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Status of tracking codes

Contents

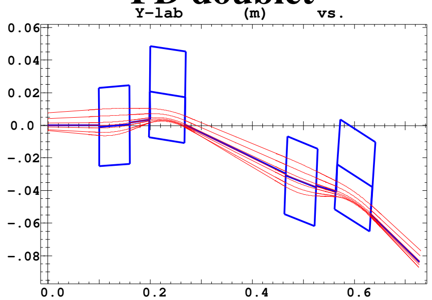
1 FFAG lattices that need large amplitude tracking

Goals in doing 6D-LAT : design and optimisation of lattice *and* magnet. DA tracking.

Linear, non-scaling

(natural $\xi_{x,z}$)

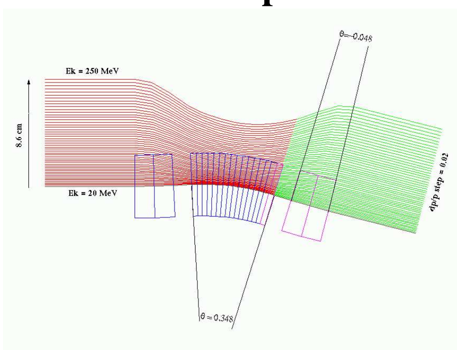
FD doublet



Concerns : muon, EMMA

Apps. : NuFact

FDF triplet



Concerns : proton, p-model

Apps. : hadrontherapy, p-driver

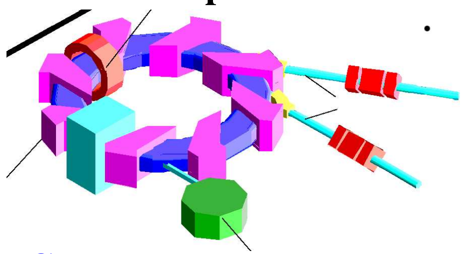
Scaling - $\frac{B}{B_0} = \left(\frac{r}{r_0}\right)^K$

(zero chromaticity, $\forall p$)

DFD triplet, doublet

SC technology

Spiral

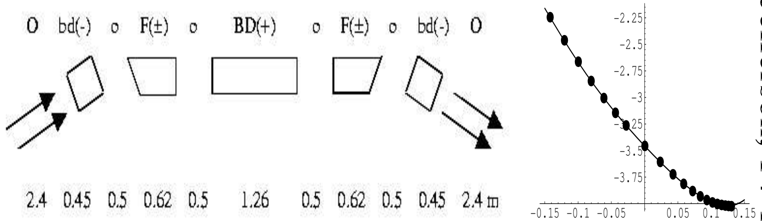


Concern : muon, e and p

Apps. : muon phys., NuFact, high power e and p, hadrontherapy, [R]Ions

Non-linear, non-scaling

Pumplet lattice

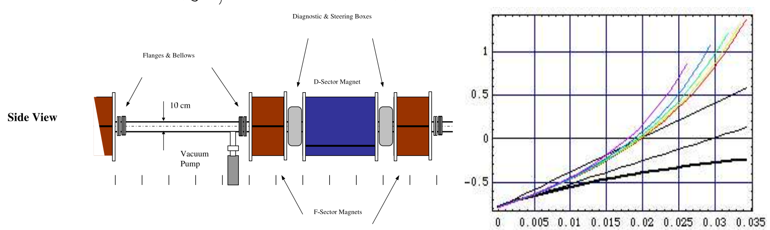


Concerns : muon and e-model (isochronous, $\xi_x > 0$, $\xi_z \to 0$) ; proton

Apps. : NuFact, p driver

Adjusted field profile ?

($\xi_{x,z} \longrightarrow$ small)



Concerns : proton

Apps. : p driver, hadrontherapy

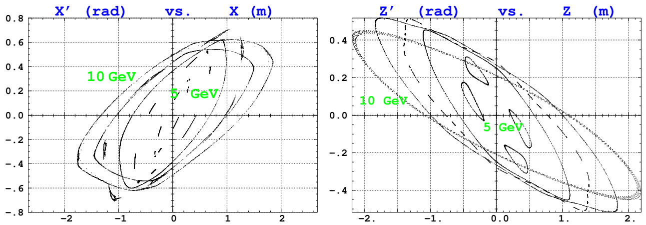
2 DA's of concern - orders of magnitude

- **DA's to explore are large, possibly *very* large - a key interest of FFAGs.**

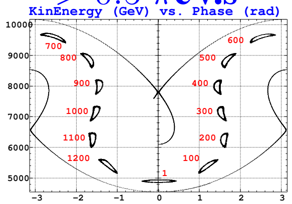
Linear, non-scaling

FD doublet

Muon :
$\gg 3\pi$cm norm.



$> 0.5\ \pi$eV.s



EMMA (electron) :
$\gg$200-300 πmm.mrad norm.

