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Flips in a Cooling Channel

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Basic Equations

- System invariant under rotations
- Two helical eigenmodes in canonical coordinates
- Emittances are

$$\epsilon \pm L/2 \qquad \epsilon^2 = \sigma_{xx}\sigma_{pp} - \sigma_{xp}^2 \qquad L = \sigma_{xp_y} - \sigma_{yp_x}$$

- Emittances evolve as

$$\frac{d(\epsilon \pm L/2)}{ds} = -\frac{m_{10}}{\beta c p}\left(1 \mp \frac{zeB_s}{2}\frac{\sigma_{xx}}{\epsilon}\right)(\epsilon \pm L/2) + \frac{S_{\mathrm{MS}}}{2}\frac{\sigma_{xx}}{\epsilon}$$

Basic Equations

- Key parameter is

$$r = \frac{zeB_s}{2}\frac{\sigma_{xx}}{\epsilon}$$

  - Ratio of beta function to constant solenoid beta function
- $r = 1$ for constant solenoid field
- Obtain $r < 1$ at absorber (only place it matters) by longitudinal variation of solenoid field
  - Cost is dynamic aperture
- With $r = 1$, one mode grows linearly, other damped
- With $r < 1$, both modes damped
- With $r = 0$, both modes treated equally

Understanding Constant Solenoid

- In physical space: one mode is helix, other is fixed transverse position
- Momentum kick (multiple scattering) induces helical oscillation *and* displacement of oscillation center
- Helical oscillation damped by energy loss, displacement unaffected
- In canonical coordinates, mode eigenvectors don't care about angular momentum
  - Which mode in physical space is helix depends on magnetic field direction
  - Flip the field, switch effect on two modes

