4-D Behavior of Balbekov Lattice

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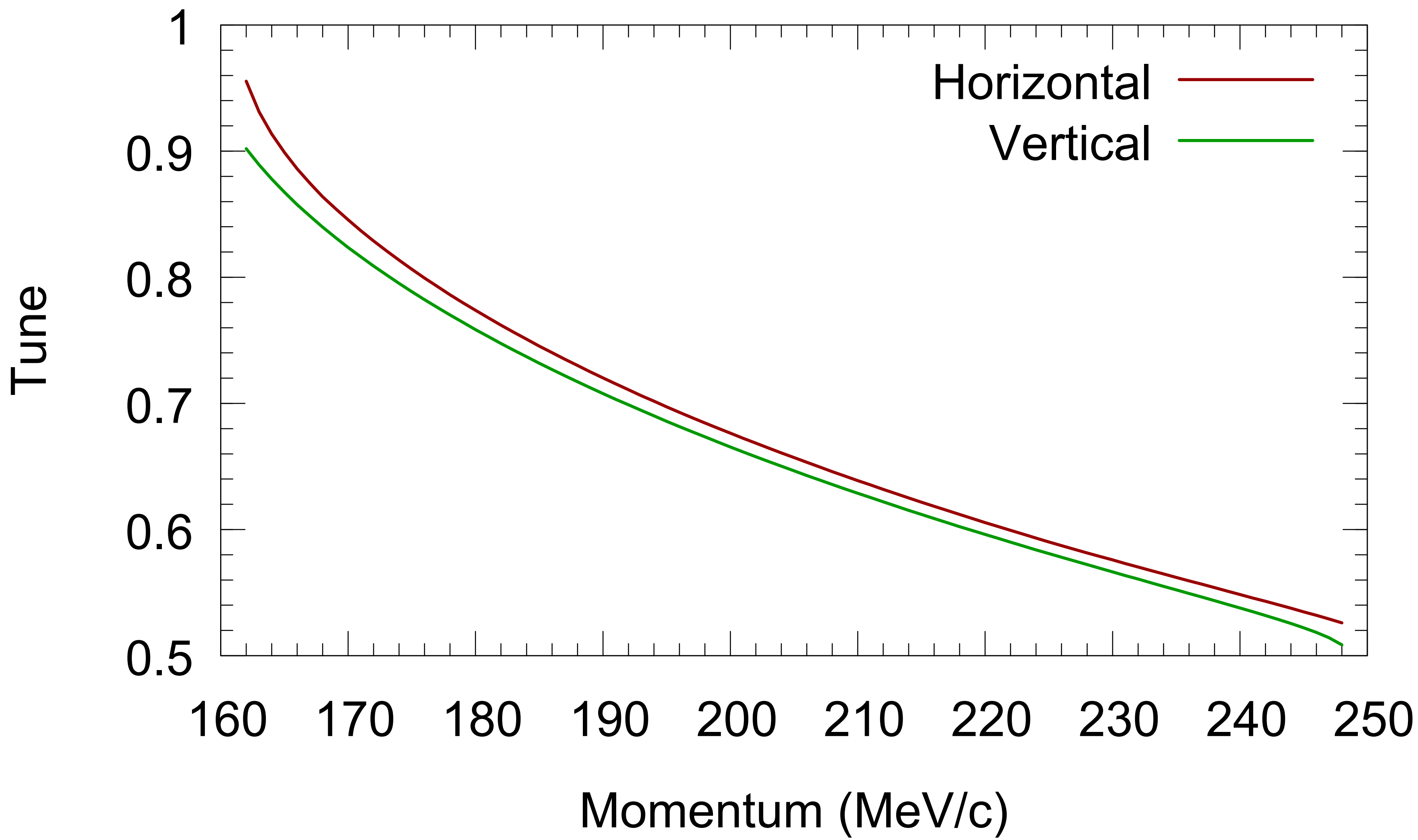
Muon Accelerator Group Meeting
16 May 2013

