Beam dynamics issues of non-scaling FFAG

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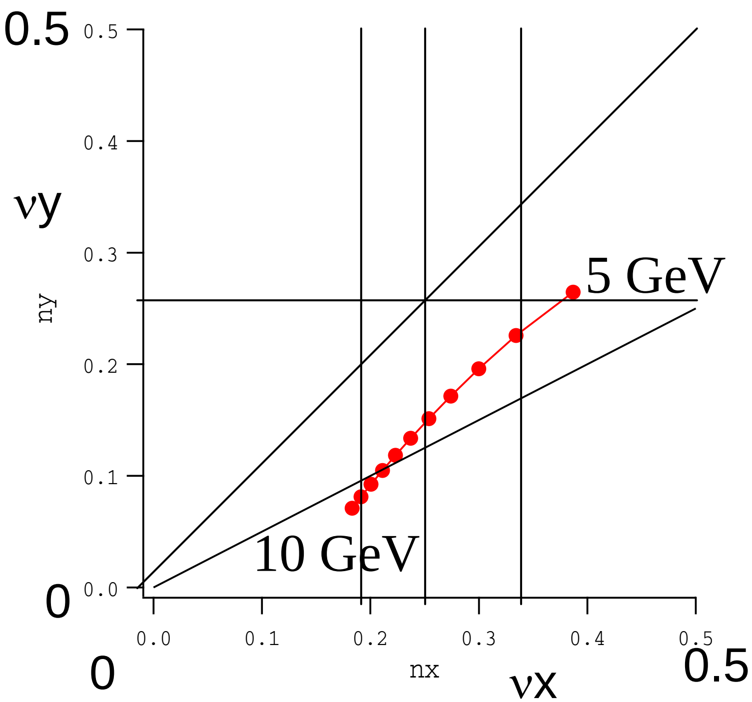
http://hadron.kek.jp/~machida/doc/nufact/ffag/machida_20060515.ppt & pdf

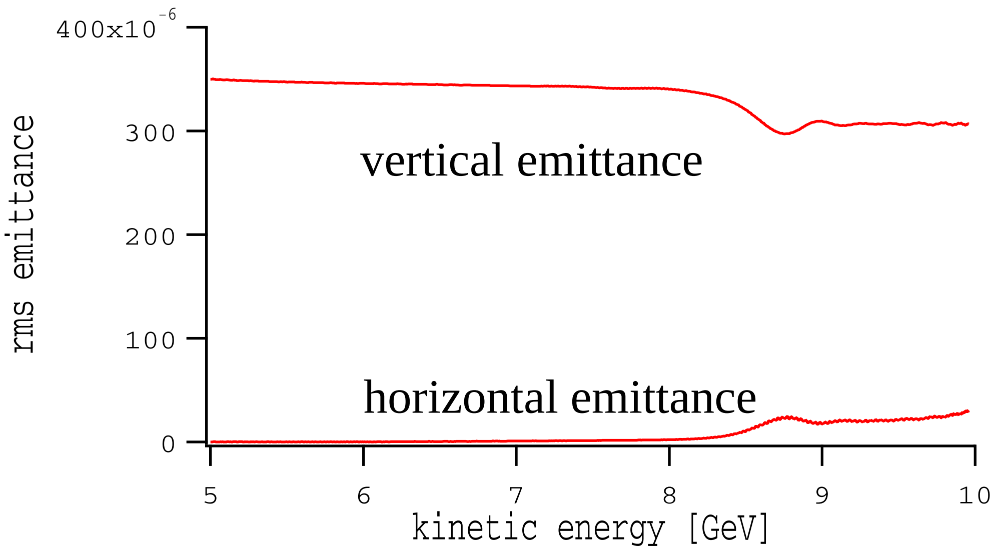
Beam dynamics issues

- Huge emittance (30,000 π mm-mrad) of muons.
  - This magnifies all the following issues.
- Crossing of many resonances during acceleration.
  - Structure resonance without any errors.
  - Integer resonances have to be considered when alignment errors exist.
- Accelerate outside of RF bucket.
  - With finite transverse amplitude.
- Matching between two FFAGs.
  - Muon accelerator assumes cascade FFAGs.
- Non-scaling FFAG with sextupole.
  - Chromaticity control and dynamic aperture.
  - Scaling FFAG does not have dynamic aperture problem.

Resonance crossing
without errors

- Vertical is 5 π mm, normalized, zero horizontal emittance.



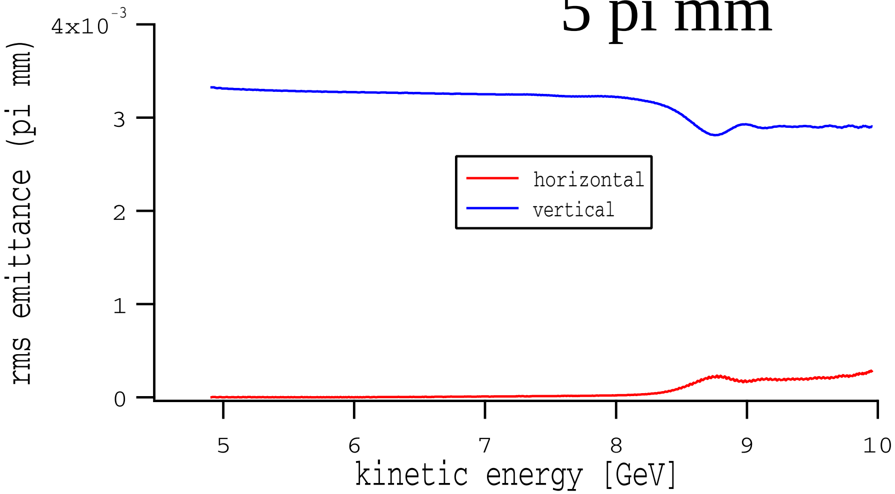


- Shows the coupling due to nx-2ny=0 (structure) resonance.
- If we start finite horizontal and zero vertical emittance, no exchange of emittance.

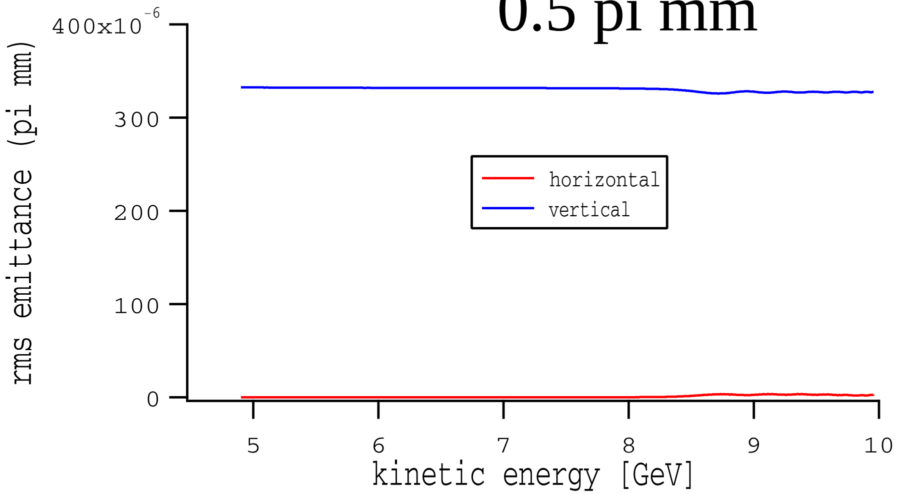
Resonance crossing

without errors, amplitude dependence

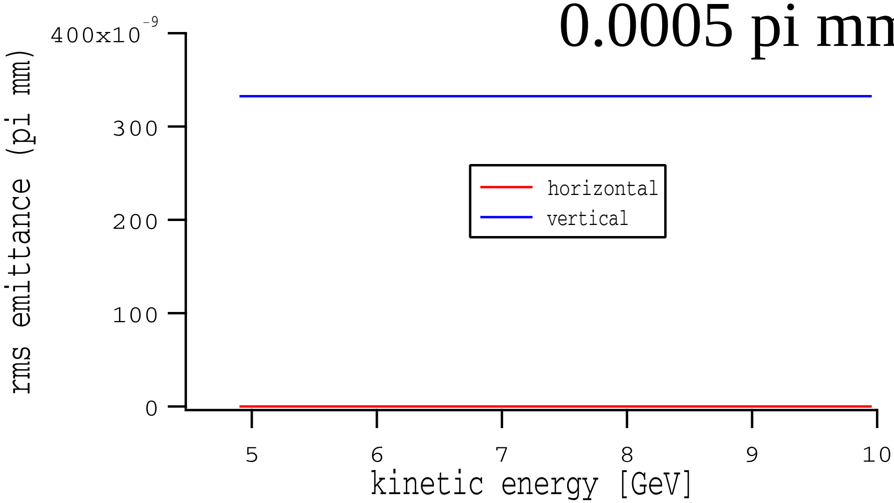
5 pi mm



0.5 pi mm

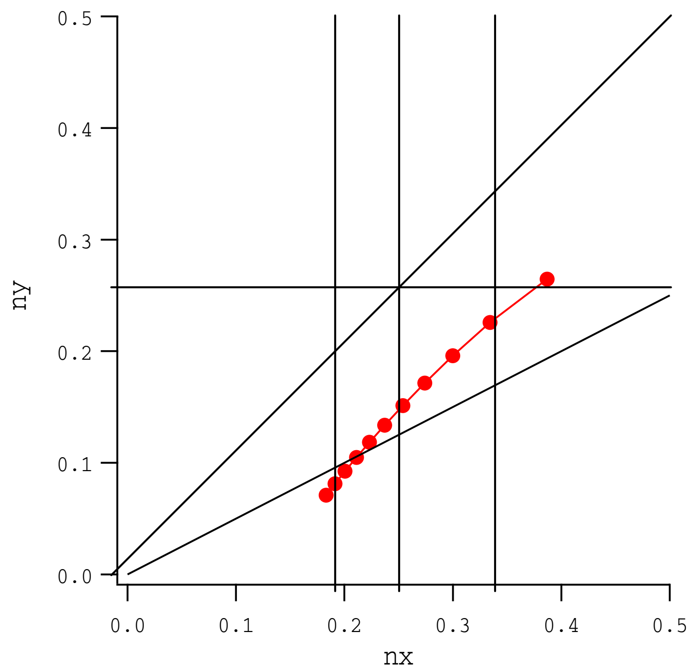


0.0005 pi mm

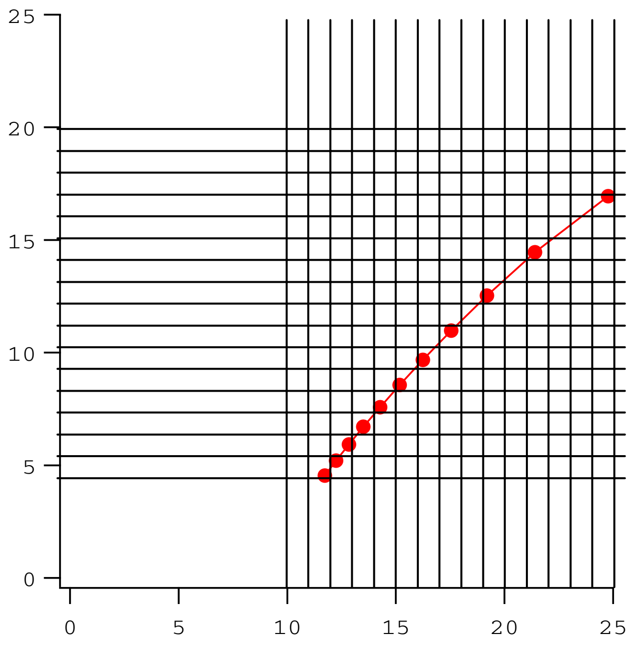


Resonance crossing
with alignment errors

Beam has to cross many integer tunes.