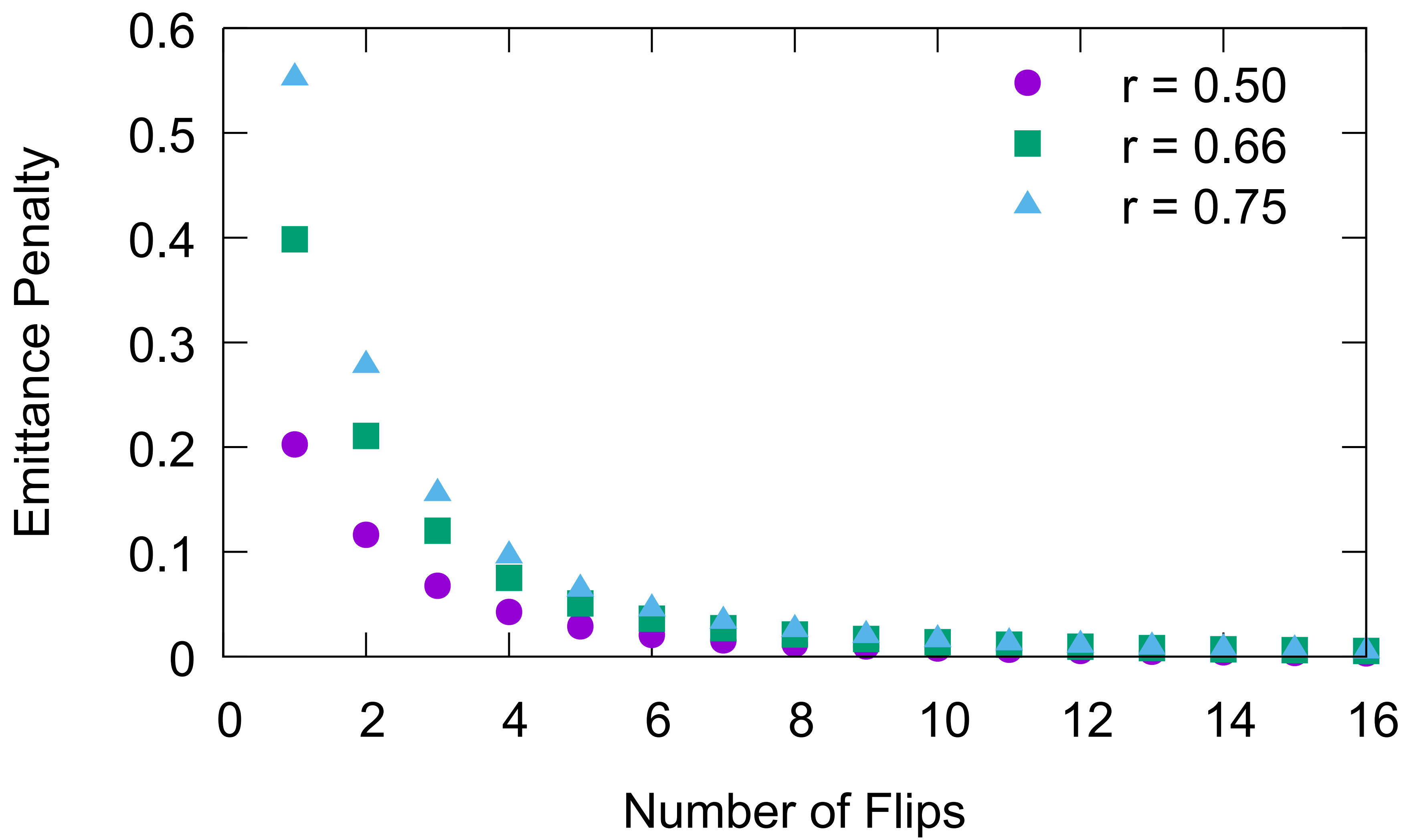
Field Flips

- Easier to get low beta without flipping fields: solenoids don't fight each other
- Generally want both modes cooled equally
- If you go far without flipping, there is a penalty over the $r = 0$ case in the emittance product for a given length of cooling
- Flip in non-periodic system may lead to emittance growth
- What is the emittance penalty vs. the frequency that you flip?

Examples

- Assume a continuously tapered channel, based on the $r = 0$ solution
- Insert a fixed number of flips
  - A single flip does not have the same length before and after the flip
  - Multiple flips start and end with half length sections; again, lengths are all different
- Length to cool by a factor of 20 for the $r = 0$ case
- Next, no flip, still taper try to make emittances differ by a factor of 10
  - Look at channel length and gain in geometric emittance average