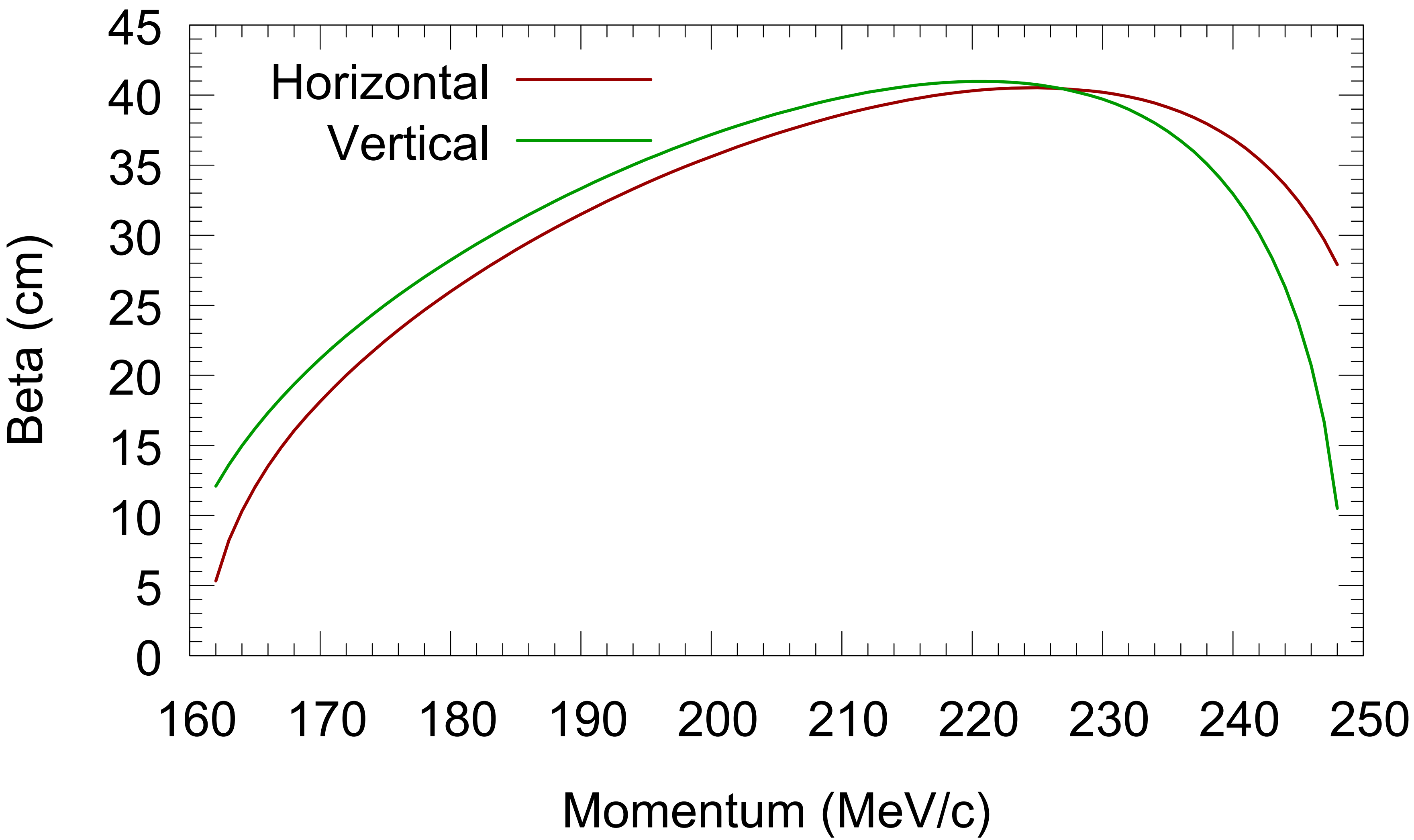
Motivation

- Understanding performance of Balbekov Ring
- Why performance worse with distribution centered at 210 MeV/c compared with 218 MeV/c