tune per cell

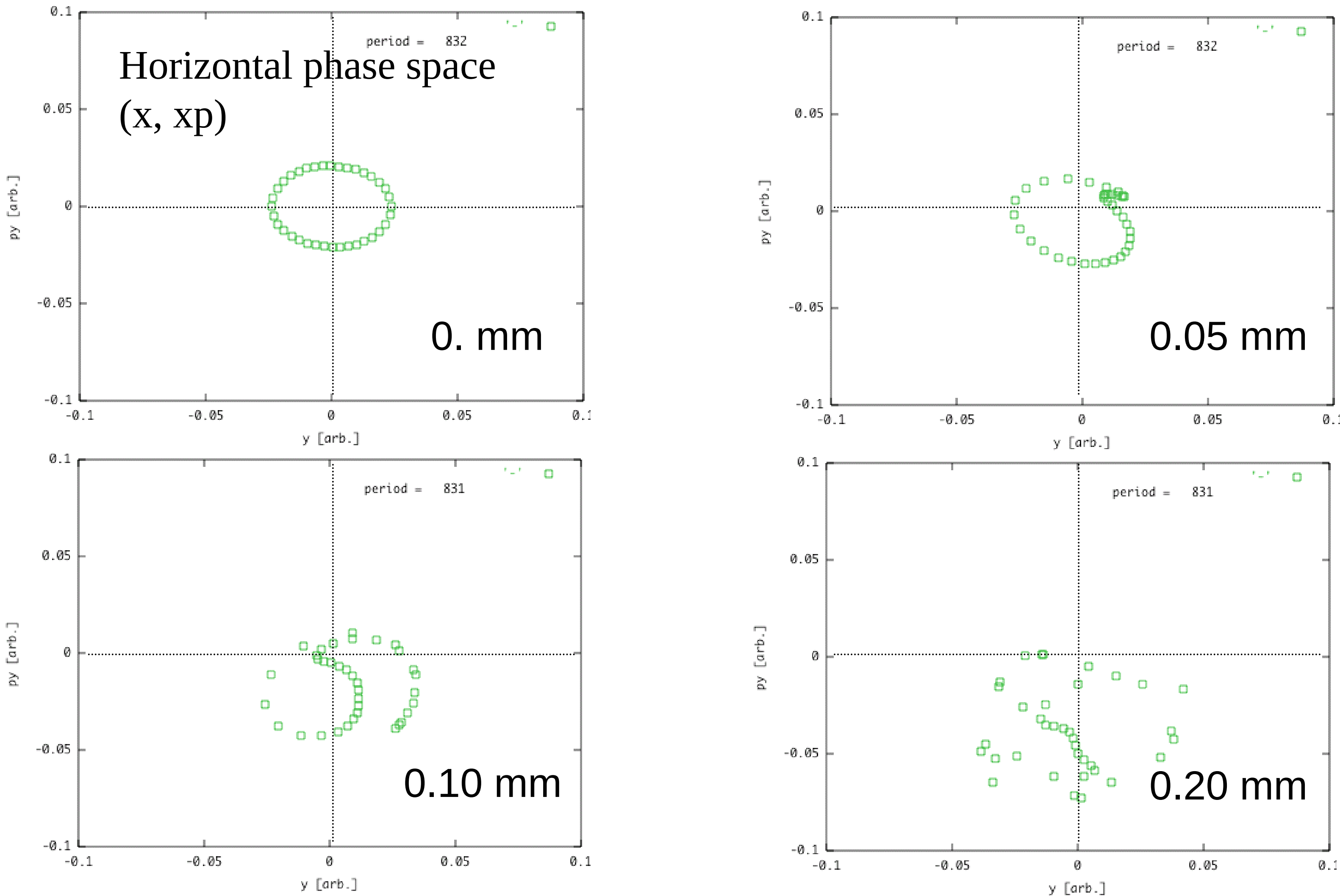


tune per ring

Resonance crossing

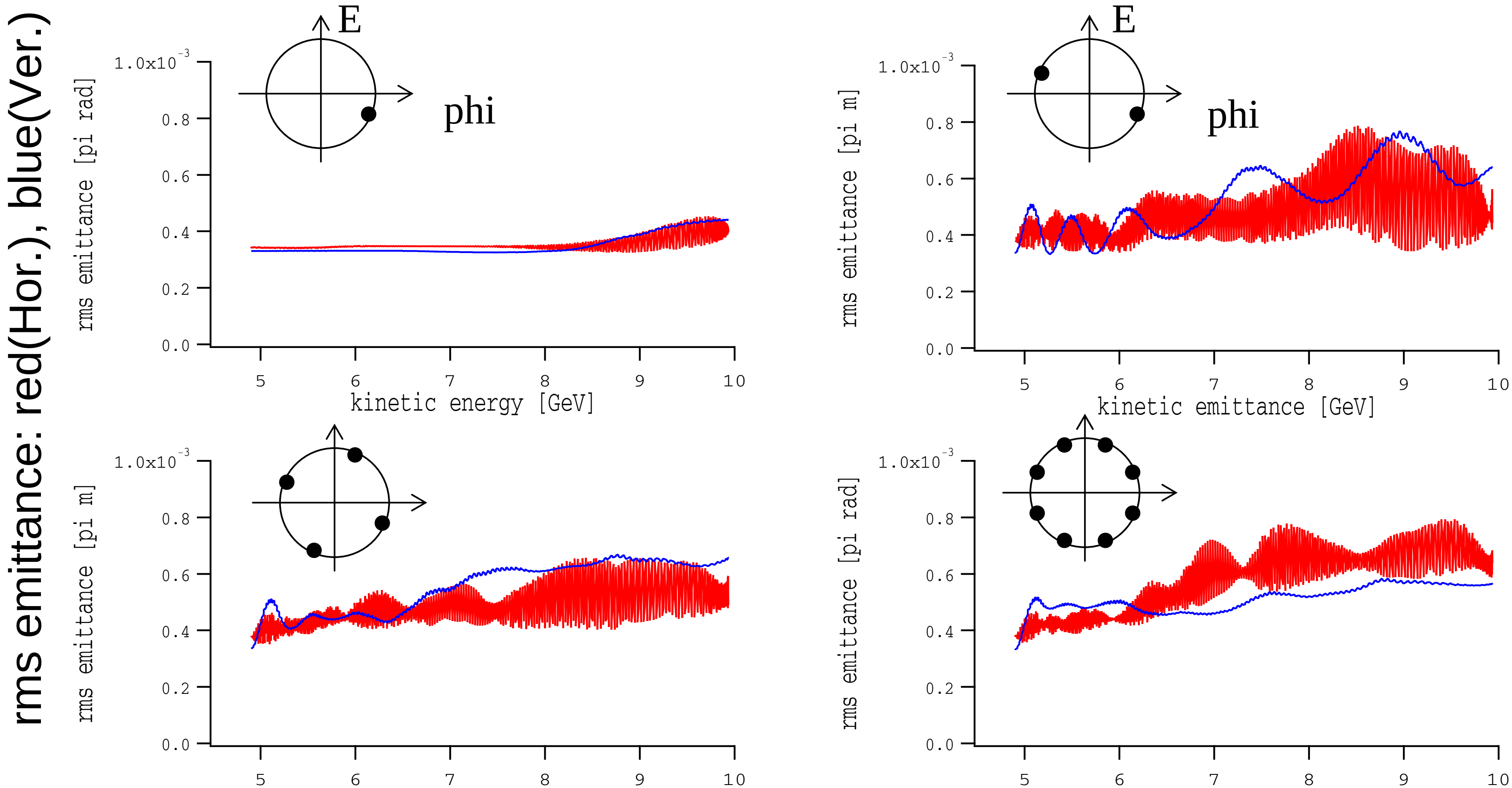
ellipse after acceleration with alignment errors

- Horizontal is 10 π mm, normalized, zero vertical emittance.
- Alignment errors of 0, 0.05, 0.10, 0.20 mm (rms).



Resonance crossing

rms emittance evolution with alignment errors (0.1mm)



- 16 turns are not enough to make decoherence.
- Each particle cross the resonance at difference time. That is another source of decoherence.

Resonance crossing

summary

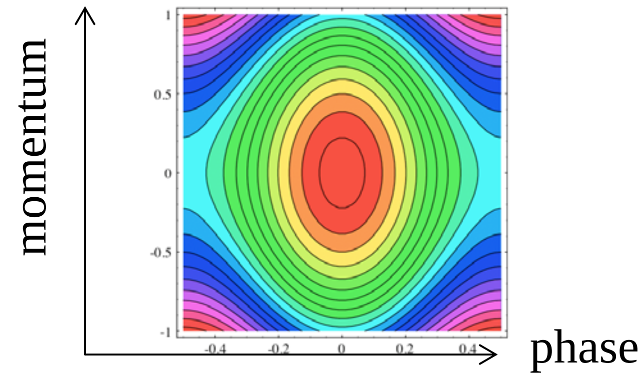
- Non-linear structure resonances is visible even though non-scaling FFAG consists of only quadrupole and dipole.
- Integer (and half-integer) resonance impose the upper limit of alignment and gradient errors.
- Particle with different momentum cross a resonance at the different time. That makes rms emittance growth.

Accelerate outside of RF bucket

scheme

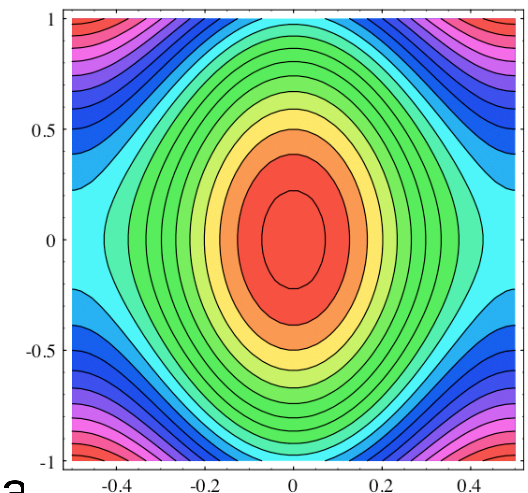
- Frequency modulation shifts RF bucket upwards and particles trapped inside are adiabatically accelerated. This is an ordinary way of acceleration.

  (Figures from Keil and Sessler)

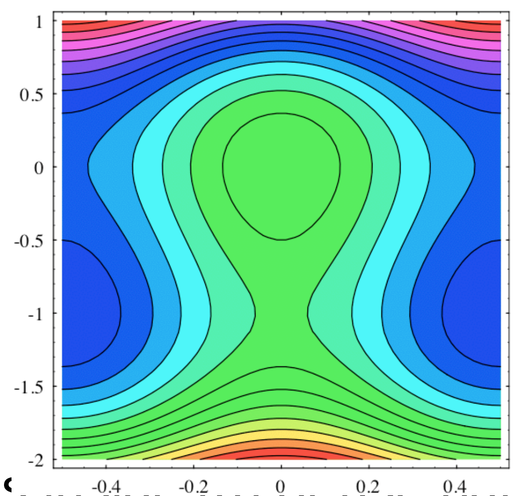


- For muon acceleration, no time for frequency modulation.

(a)



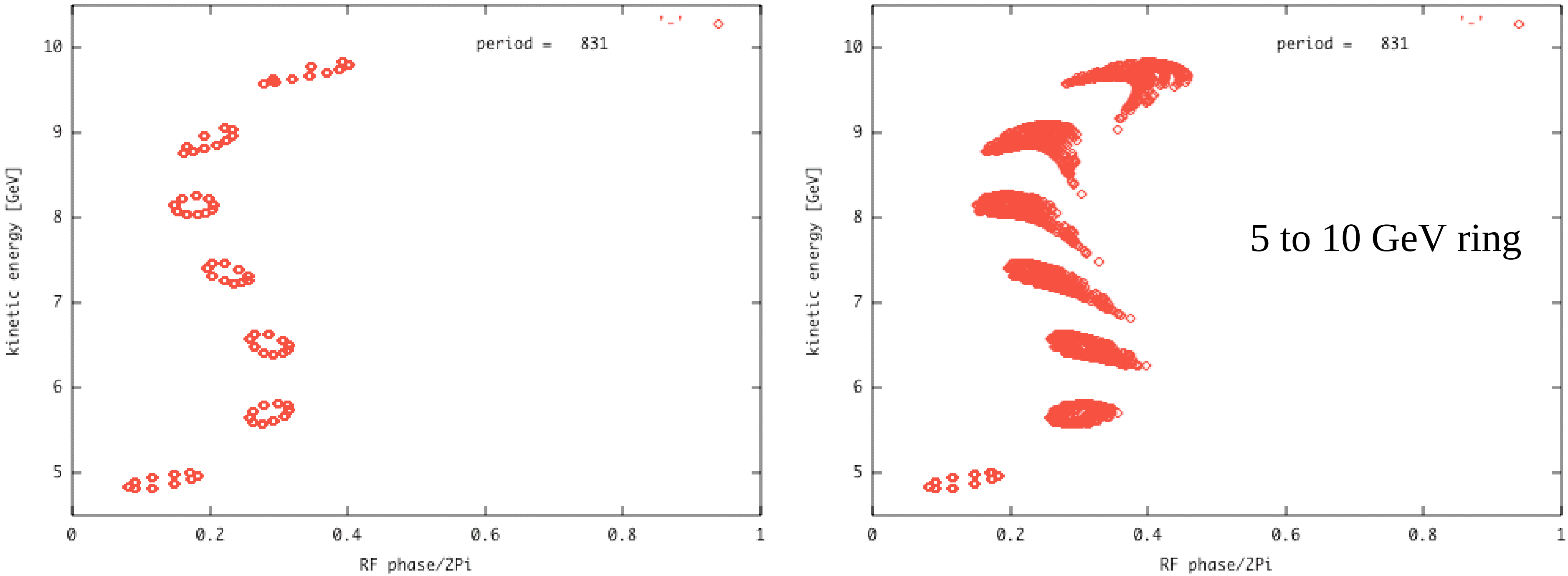
(b)



- Make a huge bucket and put particles inside (a) or use path outside of a bucket (b).

Accelerate outside of RF bucket
with finite transverse amplitude

Longitudinal phase space (phi, momentum)



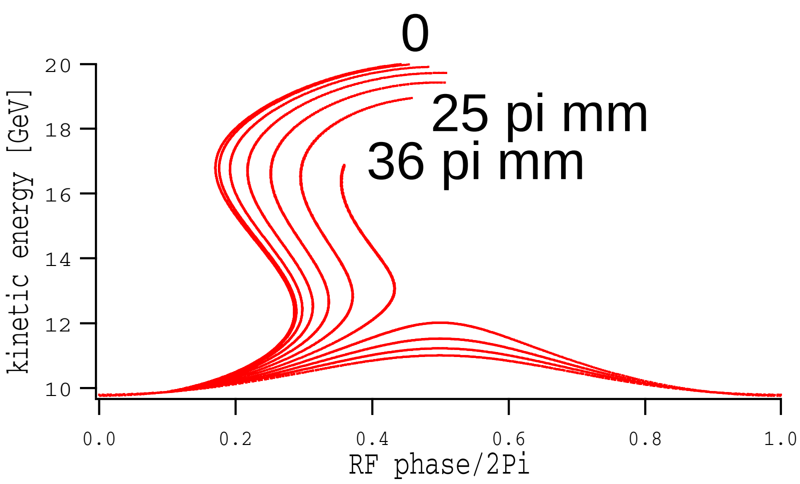
without transverse amplitude

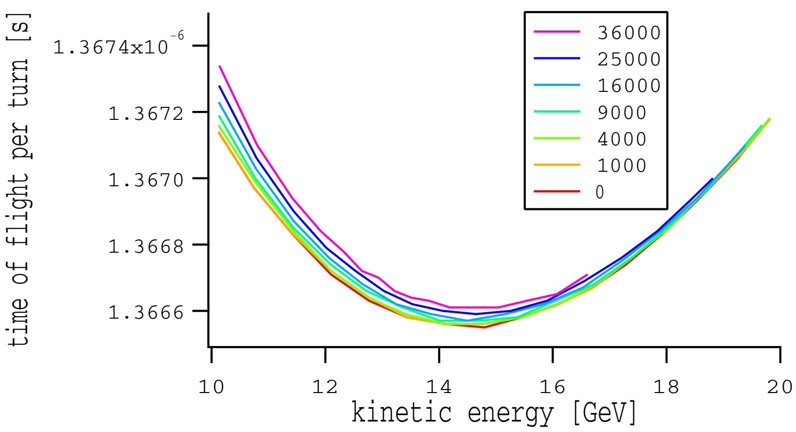
with finite transverse amplitude

Horizontal is 5 pi mm
Vertical is zero

Accelerate outside of RF bucket
time of flight variation

- Horizontal amplitude are
  (0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 π mm, normalized.)
- Vertical amplitude is zero.





- Difference of ToF becomes smaller as accelerated.

Accelerate outside of RF bucket
possible cures

Longitudinal dynamics is parameterized by

$$a = \frac{qV}{\omega \cdot \Delta T \cdot \Delta E} \quad \left(\begin{array}{l} \text{Relative energy gain} \\ \text{per phase slip.} \end{array} \right) \qquad b = \frac{T_0}{\Delta T} \quad \left(\begin{array}{l} \text{RF frequency relative} \\ \text{to revolution freq.} \end{array} \right)$$

- Cure 1
  - Increase V to increase "a".

$$a = \frac{qV}{\omega \cdot \Delta T \cdot \Delta E}$$

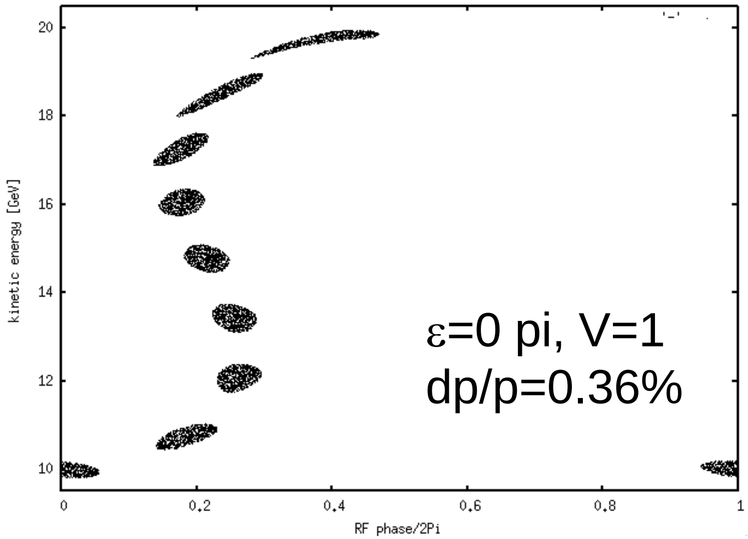
- Cure 2
  - Decrease ω to increase "a".