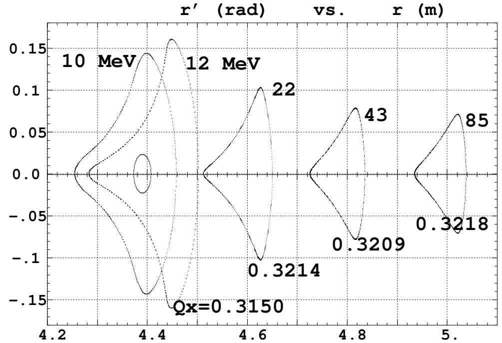
Proton :
10s πmm.mrad norm.

Non-linear, scaling

DFD triplet, doublet, spiral

$0.3 - 20$ GeV muon :
$> 3\pi$cm norm., $1.5\ \pi$eV.s

Proton :
10s πmm.mrad norm.



Non-linear, non scaling

Pumplet lattice

$8 - 20$ GeV muon
isochronous
$\approx \pi$cm norm. $-0.5\ \pi$eV.s

p-Driver :
10s πmm.mrad norm.

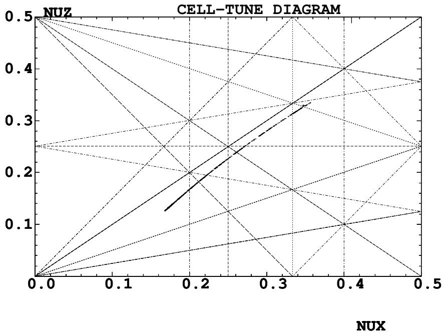
electron model :
100-300 πmm.mrad norm.

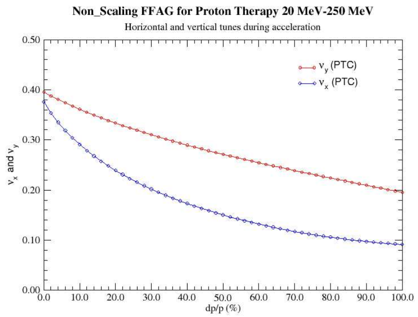
Adjusted field profile
p apps.,
10s πmm.mrad norm.

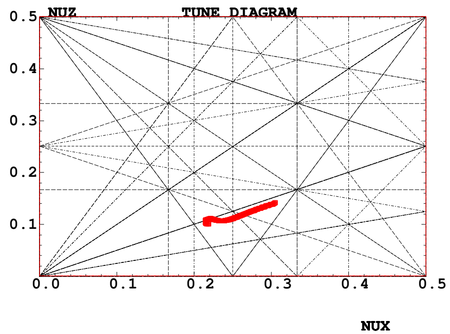
- **A straightforward remark with these types of optics : given** $\left\{ \begin{array}{l} (i) \text{ the large excursions,} \\ (ii) \text{ possibly the strong non-linearities,} \end{array} \right.$ **better use an accurate stepwise tracking method !**

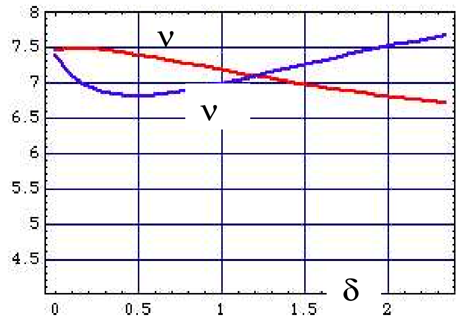
• **In all cases : FFAG tracking methods need to provide means for 6-D simulation in presence of**
- **fast acceleration**
- **orbit change with E**
- **proximity and/or crossing of resonances**

muon, EMMA, linear, non-scaling FD doublet	**250MeV p-therapy linear, non-scaling FDF triplet**	**pumplet, e-model non-linear non-scaling**	**AFP non-linear non-scaling**









• **May also need : Fringe gield overlapping, case of e.g.,**
- **scaling FFAGs, cf. PRISM**
- **linear FDF triplet, cf. 250MeV p-Therapy**

• **And also, sooner or later : will need symplectic tracking using** ***magnetic field maps*****.**
Cf. for instance present R&D in Japan

3 Tracking codes known to (or to have) handle(-d) FFAG problems

code	seen in company of (sort of POP)	allows FF overlap	allows field map	method
COSY	linear FFAG			Taylor expansion, needs ref. orbit
ICOOL	muon ; scaling DFD	yes ●		RK4
MAD-PTC	muon, EMMA		no	kick-drift, symplectic, z-type
J-RK4	typical of J R&D		yes	
S	linear ; scaling		yes	kick-drift, sympl., s-type
Zgoubi	all types of FFAGs	yes ●	yes	Taylor series, s-type