Tapered Channel, $\epsilon_\infty/\epsilon_0 = 0.66$



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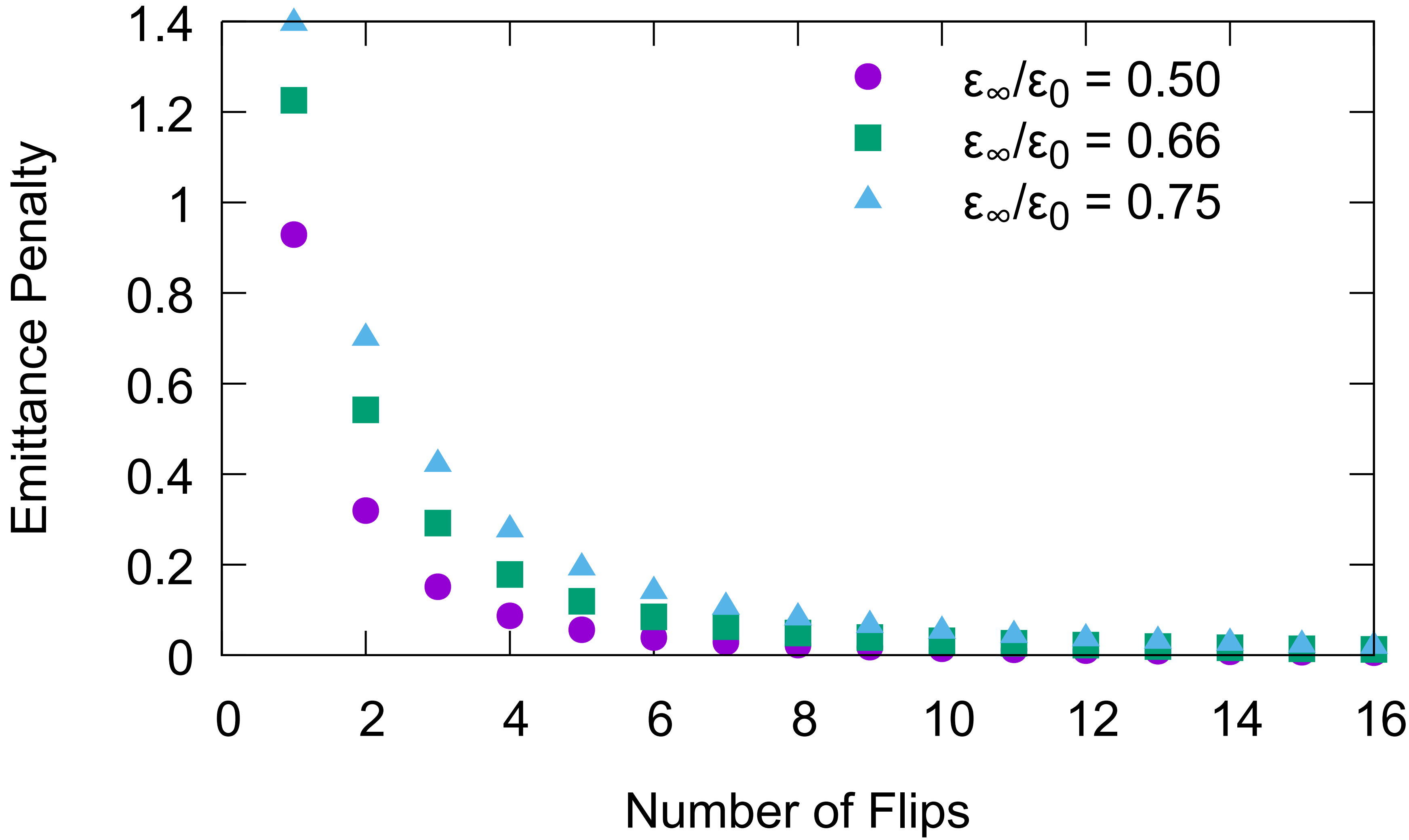
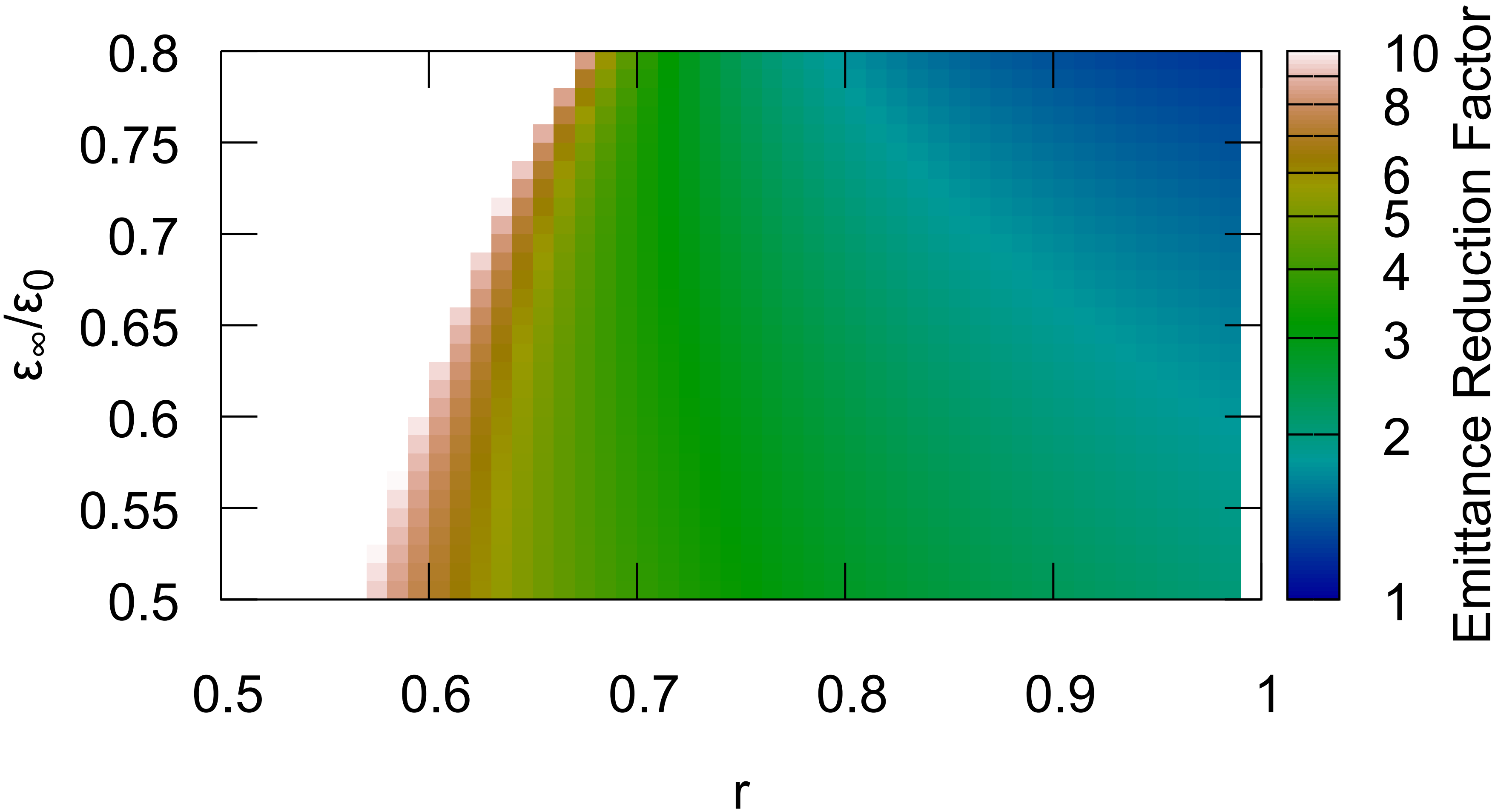
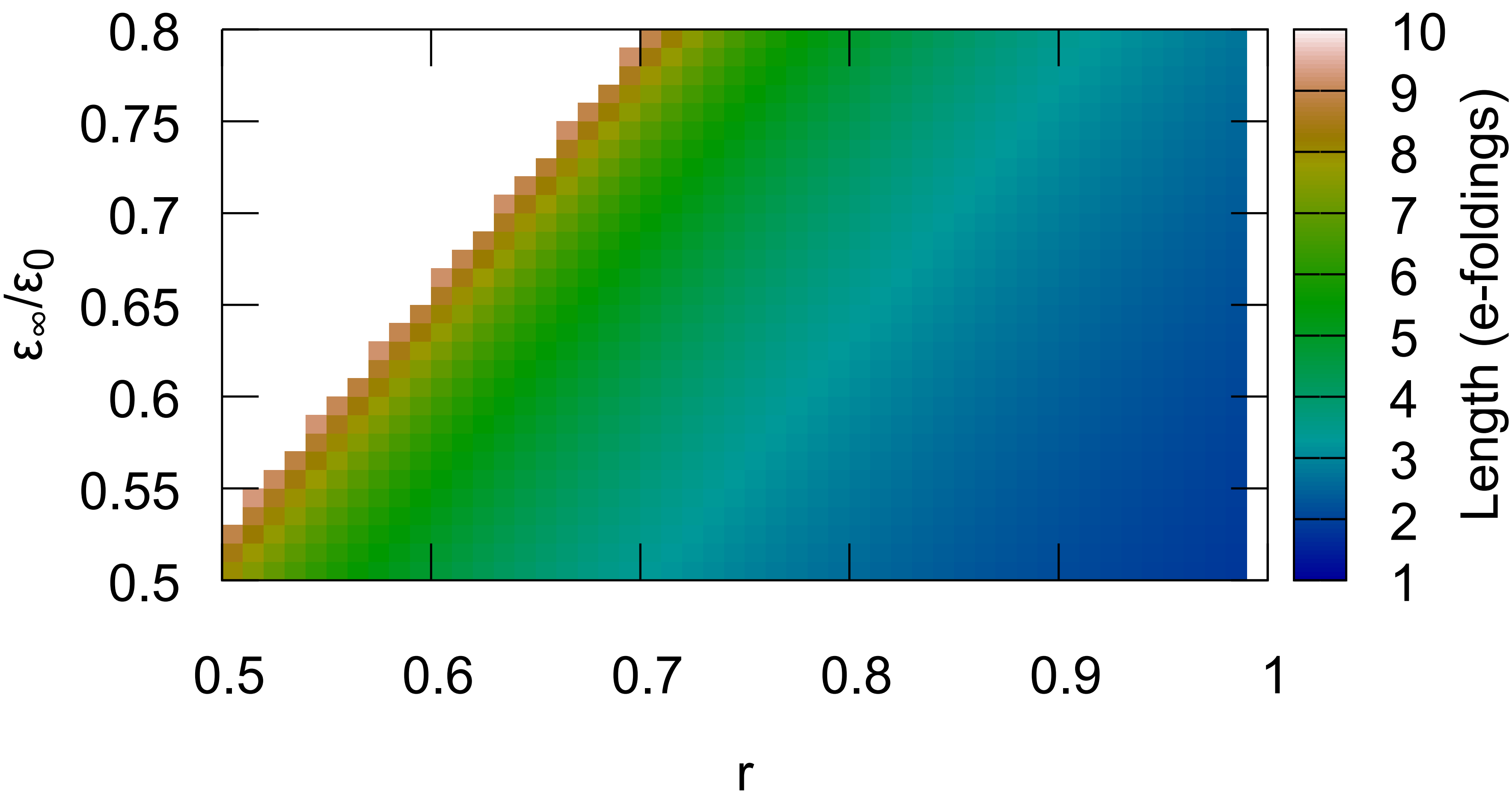
Tapered Channel, $r = 1$



No Flip: Emittance Gain



No Flip: Channel Length



Conclusions

- Can get some analytic estimates of penalty for infrequent flips
- Don't need to flip that often
  - The flip itself may have a penalty
- If a emittance ratio is desired (for flat beams), clear advantages to doing this with low beta at absorbers. Cheapest system probably has $r = 1$ and highest fields you can manage
- Should use to guide the designs of real systems
  - See what flipping does to the beam
  - See what dynamic aperture and losses look like as $\epsilon_\infty/\epsilon_0$ and r vary