  - 210 MeV/c appears to be better for momentum acceptance
  - May be issues with missing longitudinal-transverse correlation in initial distribution

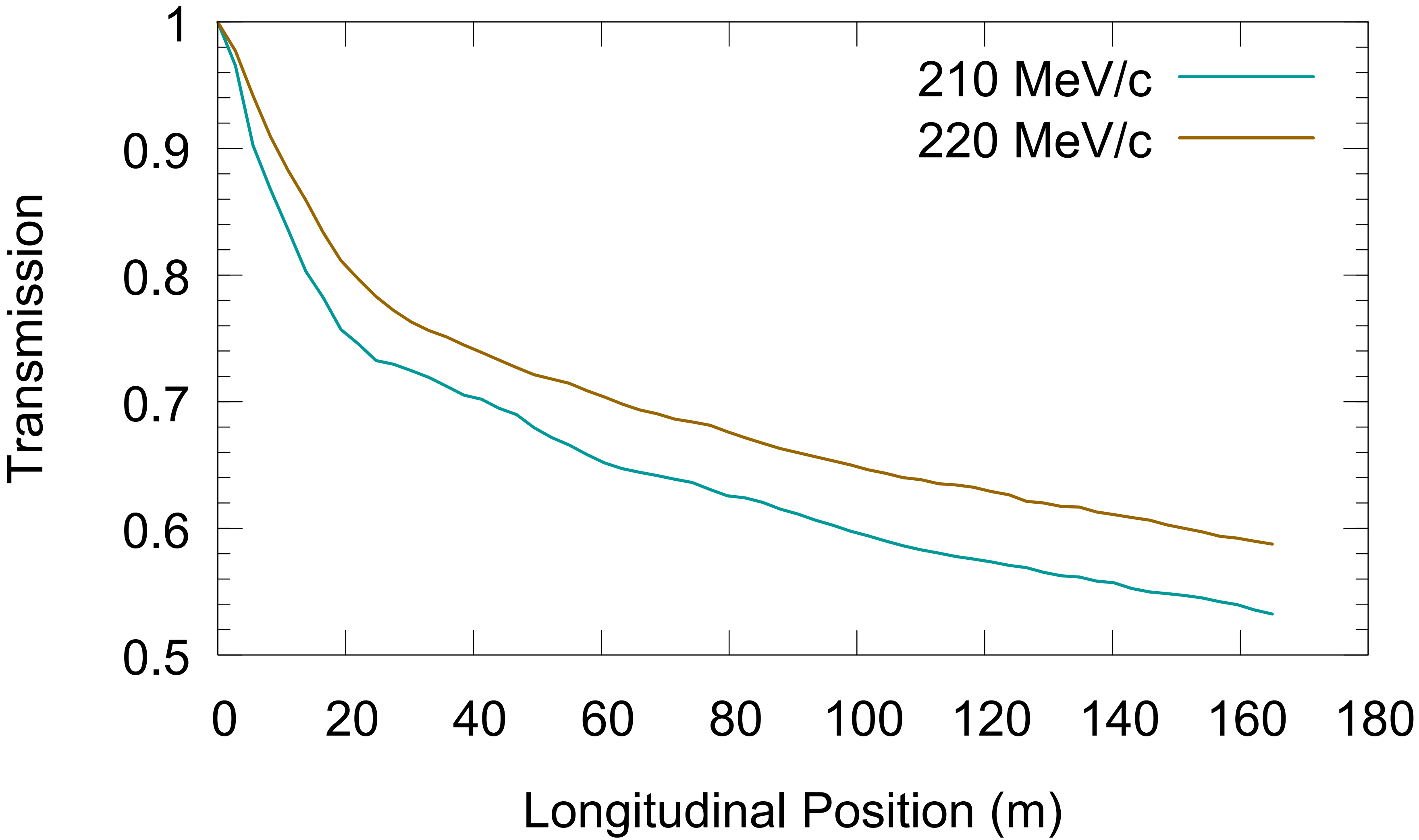