Probably needs completion

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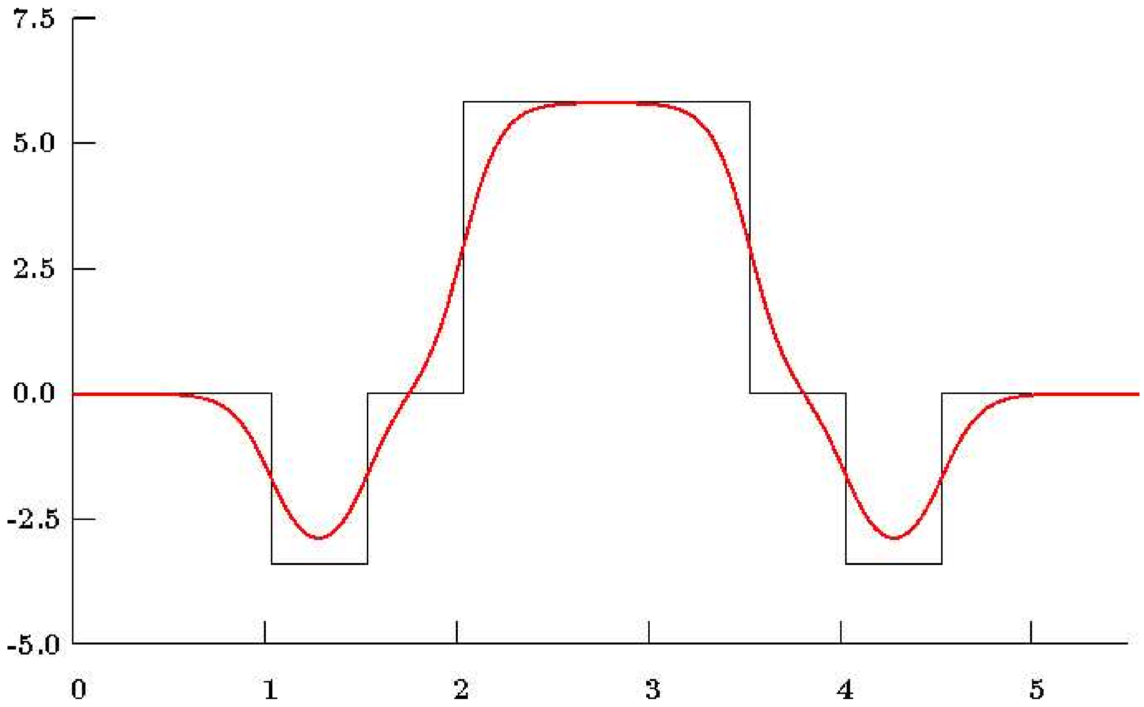
Optics investigations, SYNCH, TEAPOT, MAD8, etc. : see Trbojevic et als., PAC03

ICOOL

Example : overlapping fringe fields in an FFAG triplet

- **Fringe field model, with two asymptotes :** $B = \frac{1}{2}B_o\left(1 - \frac{e^{z/\Gamma}-e^{-z/\Gamma}}{e^{z/\Gamma}+e^{-z/\Gamma}}\right)$
- **Overlapping based on** $\sum FT\left(B(s)\right)$**, also avoids discontinuities**

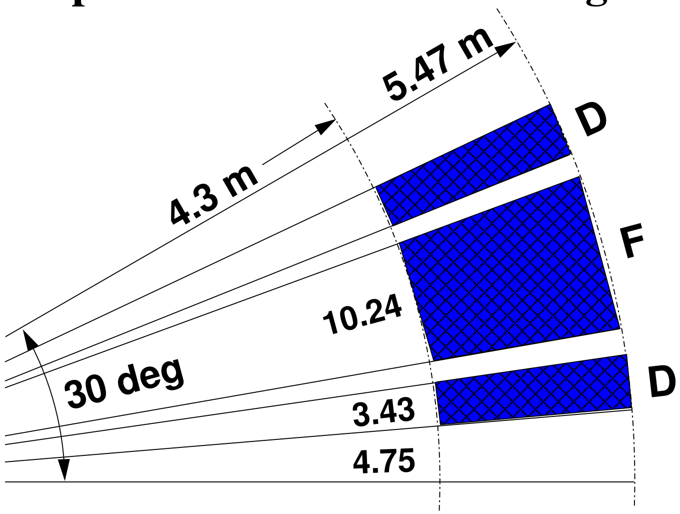


Zgoubi - the "FFAG" and "DIPOLES" procedures

- **Two main goals :** $\begin{cases} (i) \text{ simulate } B_{zi}(r,\theta) = B_{z0,i}\,\mathcal{F}_i(r,\theta)\,\mathcal{R}_i(r) \text{ (e.g., scaling, pumplet)} \\ (ii) \text{ allow for possible overlap of fringe fields} \end{cases}$

  Main apps : scaling and isochronous FFAGs.

