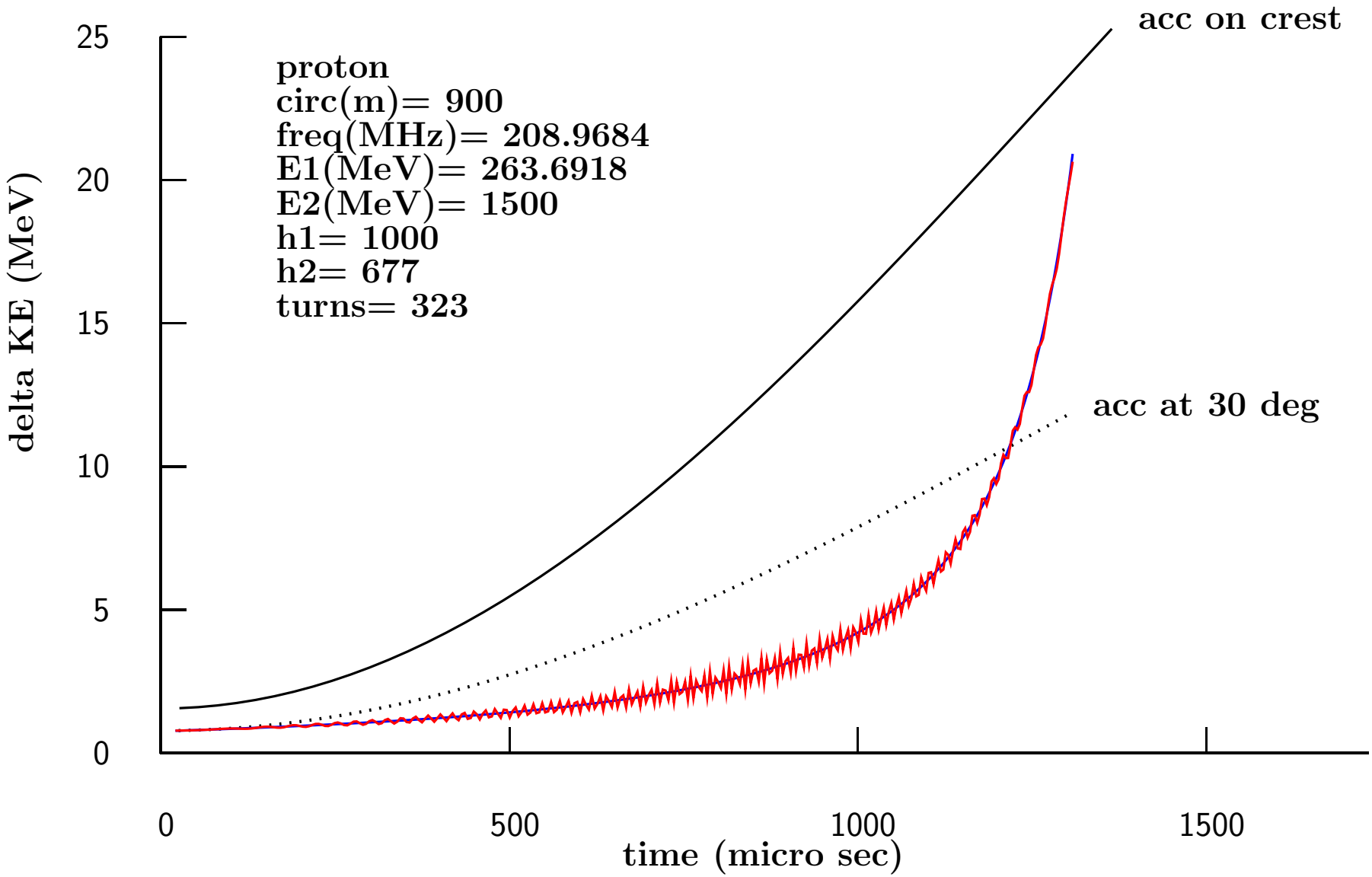


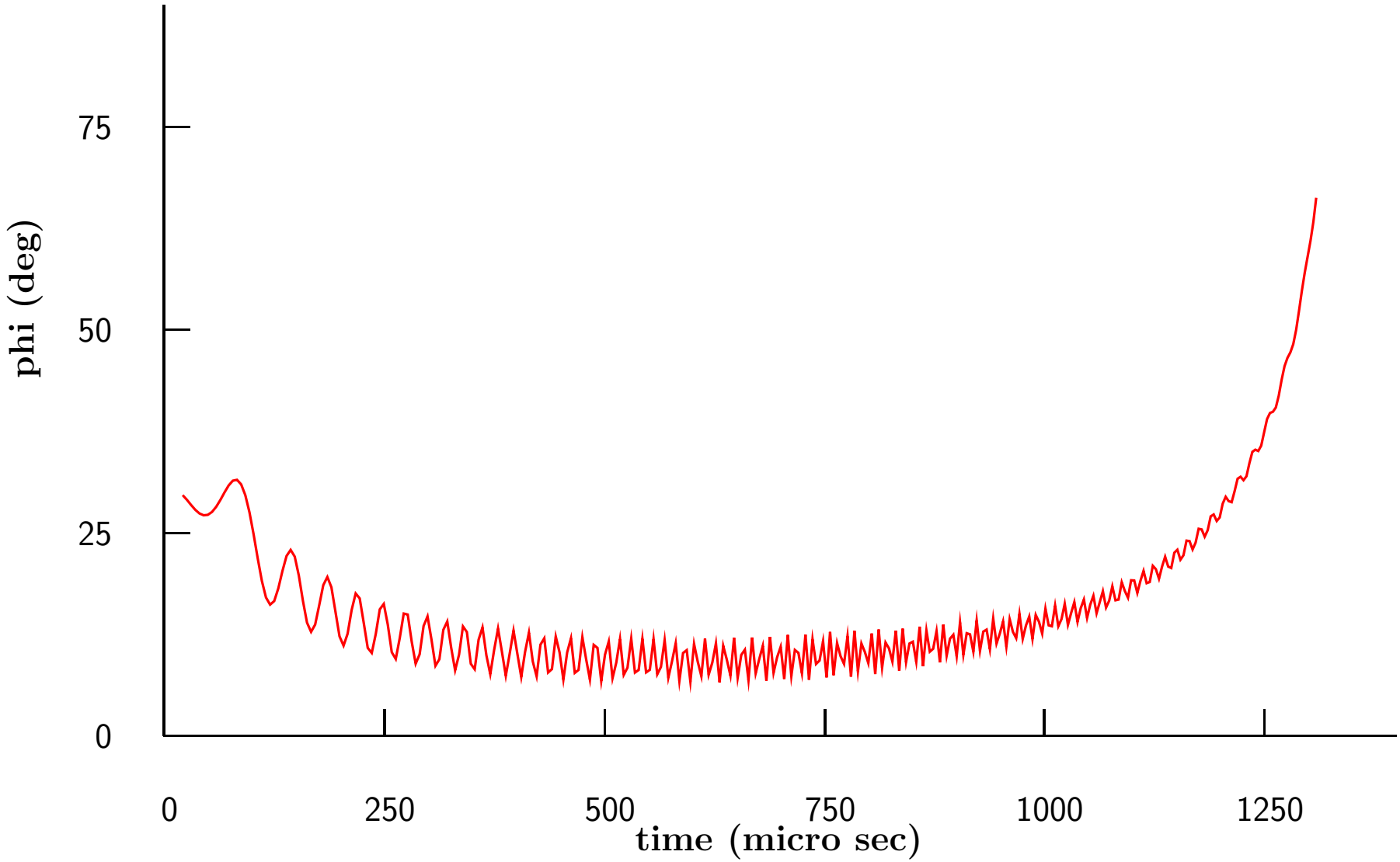
So it works fine Note $\Delta E = 25$ KeV is very easy