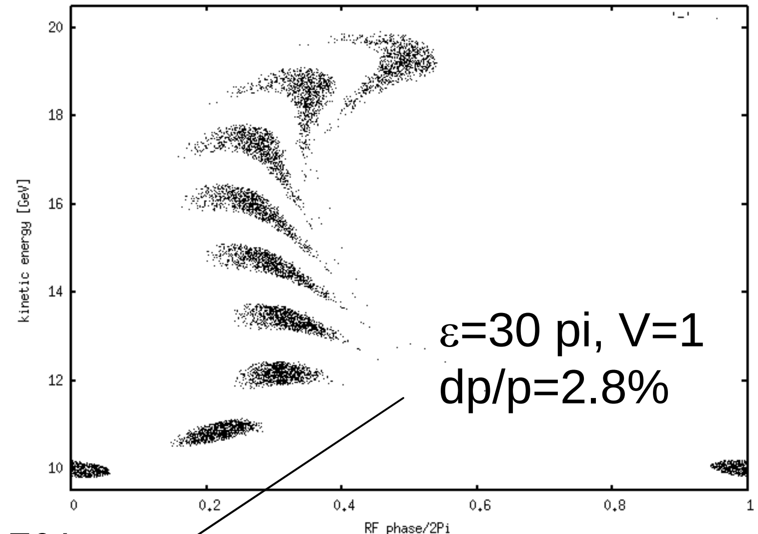
$$a = \frac{qV}{\omega \cdot \Delta T \cdot \Delta E}$$

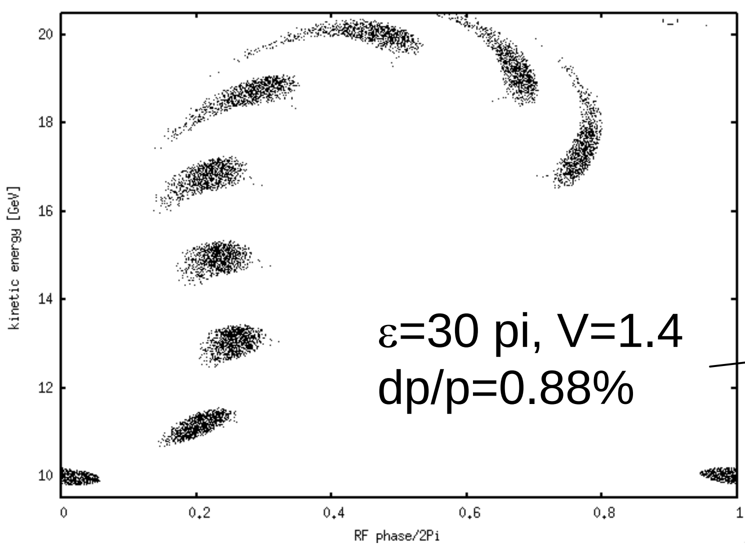
- Cure 3
  - Flatten crest by introduction of higher harmonics.

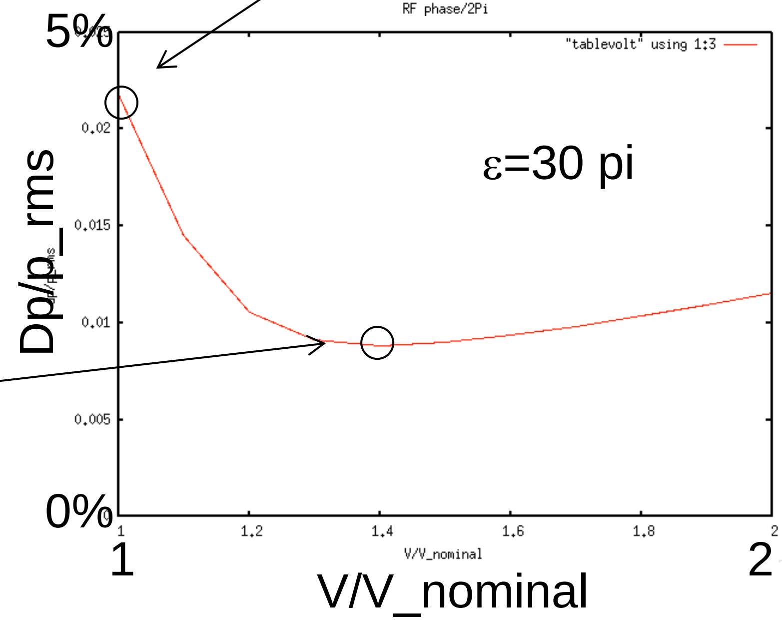
Accelerate outside of RF bucket

with higher voltage (1)

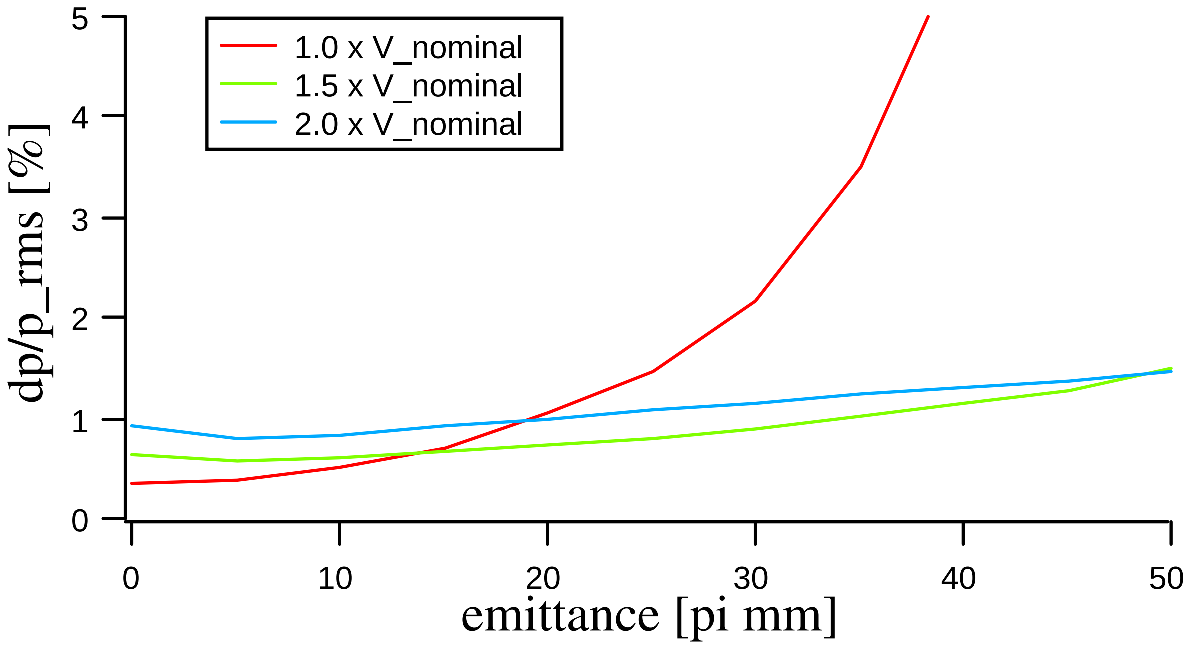








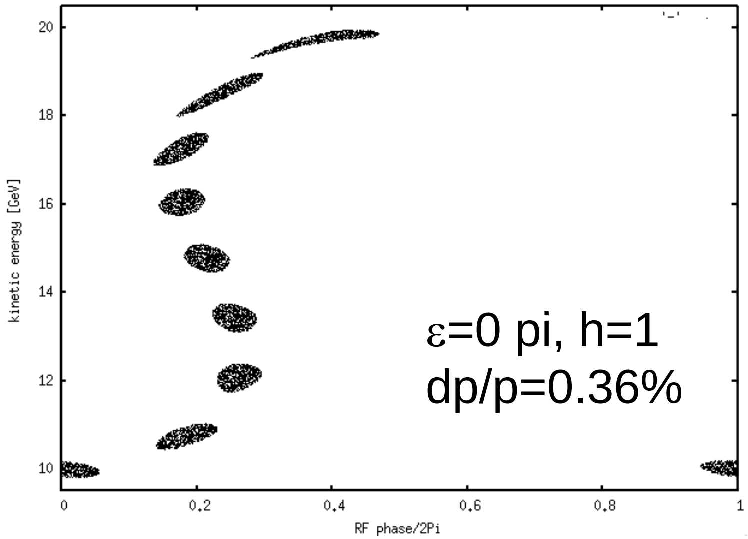
Accelerate outside of RF bucket
with higher voltage (2)

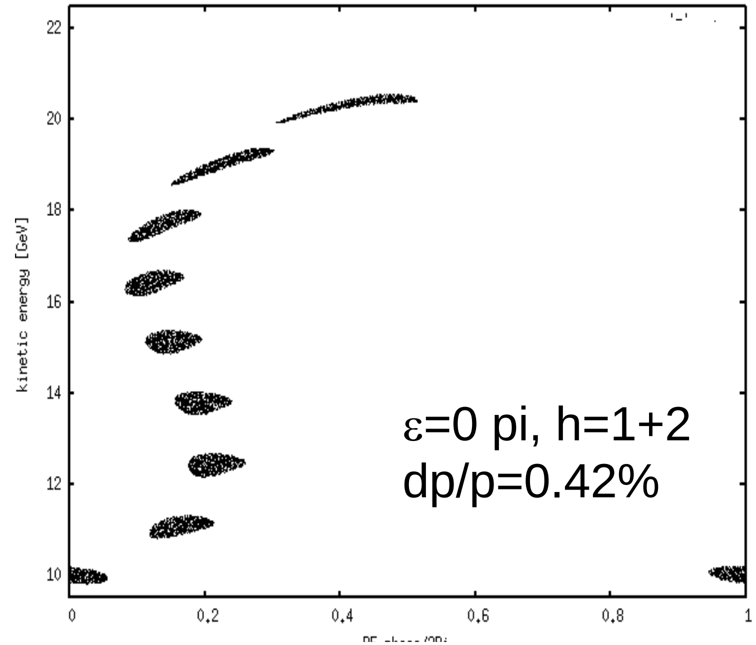


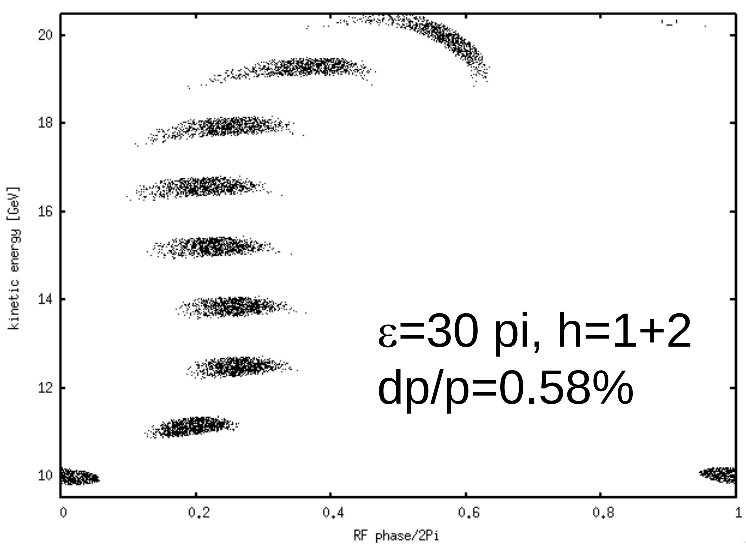
- 50% increase of RF voltage makes dp/p around 1% up to 50 pi mm.

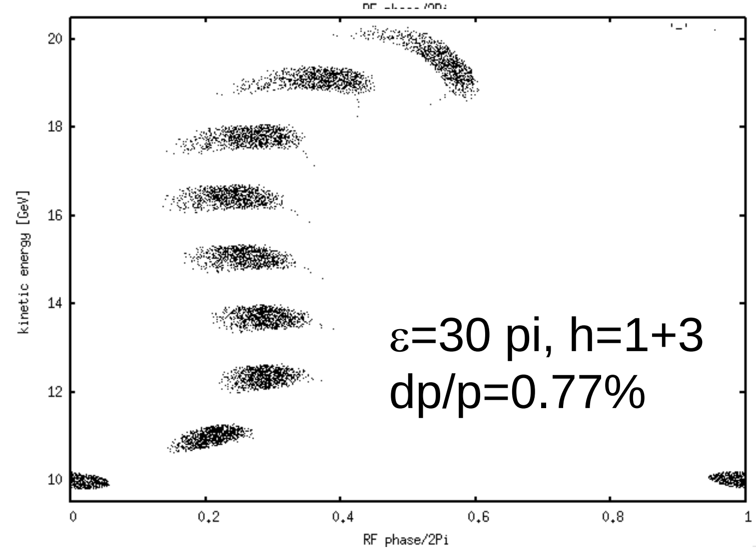
Accelerate outside of RF bucket

with higher harmonics (1)