- **An example : simulation of a scaling FFAG triplet :**

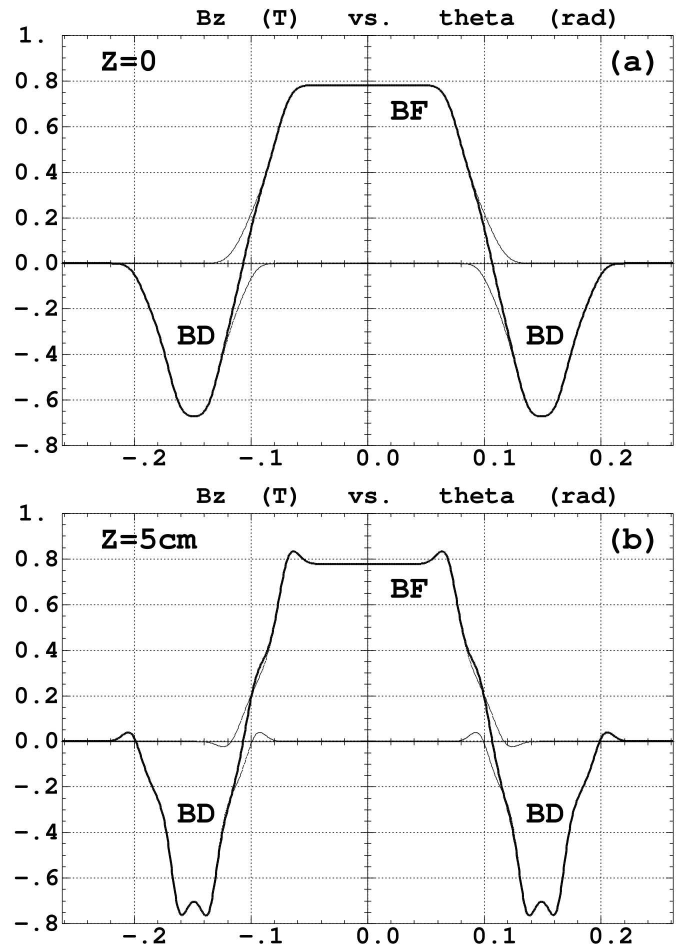


DFD triplet.

The geometrical model is based on the superposition of the independent contributions of the N dipoles :

$$\boxed{B_z(r,\theta) = \sum_{i=1,N} B_{z0,i}\,\mathcal{F}_i(r,\theta)\,\mathcal{R}_i(r)}$$

at all (r,θ) in the mid-plane.
Field off mid-plane is obtained by Taylor expansion accounting for Maxwell's eqs.



Field experienced for $r_0 = 4.87$ m in a DFD dipole triplet.

Codes known to (or to have) handle(-d) FFAG problems

code	seen in company of (sort of POP)	allows FF overlap	allows field map
COSY	linear FFAG		
ICOOL	muon ; scaling DFD	yes	
MAD-PTC	muon, EMMA		no
J-RK4	typical of J R&D		yes ●
S	linear ; scaling		yes
Zgoubi	all types of FFAGs	yes	yes ●
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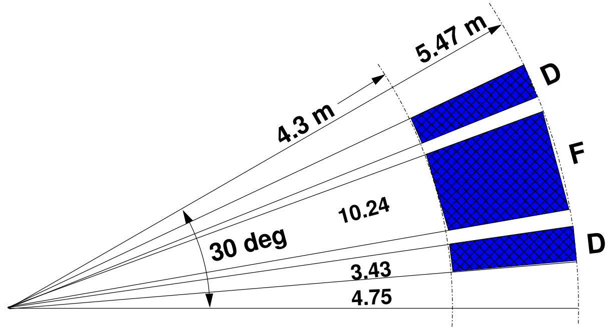
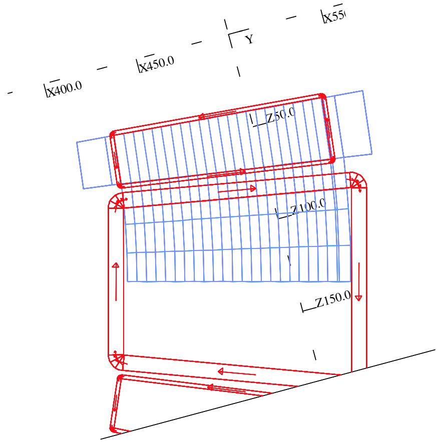
. . .			
. . .			