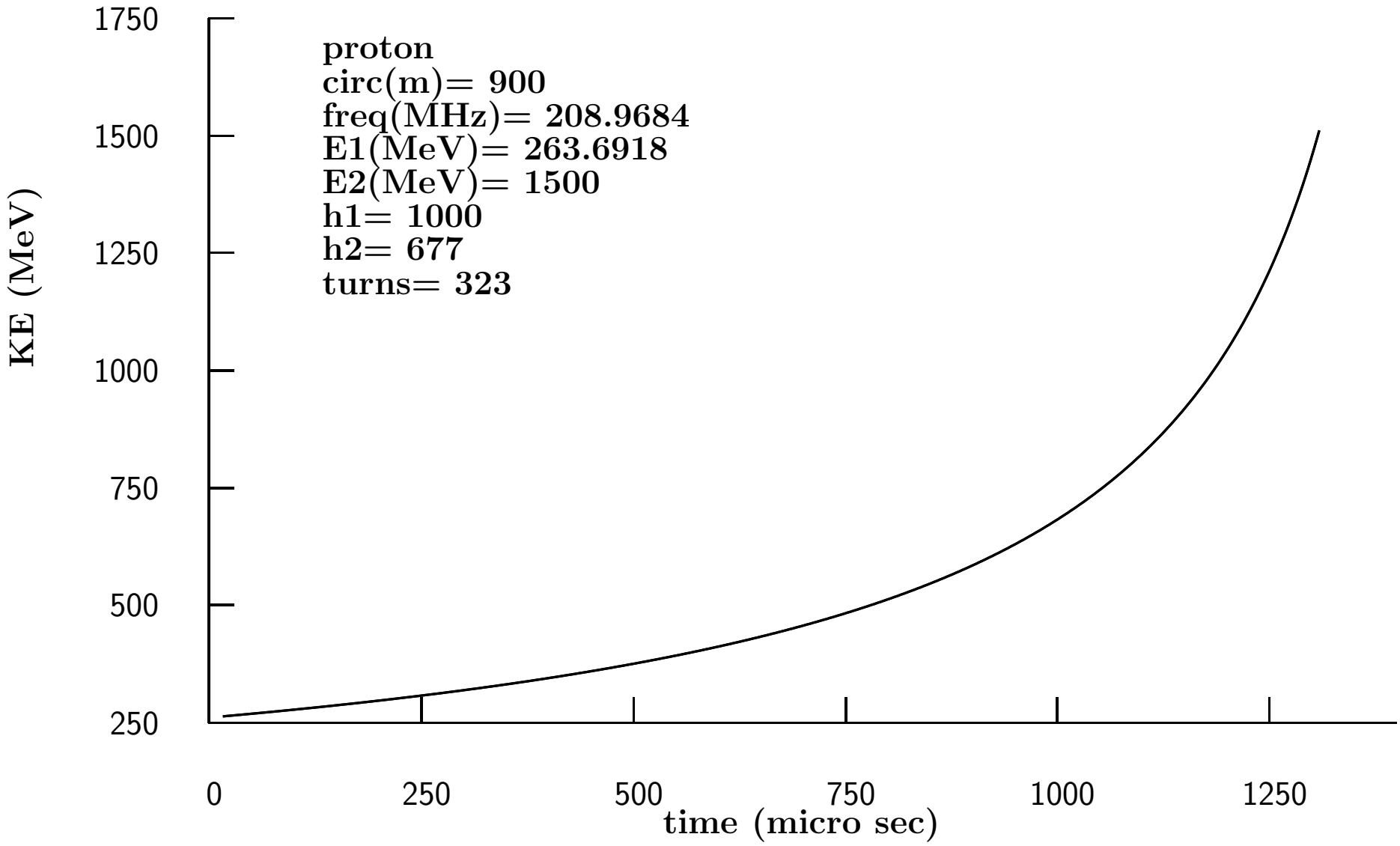
Proton Case

- If in AGS Ring then $L \approx 900$ (m)
- the above recipe gives $f \approx 100$ MHz
- which is a bit low
- Raise h_1 from 500 to 1000
- Then $f \approx 200$ MHz
- a smaller ring might be prefered, but try this

Proton Simulation




phi (deg)
75
50
25
0
0
250
500
750
1000
1250
time (micro sec)


proton
circ(m)= 900
freq(MHz)= 208.9684
E1(MeV)= 263.6918
E2(MeV)= 1500
h1= 1000
h2= 677
turns= 323
KE (MeV)
1750
1500
1250
1000
750
500
250
0
250
500
750
1000
1250
time (micro sec)

Conclusion

- So proton ring works fine too
- But 25 MeV Energy gain may not be so easy
  - 20 MV/m maybe ok
  - But acceleration must be within $\lambda/2$ and $\lambda \propto 1/h$ is NOT constant
- This may be good
  - If phases right for final harmonic
  - Then acceleration is less for earlier harmonic
  - We may get our time dependent effectice gradient automatically

Needs more study, including longitudinal acceptance