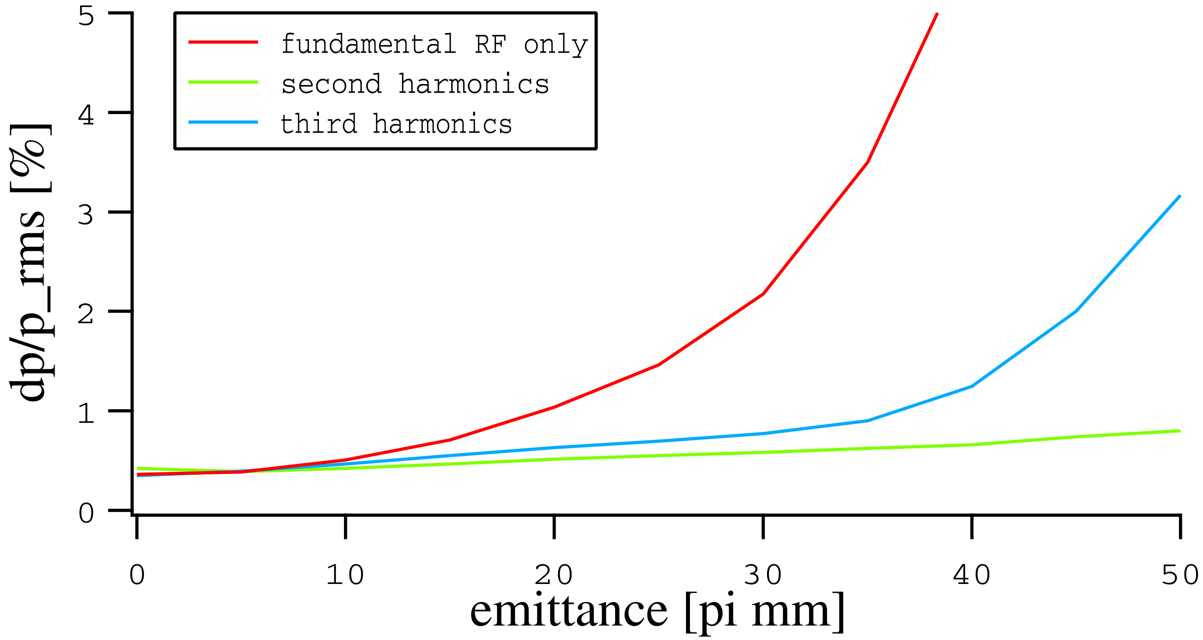








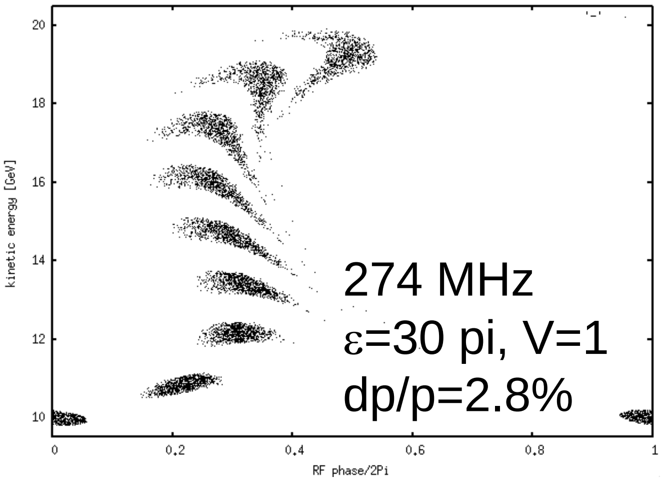
Accelerate outside of RF bucket
with higher harmonics (2)

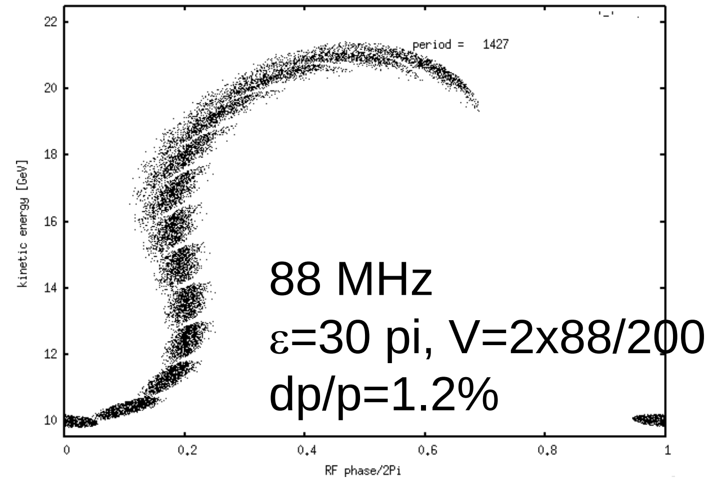


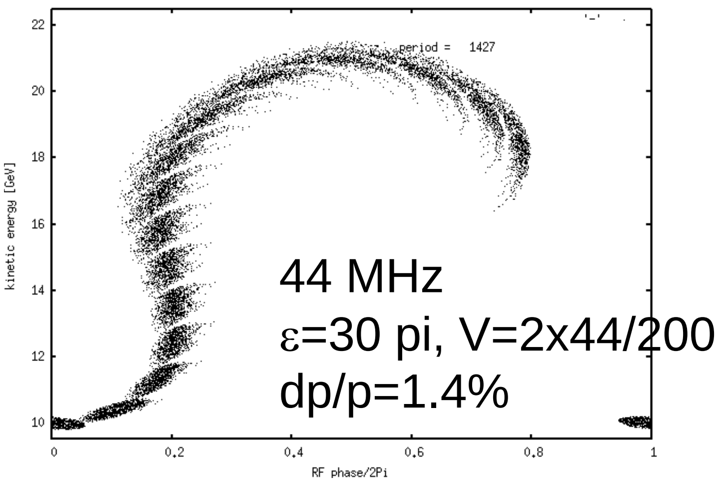
- Second harmonics makes dp/p around 1% up to 50 pi mm.
- It requires more RF power because second harmonics reduce peak voltage 25%.

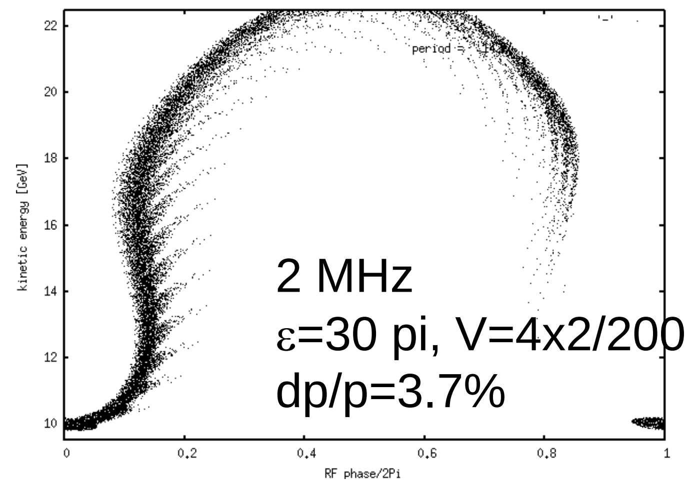
Accelerate outside of RF bucket

with lower frequency RF (1)

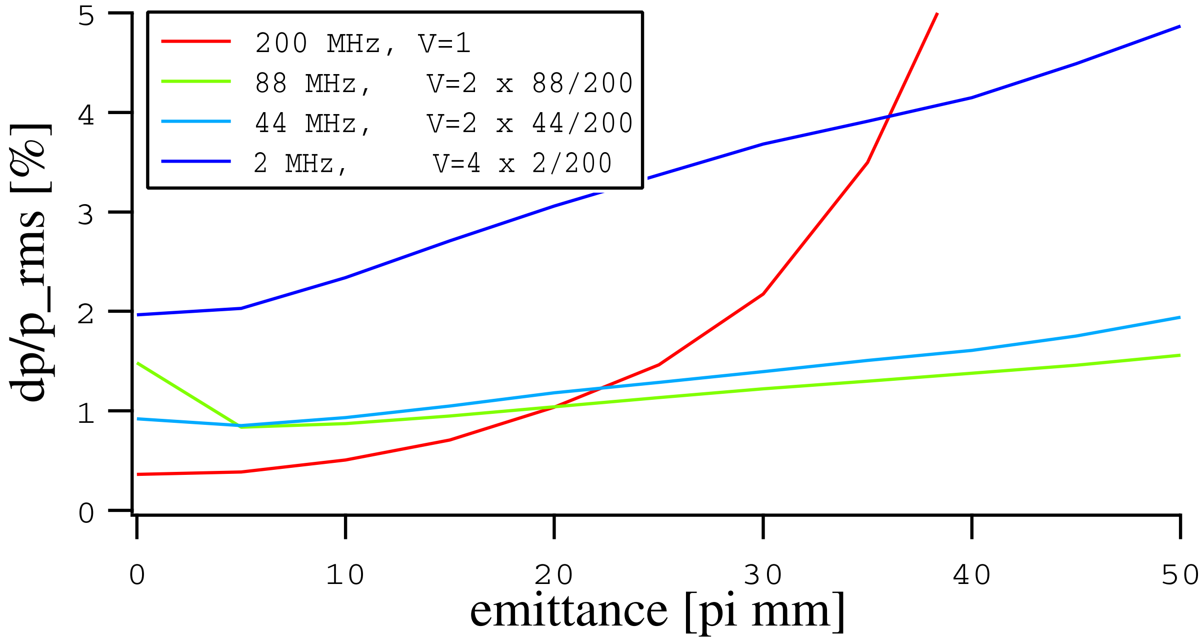








Accelerate outside of RF bucket
with lower frequency RF



- Lower frequency helps.
- However, it requires relatively higher voltage and time to complete acceleration.

Accelerate outside of RF bucket
summary

- Increase of a few 10% of RF voltage or second harmonics makes dp/p around 1% up to 50 pi mm beam.

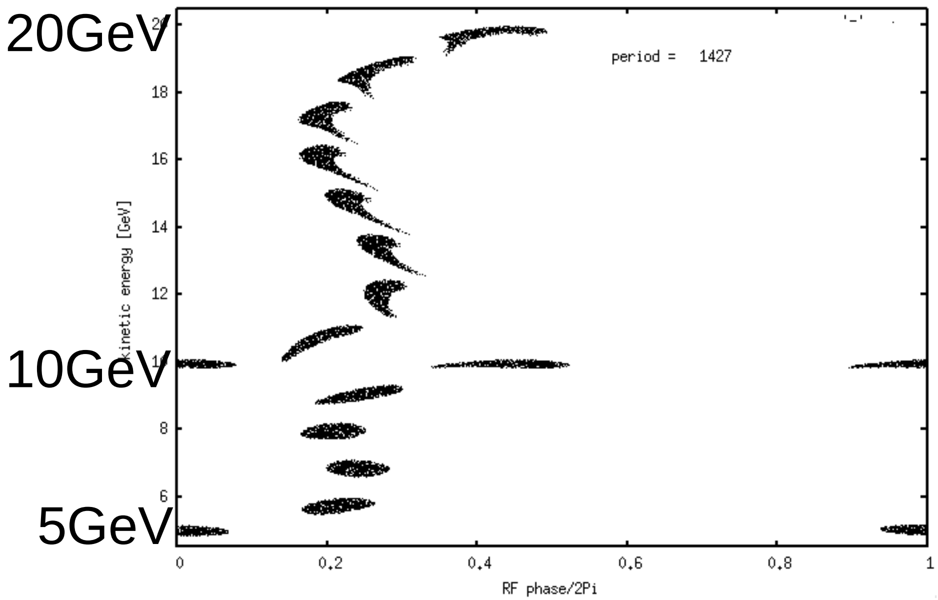
- That requires additional RF power.

- Amplitude effects can be cured in principle.

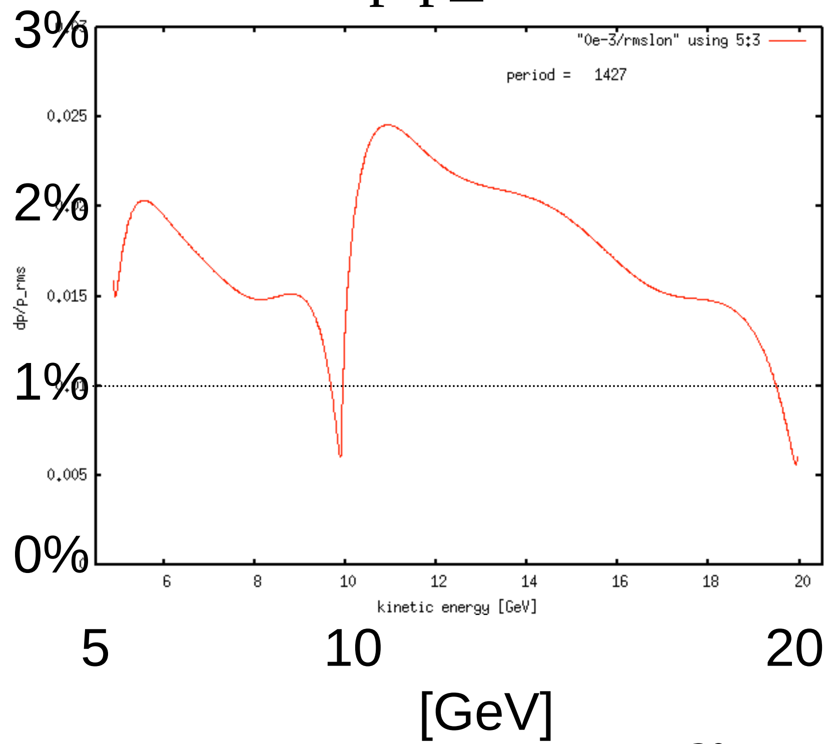
Matching between two FFAG rings
longitudinal

- Zero transverse emittance beam has no problem of longitudinal matching.

longitudinal phase space

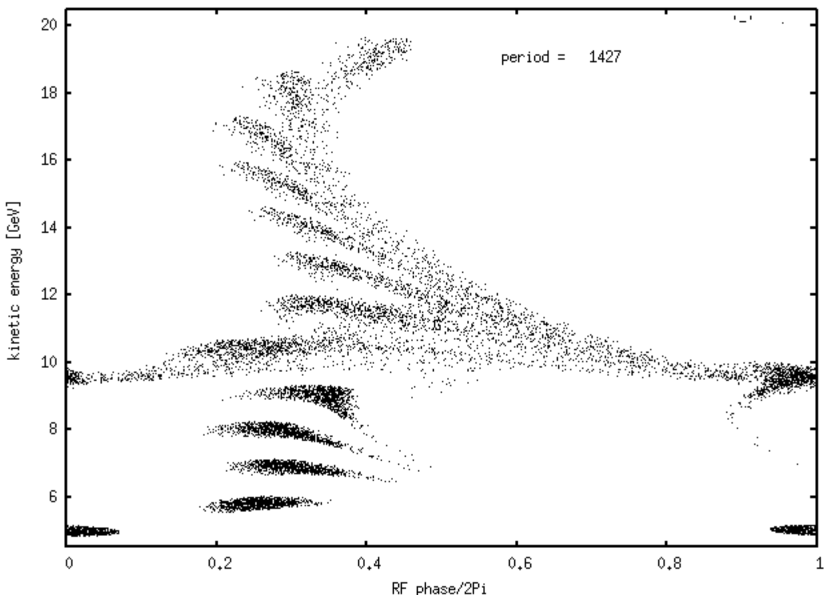


dp/p_rms

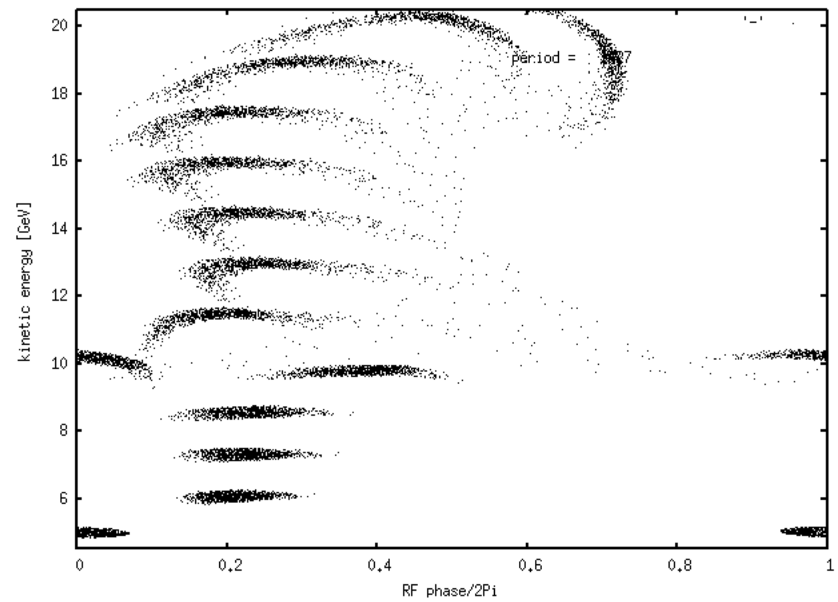


Matching between two FFAG rings
with finite transverse amplitude (1)