Probably needs completion

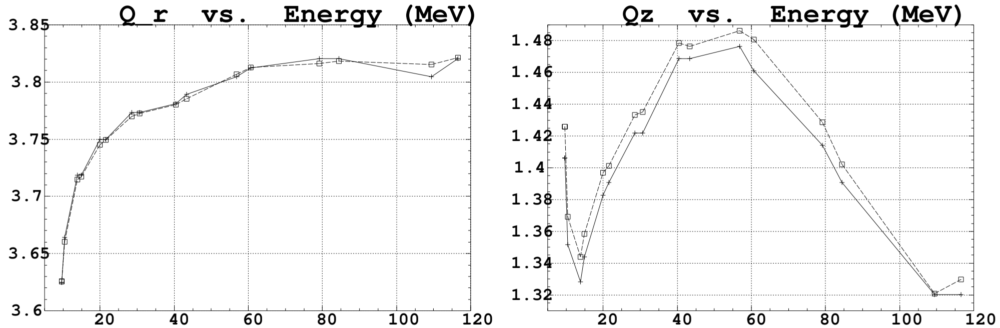
Tracking in 3-D field maps



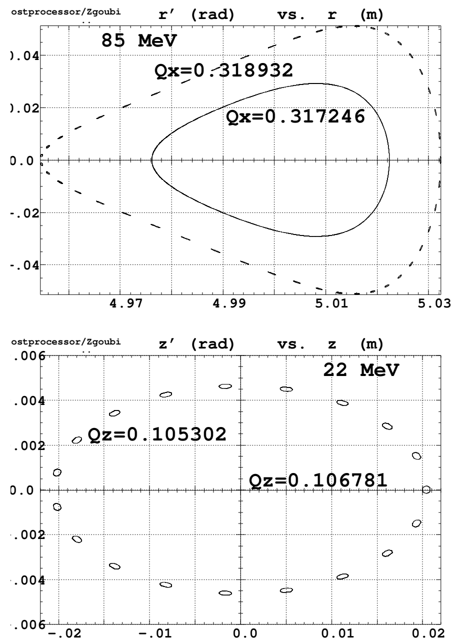
DFD sector triplet constituting a 30 degrees sector cell.



Geometry of TOSCA field map, covering half the angular extent.



Radial tune (left plot) and axial tune (right) as a function of energy, as obtained using
RK4 integration (solid lines/crosses),
or using Zgoubi (dashed line/squares).



Sample multiturn tracking using field maps, more than 3000 passes in a DFD triplet cell.
The accuracy (“symplecticity”) appears to be very good.

Codes known to (or to have) handle(-d) FFAG problems

code	seen in company of (sort of POP)	allows FF overlap	allows field map	has performed 6-D tracking
COSY	linear FFAG			no
ICOOL	muon ; scaling DFD	yes		yes
MAD-PTC	muon, EMMA		no	linear FFAG ●
J-RK4	typical of J R&D		yes	yes
S	linear ; scaling		yes	yes ●
Zgoubi	all types of FFAGs	yes	yes	all types ●

Probably needs completion

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...
...

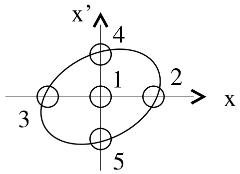
4 Comparisons between codes

S / Zgoubi comparison (1)

Time of flight in muon linear FFAG. **Some work left...**

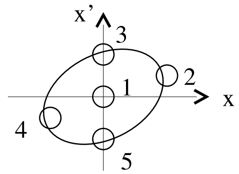
Time of flight of 30 π mm particles per turn

Zgoubi



PID	Trev	ΔTrev
1	.964504	
2	.964807	.000303
3	.964819	.000315
4	.964813	.000309
5	.964804	.000303

S-code

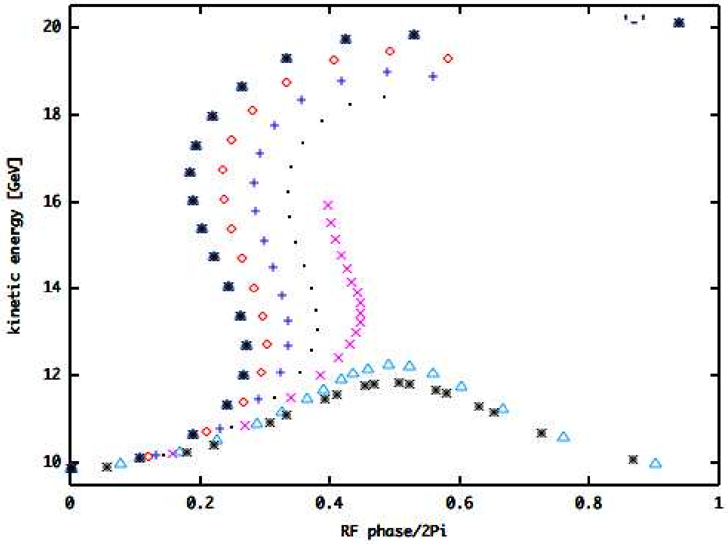


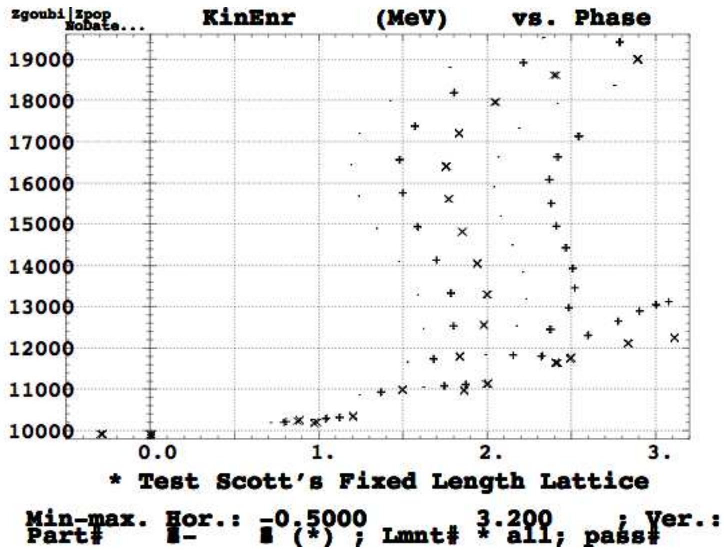
PID	Trev (μs)	ΔTrev
1	.950106	
2	.950416	.000310
3	.950406	.000300
4	.950398	.000292
5	.950394	.000288

S / Zgoubi comparisons (2)

Acceleration in muon linear FFAG. **Results seem promissing. Check in detail ?**

Trajectory in longitudinal phase space for different transverse amplitudes (10 to 20 GeV muon ring)





Markers (colors) corresponds to the horizontal amplitude of 0, 10, 20, 30, 40, 50, 60 π mm (normalized), respectively.
40 π mm particle is barely accelerated, but not 50 π mm.

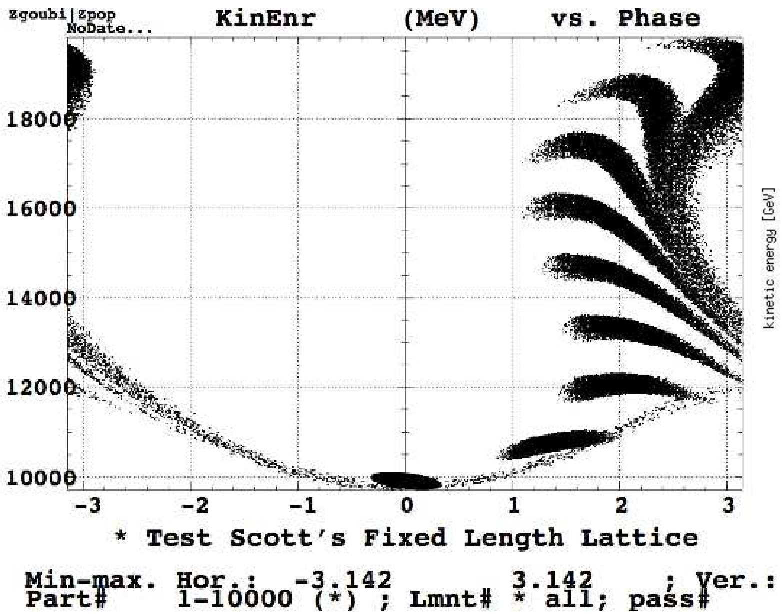
S / Zgoubi comparisons (3)

6-D cceleration in muon linear FFAG.

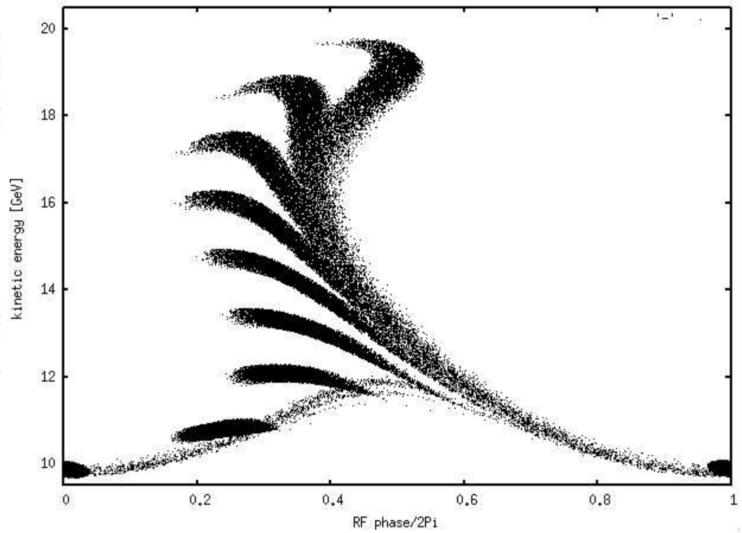
10 to 20 GeV muon ring

Particles are uniformly filled in each phase space independently.

Zgoubi



S-code



(30 p mm in transverse, 0.05 eVs in longitudinal.)