ε=30 pi, h=1, V=1



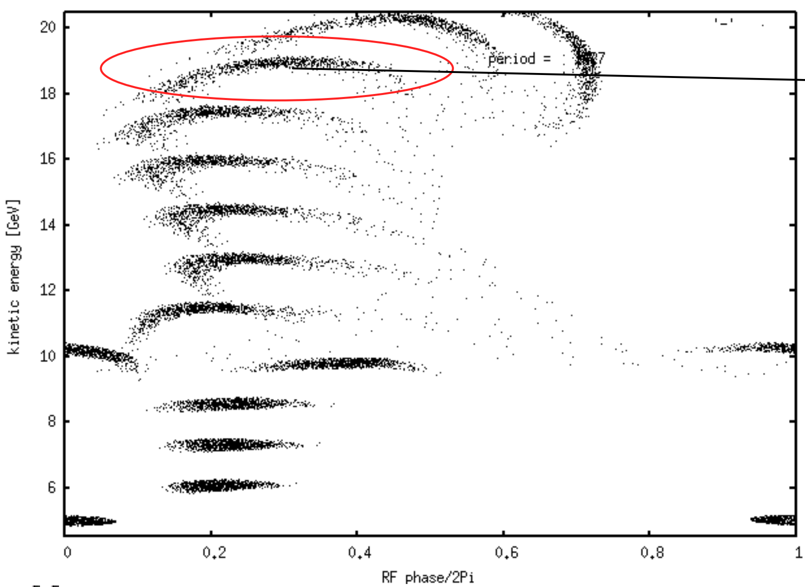
ε=30 pi, h=1+2, V=1.1

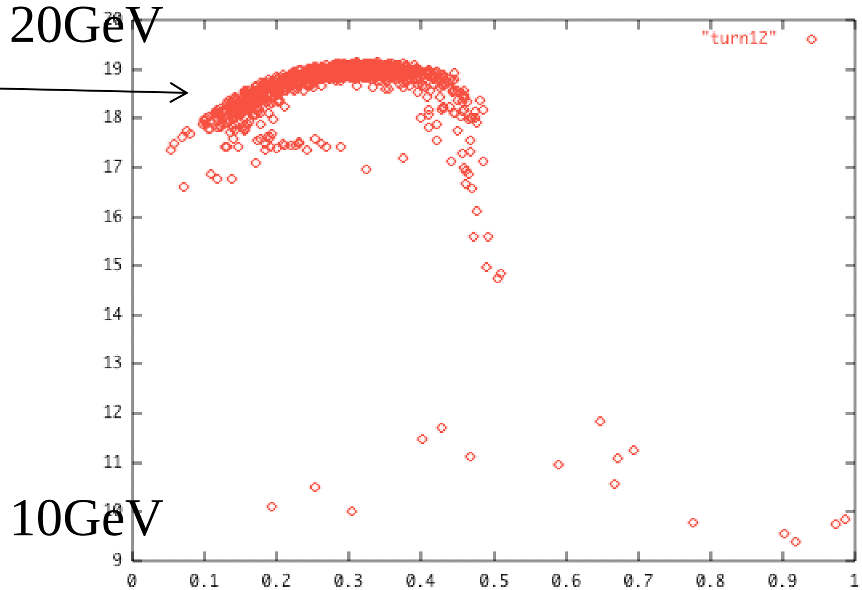


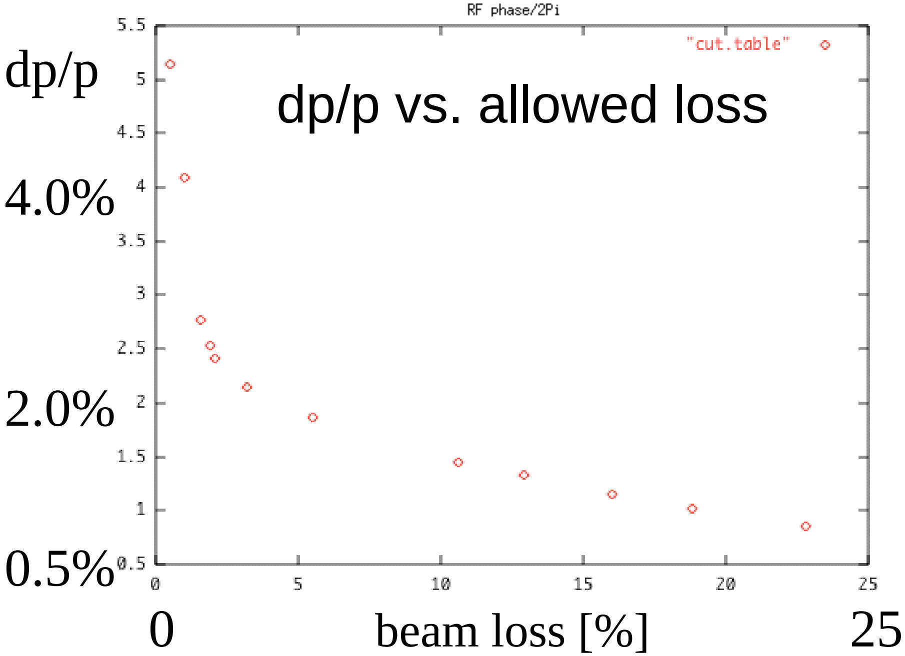
- Second harmonics and 10% increase of RF voltage only partially cure the problem.
- Is it good enough for the downstream accelerator?

Matching between two FFAG rings

with finite transverse amplitude (2)







allowed loss [%]	dp/p_rms [%]
0.5	5.1
1.0	4.1
1.9	2.5
5.5	1.9
10.6	1.5
16.0	1.2
22.8	0.9

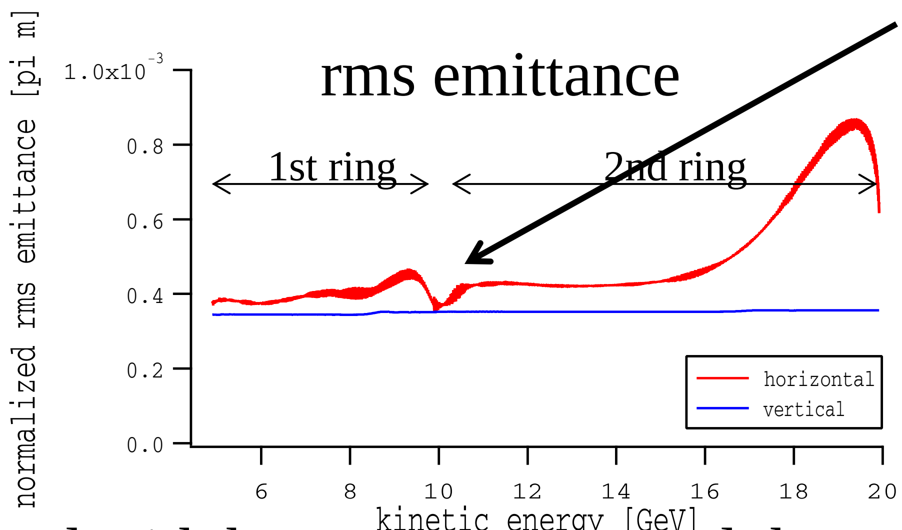
Matching between two FFAG rings

transverse

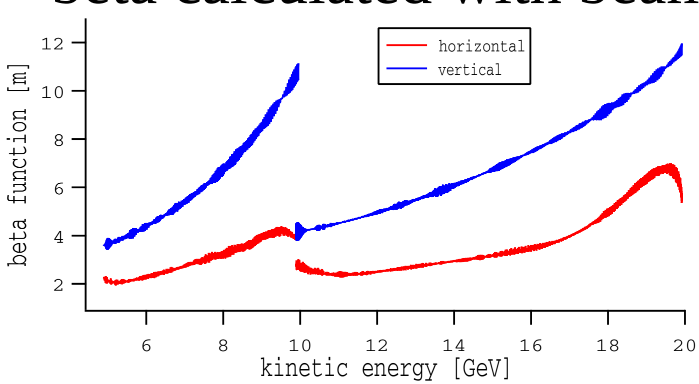
- Optical matching can be done.

Mismatch is small
maybe because of
small emittance (0.5 pi mm).

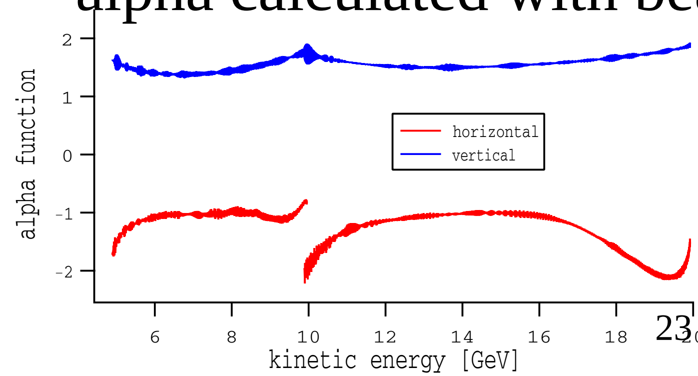
rms emittance



beta calculated with beam



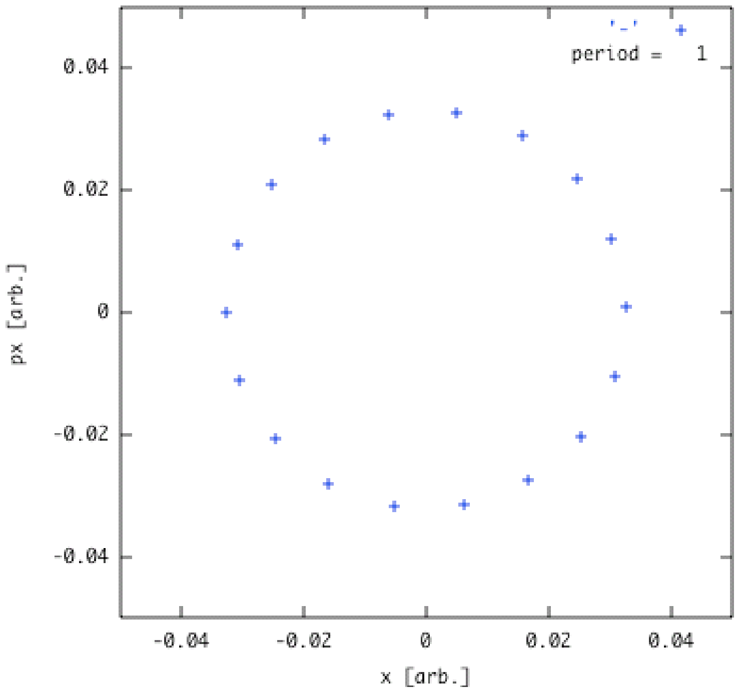
alpha calculated with beam



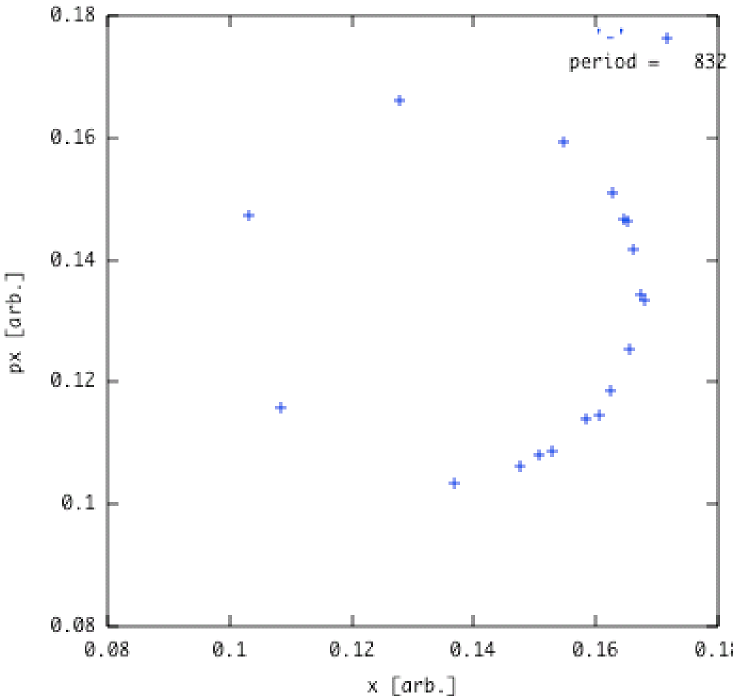
Matching between two FFAG rings

effects on betatron oscillations

- Starting with a circle of 10 pi mm (equally spaced).
- Accumulated betatron phase depends on initial betatron phase.
- Distribution becomes non-uniform after acceleration.
- Does distribution mismatching make any problem?



at 5 GeV (injection)



at 10 GeV (extraction)

Matching between two FFAG rings

summary

- Acceleration outside of RF bucket is a scheme that imposes strong longitudinal and transverse coupling

  (due to finite chromaticity, small dispersion, and no synchrotron oscillations).
  - Matching between 2 rings (both longitudinal and transverse) is not trivial.

- We can still play with many parameters to cure matching problem between two FFAGs.
  - Injection phase of the second ring (partially done).
  - Optical parameters of the first ring at injection (partially done).
  - Combination of 2 or more cures (partially done)

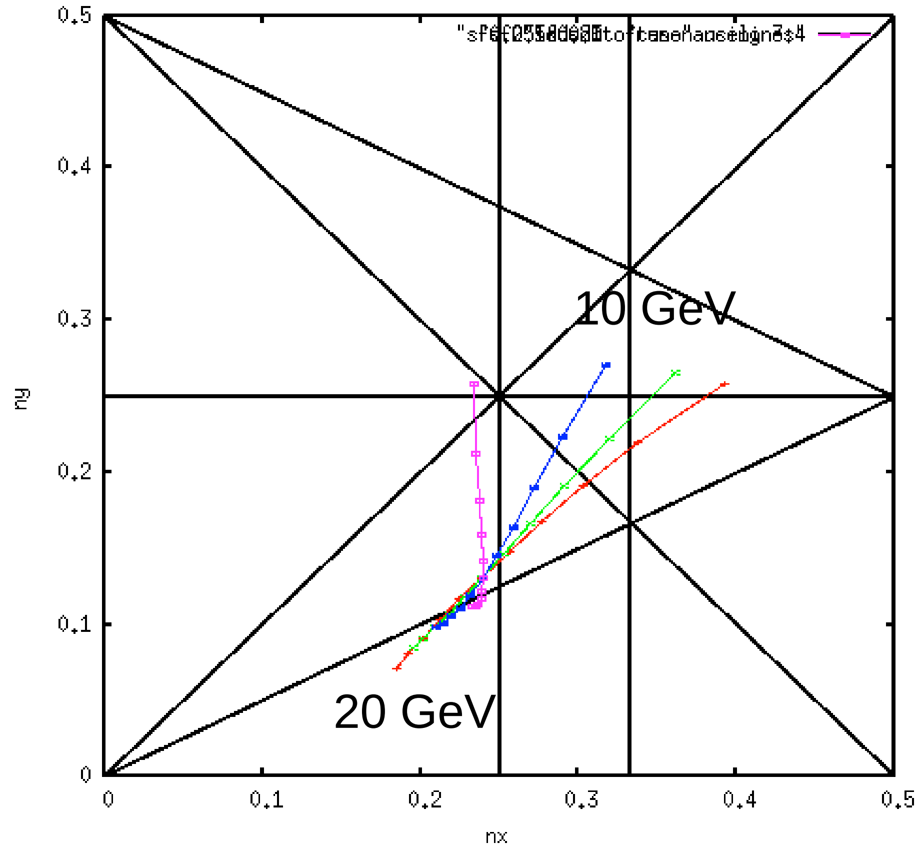
Non-scaling FFAG with sextupole

tune excursion

Both SF and SD sextupoles are installed at QF and QD*.

Strength are relative to the one required for full chromaticity correction in a SAD (MAD) model.

*Although SF is more effective than SD as pointed out by Koscielniak(tridn-2005-98.pdf), SF only makes vertical tune unstable at high momentum.

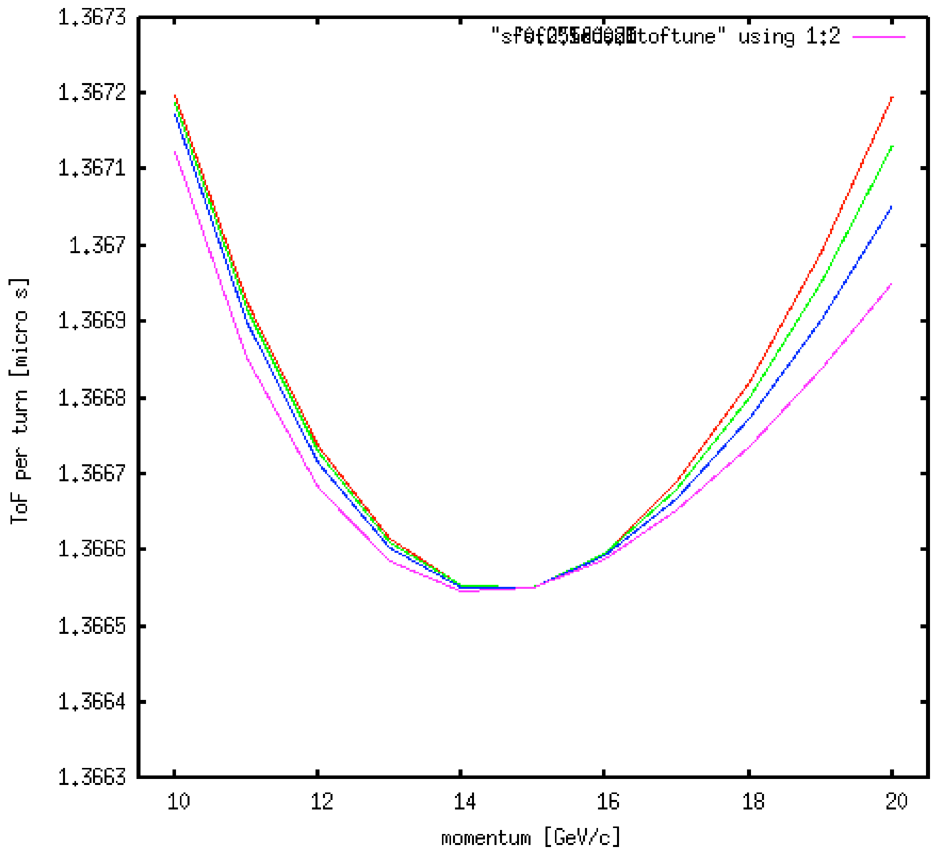


Sextupole strength:

Red	0%
Green	10%
Blue	25%
Magenta	50%

Non-scaling FFAG with sextupole
time of flight with sextupole

Although there are slight differences with sextupole, we use same voltage and frequency to do gutter acceleration in the following page.

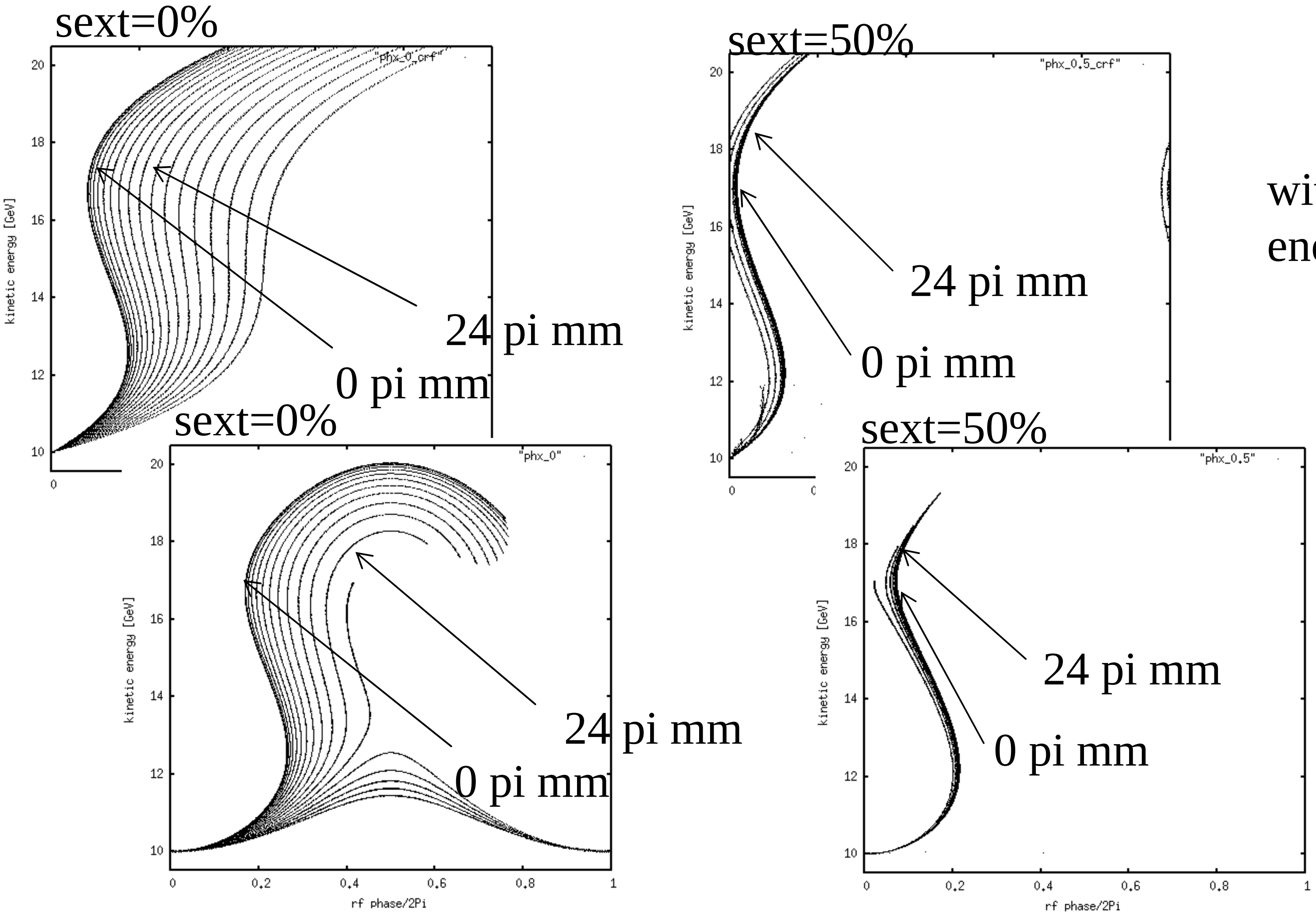


Sextupole strength:

Red	0%
Green	10%
Blue	25%
Magenta	50%

Non-scaling FFAG with sextupole

time of flight variation



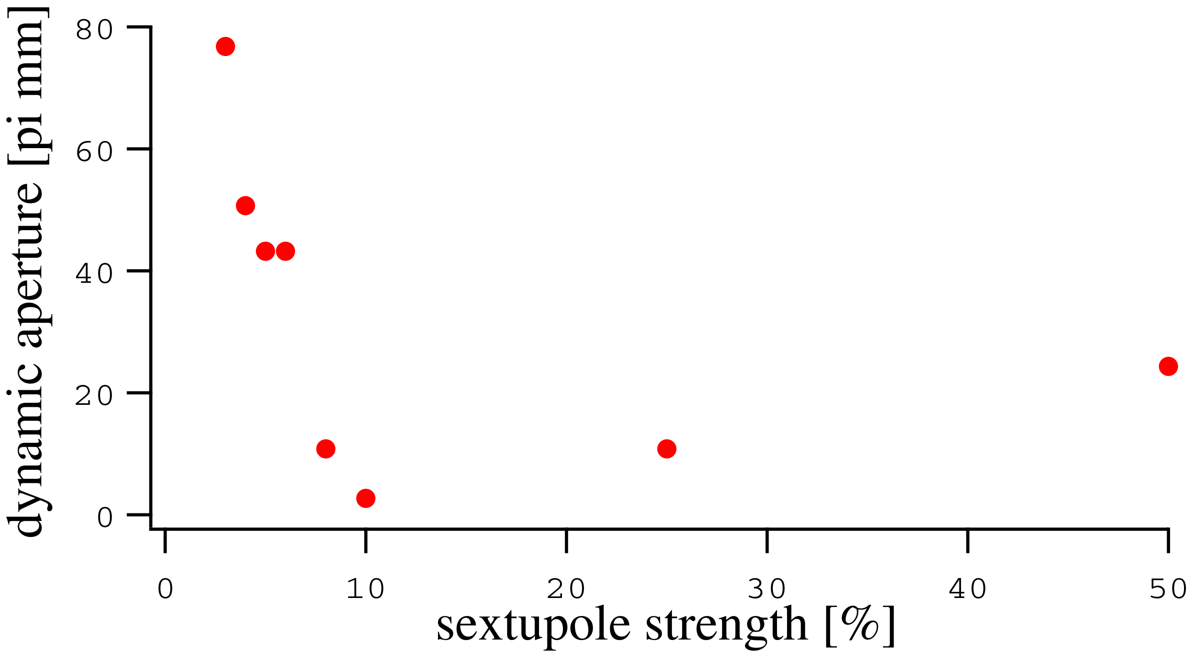
with constant energy gain

Accelerate outside of bucket

- Path length variation is reduced with (Hor.) chromaticity correction.

Non-scaling FFAG with sextupole

dynamic aperture at injection



- Sudden decrease of dynamic aperture with sextupole.
- Gradual increase with stronger sextupole.

Non-scaling FFAG with sextupole

summary

- Sextupole reduces time of flight variation with transverse amplitude.

- There is reduction of dynamic aperture with sextupole.

More on dynamic aperture of non-scaling FFAG with sextupole

Not only for muon application,
also for proton FFAG.

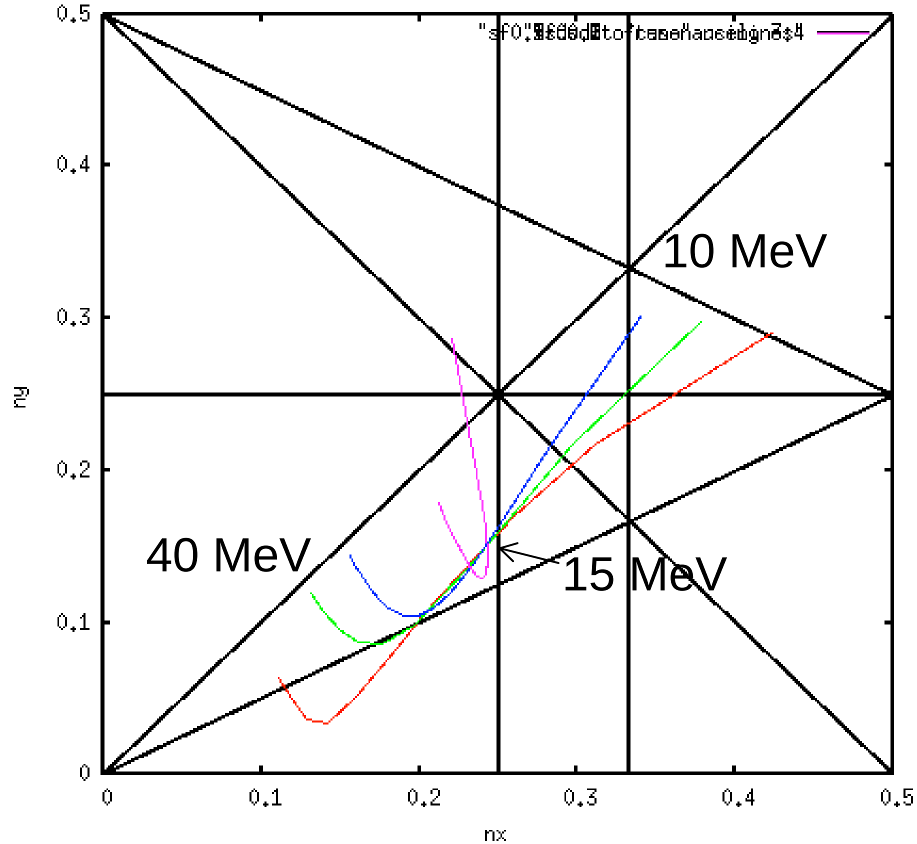
More on dynamic aperture

electron model with different sextupole strength

Question:
What is a reason of a small dynamic aperture?

Both SF and SD sextupoles are installed at QF and QD.

Strength are relative to the one required for full chromaticity correction in a SAD (MAD) model.