S / Zgoubi comparisons (4) : more wanted

Analytical modeling for non-scaling magnet (1)

- Shifted quadrupole
  - Soft edge model with Enge type fall off.
  - Scalar potential in cylindrical coordinates.

$$P_2(r, q, z) = \frac{r^2 \sin 2q}{2}\left[G_{2,0}(z) + G_{2,2}(z) r^2 + ¿\cdot¿\right]$$

where

$$G_{2,2k}(z) = (-1)^k \frac{2}{4^k k!(2+k)!} \frac{d^{2k} G_{2,0}(z)}{dz^{2k}}$$

and

$$G_{2,0}(z) = \frac{G_0}{1+\exp\left(\sum_{i=0}^{5} C_i z^i\right)} \qquad z = \frac{s}{g}$$

s: distance from hard edge.

g: scaling parameter of the order of gap.

C_i: Enge coefficient.

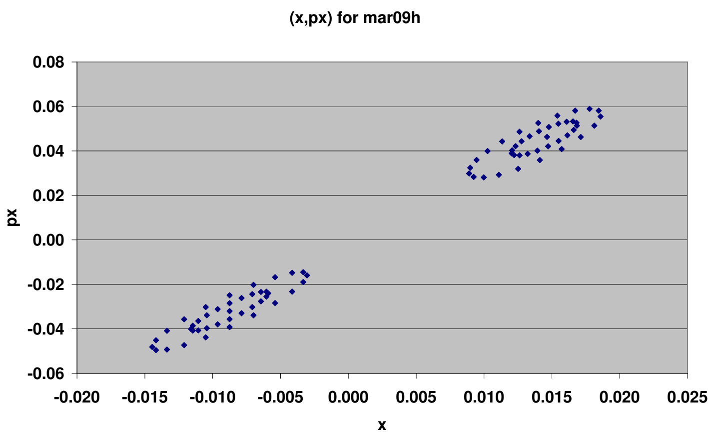
Analytical modeling for non-scaling magnets (2)

- Up to G_{20} and G_{21}
  - Edge focusing
- Up to G_{22} and G_{23}
  - Octupole components of fringe fields
- Up to G_{24} and G_{25}
  - Dodecapole …

- Feed-down of multipole (octupole) has large effects when G_{22} and higher order is included.
- It is not clear if it is real or numerical defects due to subtraction of two large numbers.

Potential for cross-checks with MAD-PTC (1)

Horizontal Phase Space Trajectories vs. Horizontal Emittance

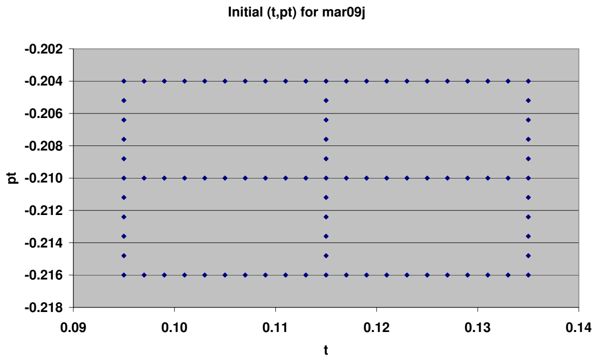


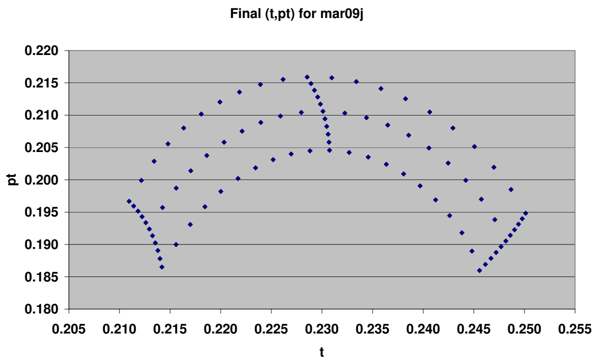
- Graph in (x, p_x), not in normalised phase space
- Abscissa is horizontal offset x from reference orbit
- Ordinate is p_x divided by reference momentum $p_0 = 15$ MeV
- Initial beam in lower left corner, final beam in upper right corner
- Launch the same 41 particles as before

- Rather wide initial beam → choose smaller ε_{xn}?
- Final beam has elliptical outline
- Ellipses not centred around particle launched on closed orbit
- Concentration of dots on right side of final beam, c.f. Machida's talk

Potential for cross-checks with MAD-PTC (2)

Longitudinal Distortion





- Graphs of coordinates in (t, p_t), not in normalised phase space
- Abscissa is longitudinal offset t from reference orbit, multiplied by c
- Ordinate is p_t divided by reference momentum $p_0 = 15$ MeV/c
- Launch 87 particles in rectangle on closed orbit of central one
- Possible alternative: Launch particles on closed orbit of their $\delta p/p$
- $\Delta t \approx 40$ mm, $\Delta p_t \approx 180$ keV/c, close to Méot's data on 20 Dec 2005
- Enclosed area $\varepsilon_{\|} \approx 15$ mm $\approx 2.4 \cdot 10^{-5}$ eVs
- Rings with $b = 0$ have smaller distortions than rings with $b \neq 0$

5 Remarks on s-type integrators and precision

• Final ITER-ation

In s-type integration, the downstream boundary of an optical element is in general not reached exactly after an integer number of steps Δs. Therefore, s-type integration needs a terminator algorythm.

The final step $\delta s < \Delta s$ for the trajectory to intecept the downstream boundary being not know, that intercept will result from an iteration process.

In Zgoubi for instance, the ITER algorythm writes :

```
      SUBROUTINE ITER(A,B,C,DS,COSTA,KEX,*)
      PARAMETER (EPS=1.D-6,ITMAX=1000)
      PARAMETER (EPS2=1.D-10)
      DM=1.D30
CALCUL INTERSECTION DE LA TRAJECTOIRE AVEC DROITE AX+BY+C=0
      DO 1 I=1,ITMAX
        D=A*XF(1)+B*XF(2)+C
        ABSD = ABS(D)
        IF(ABSD .LE. EPS)  THEN
          IF(ABSD .LE. EPS2)  THEN
            RETURN
          ELSEIF(D.GT.DM.OR.D.EQ.0.D0 ) THEN
            RETURN
          ENDIF
        ENDIF
        DM=ABSD
        DS=DS-D/COSTA
C One more push
        CALL DEPLA(DS)
 1    CONTINUE
```

In that manner, the boundary is reached with accuracy at machine precision, in general in no more $IT = 2 - 5$ iterations.

- **Order of final ITER-ation integrator**

It can be shown that the overall precision of the integration from end to end over an optical element is liable to fall *below that of the final iteration integrator.*

For that reason, it is *fundamental* **that the latter be of the same order in Δs as that of the body integrator.**

In Zgoubi for instance, it is simple : the ITER integrator is the same as for body (DEPLA(DS), above).

These considerations are illustrated in the next slide

Tracking : effects of integrator order, ITER-ation order

ZGOUBI

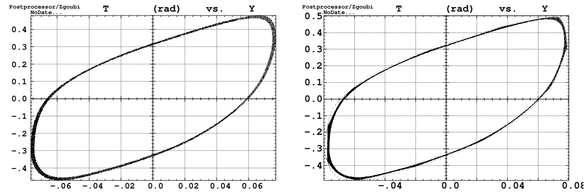


Figure 1: Zgoubi, order 4 with 20 steps (left) or 6 with 5 steps (right).

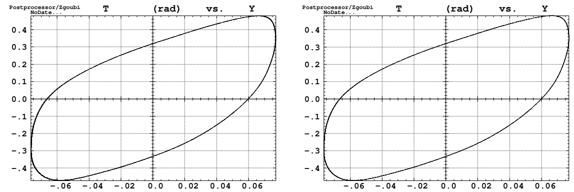


Figure 2: Zgoubi, order 4 with 40 steps (left) or 6 with 10 steps (right).

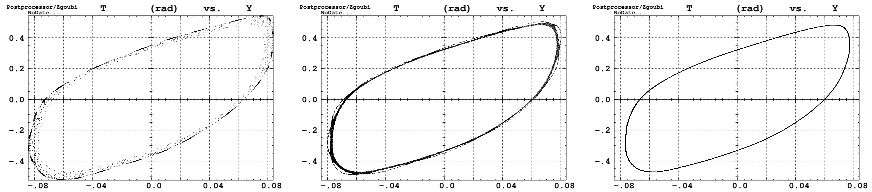


Figure 3: Effect of loss of precision in ITER. Zgoubi is used at order 6 with 20 steps, so to insure good precision at all steps but the last one. (This should produce an invariant at least as good as in Fig. 2-right). Here however we purposely mishandle the last step, lowering the order of ITER (χ) to 2, 3 or 4 from left to right. Conclusion: spoiling the precision in the last step is enough to spoil the overall symplecticity. It decreases the precision to respectively order 2, 3 and 4.

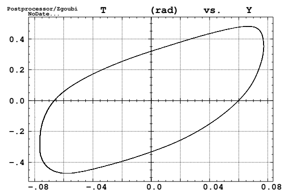


Figure 4: Starts from the conditions in Fig. 3, namely, order 6 at all steps except for the last step where order 2 is taken. The difference is in the number of steps, 2000 instead of 20. The precision is regained. The overall order is unchanged, still spoiled down to order 2 due to the order 2 in the last step. However the accuracy is better in account of the high number of steps.

6 (Tentative) conclusions

- **We dispose of three-four 6D-LAT codes**
- **We dispose of the trackers (sort of sacerdotical life type of folks...)**
- **It's probably enough, it's not too much, the difficulty of the FFAG problem deserves it**
- **Carry on cross checks**
- **Carry on upgrade of codes and optics libraries**

Thank you