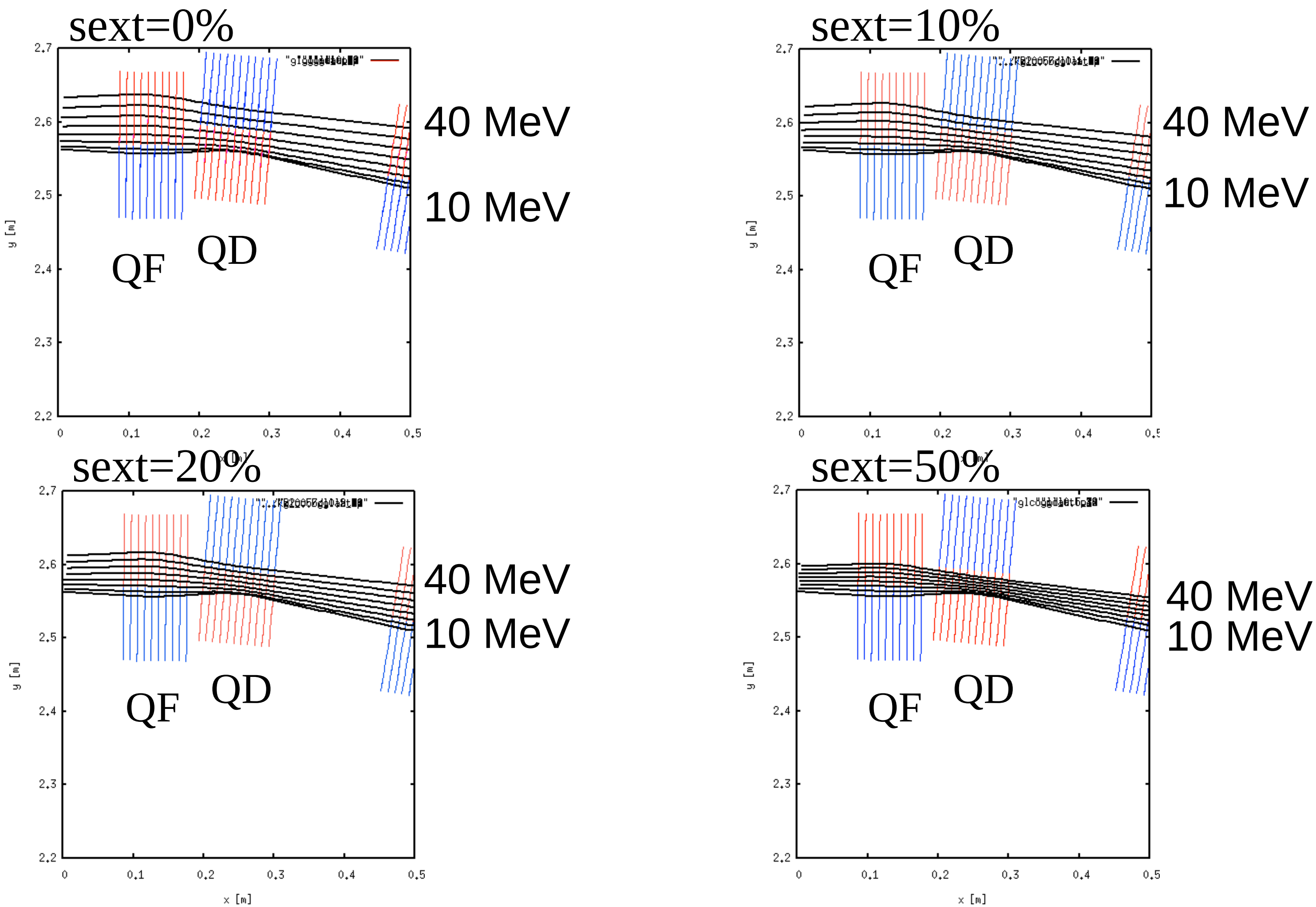


Sextupole strength:

Red	0%
Green	10%
Blue	20%
Magenta	50%

More on dynamic aperture

orbit of different momentum

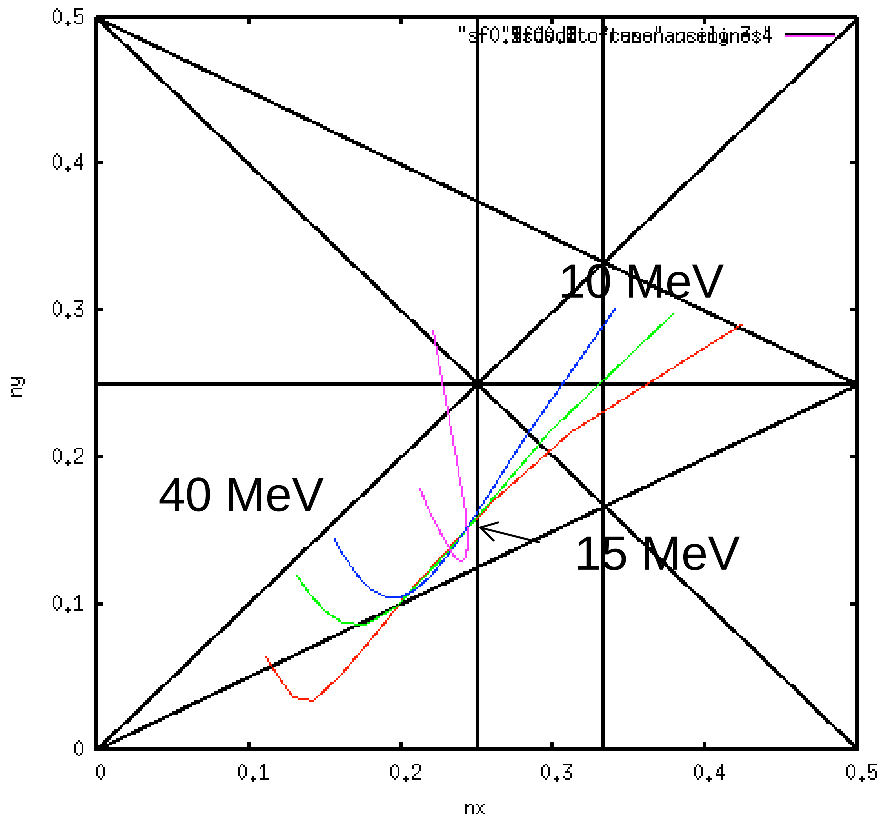


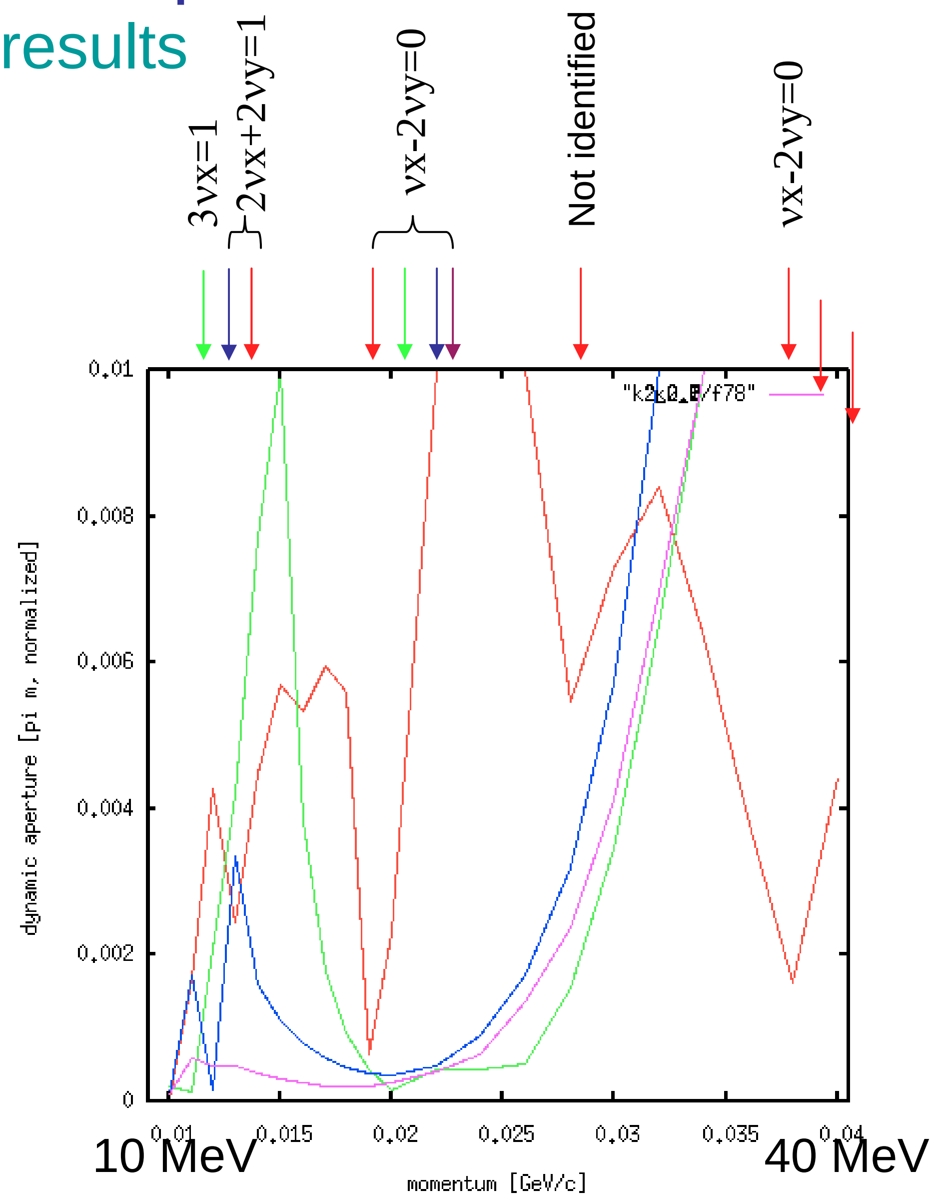
Stronger sextupole makes orbit excursion (dispersion function) smaller.

More on dynamic aperture

tracking results

100 turns tracking to estimate dynamic aperture.



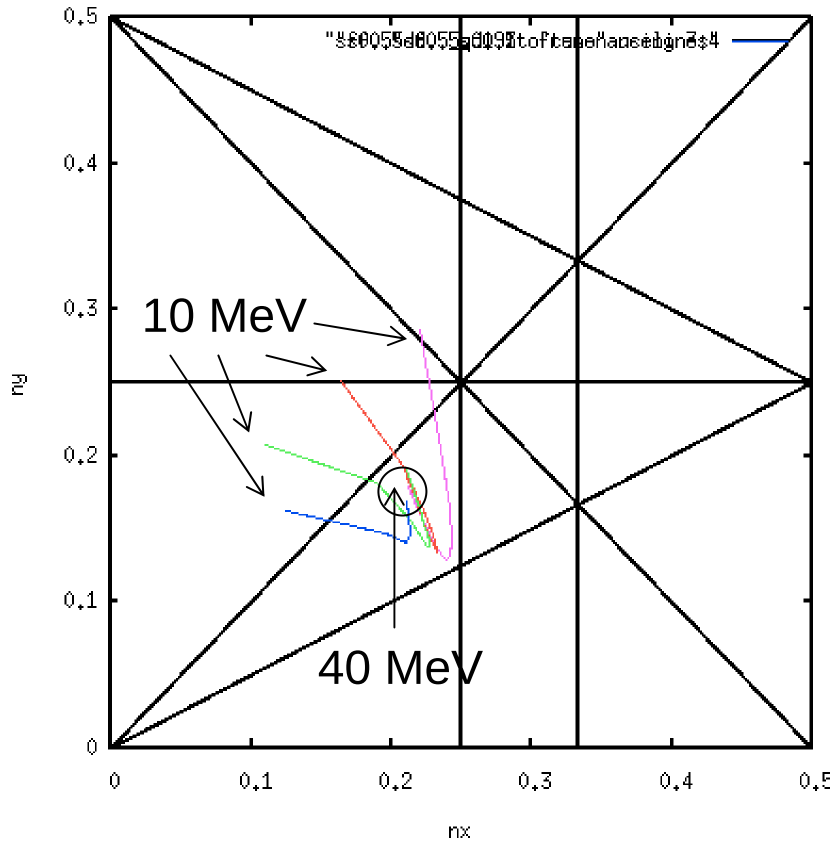


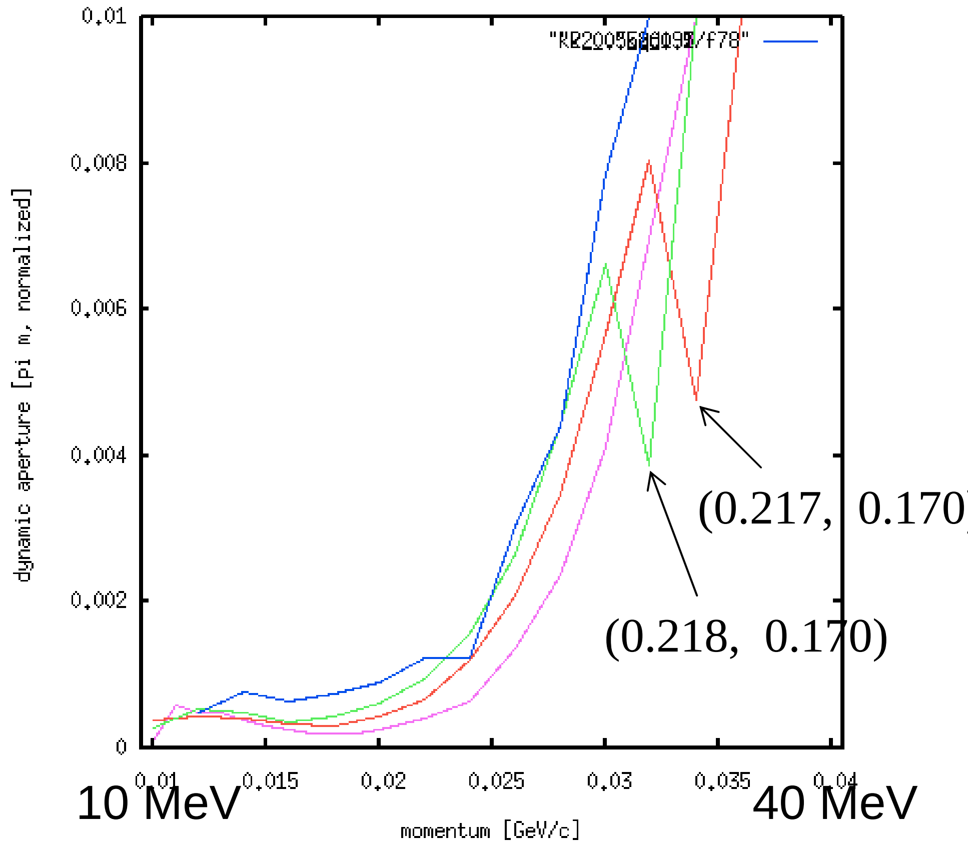
Sextupole strength:
Red 0%, Green 10%, Blue 20%, Magenta 50%

More on dynamic aperture

another operating point (1)

With the same sextupole strength (50%), change quadrupole slightly.



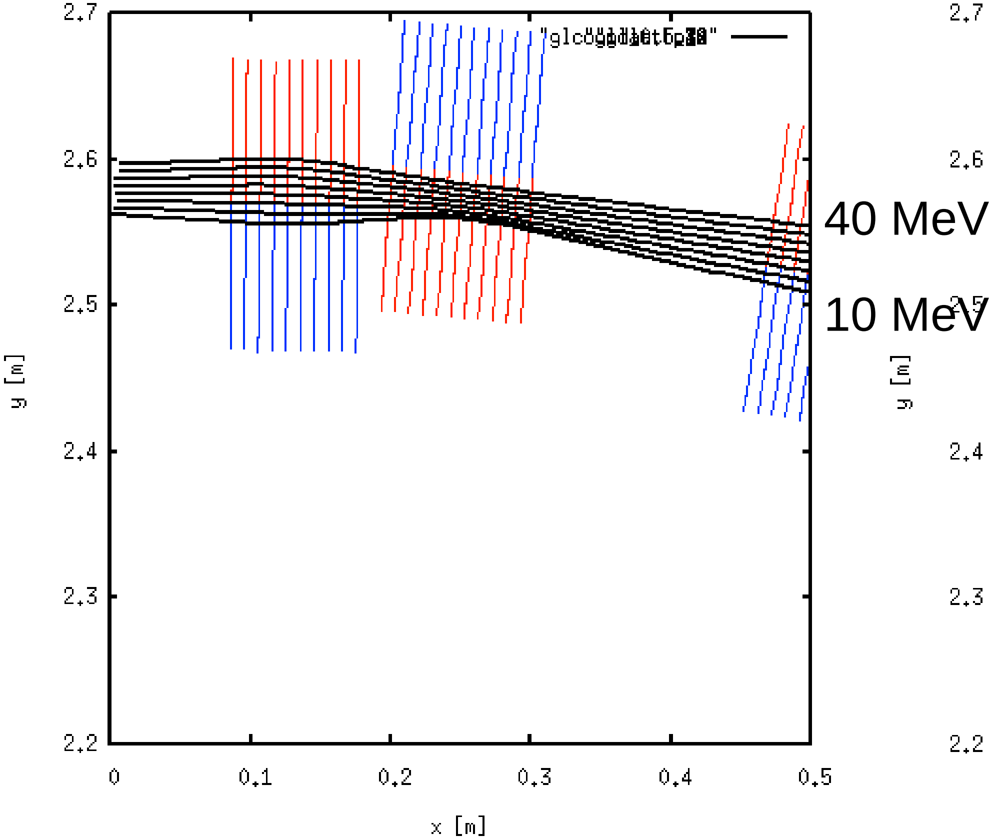


- Small dynamic aperture around 10 MeV is independent of operating points.
- The dip at (0.217~8, 0.170) is not associated with resonance.

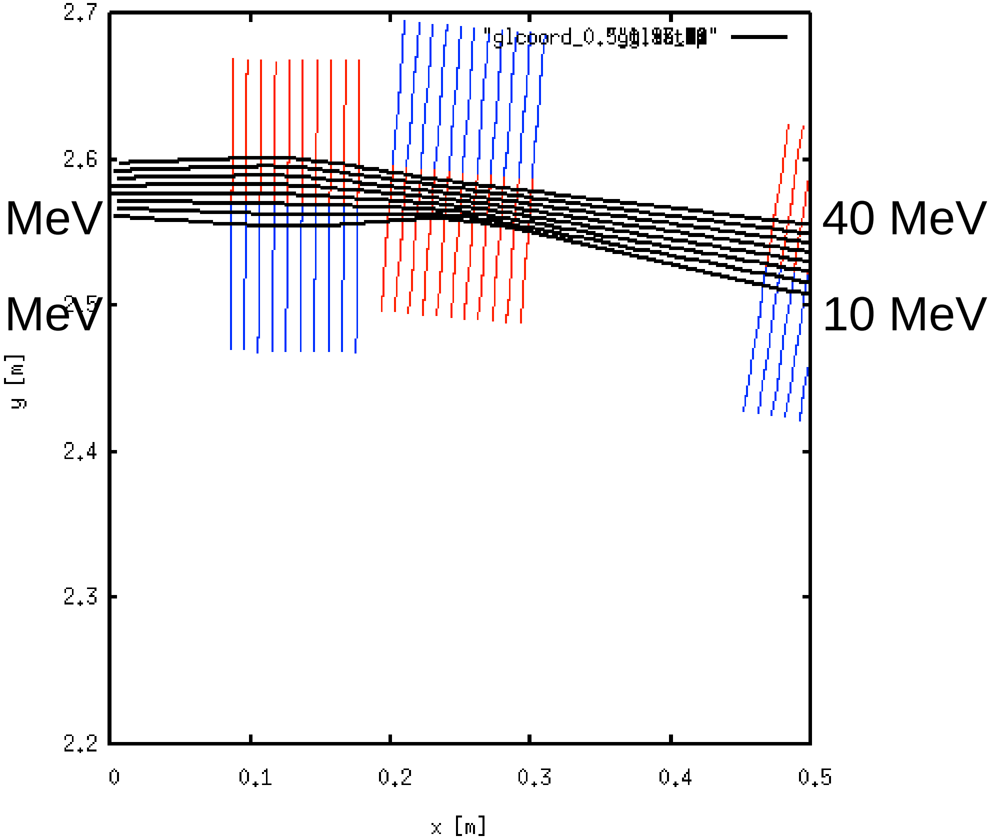
More on dynamic aperture

another tune (2)

QF=100% (magenta)



QF=97% (red)



Orbit excursion is about the same with different quadrupole setting.

More on dynamic aperture
summary

- Some of small dynamic aperture can be attributed to resonance.

- Around 10 MeV/c where it makes typical non-scaling optics, namely dispersion is minimum and QF gives reverse bend and QD does normal bend, dynamics aperture is always small independent of operating points.

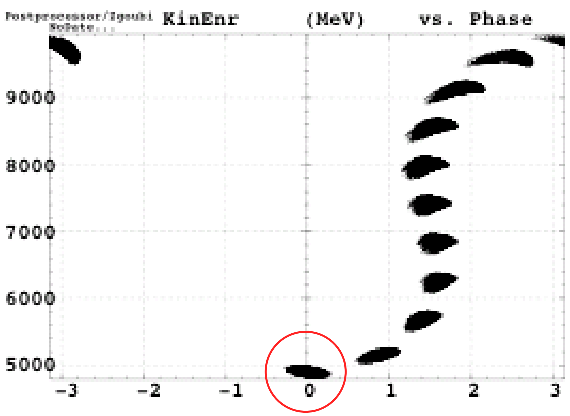
- At higher momentum, where optics looks like scaling, namely dispersion is relatively large and QF gives normal bend and QD does reverse bend, dynamic aperture becomes large.

Appendix

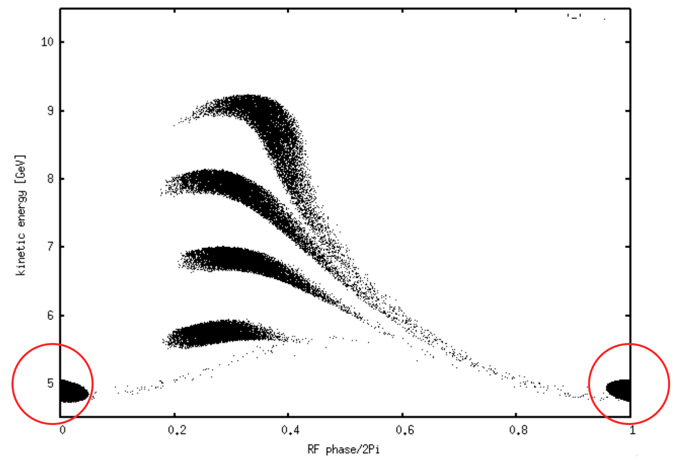
Code benchmarking

Code benchmark (1)

- Zgoubi and S(hinji's)-code had discrepancy (?)



Zgoubi



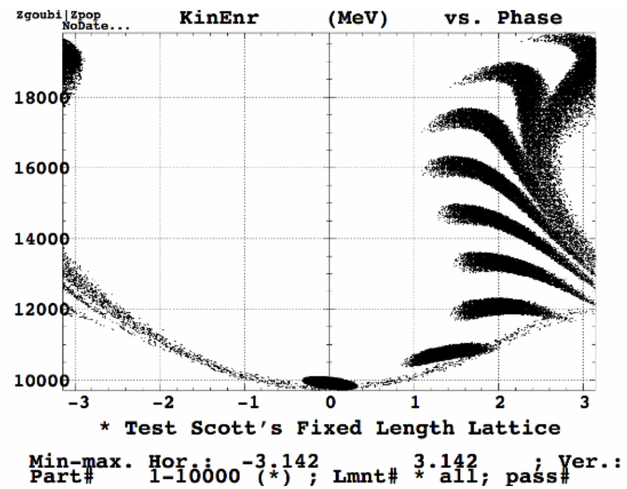
S-code

(5 to 10 GeV: 30 π mm in transverse, 0.05 eVs in longitudinal.)

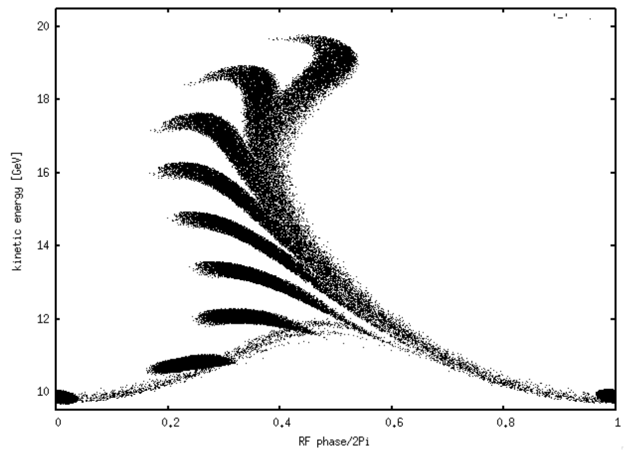
- Initial distributions were different.
  - Zgoubi assumed 6-D ellipsoid. S-code assumed 2-D ellipse independently in horizontal, vertical, and longitudinal space.

Code benchmark (2)

- With the same initial condition: independent in each plane.



Zgoubi

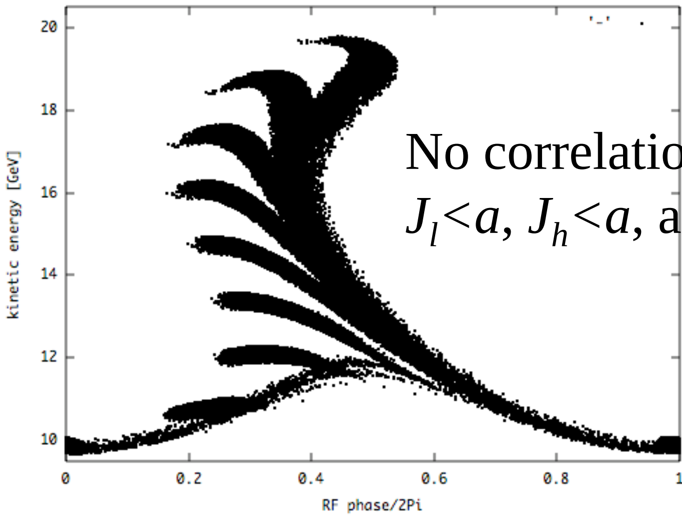


S-code

(10 to 20 GeV: 30 π mm in transverse, 0.05 eVs in longitudinal.)

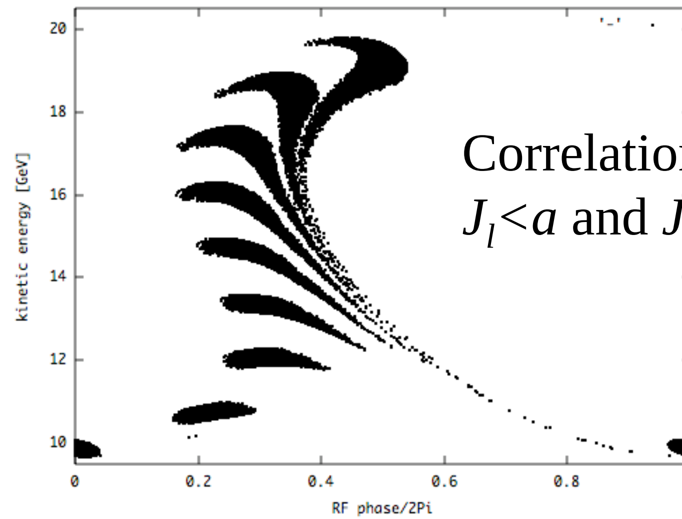
- Large amplitude particles make a problem also on Zgoubi.
- Another confirmation of the problem by Keil with MAD.

Code benchmark (3)

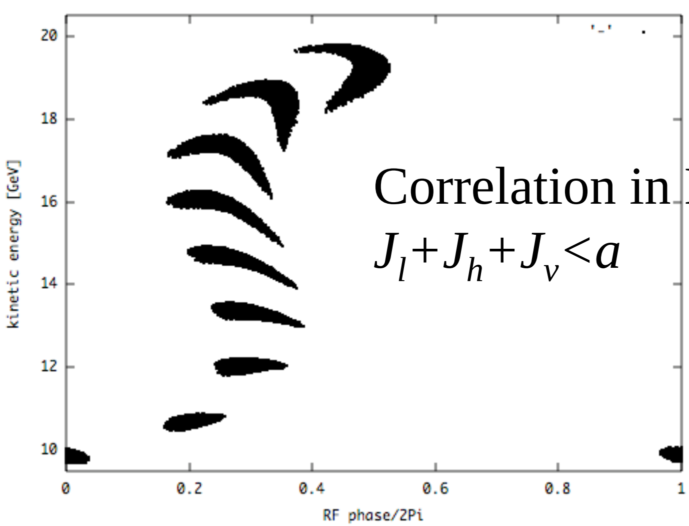


No correlation
$J_l<a$, $J_h<a$, and $J_v<a$

Some particles are not accelerated.



Correlation in T only
$J_l<a$ and $J_h+J_v<a$



Correlation in L and T
$J_l+J_h+J_v<a$

Correlation between transverse and longitudinal space eliminates large amplitude particles in both planes.

(10 to 20 GeV: 30 π mm in transverse, 0.05 eVs in longitudinal.)

a is amplitude in normalized phase space.

Code benchmark (4)

- In real life, there will be some correlation between horizontal and vertical, but probably independent of longitudinal.

- The best way is to take particle distribution at the end of linac or RLA. Use it as an initial distribution to FFAG.