Tunes


Horizontal
Vertical
Tune
1
0.9
0.8
0.7
0.6
0.5
160
170
180
190
200
210
220
230
240
250
Momentum (MeV/c)

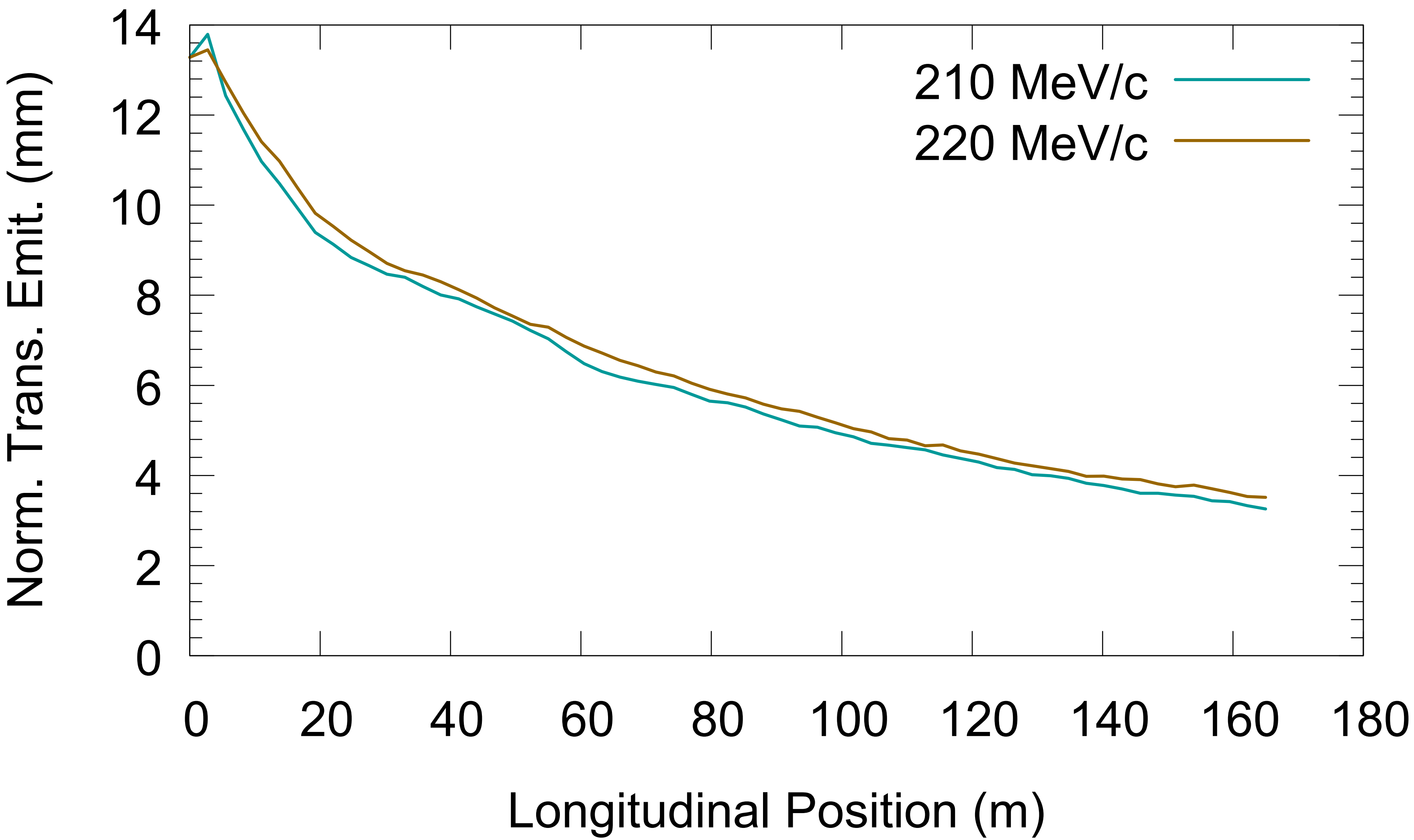
Beta Functions



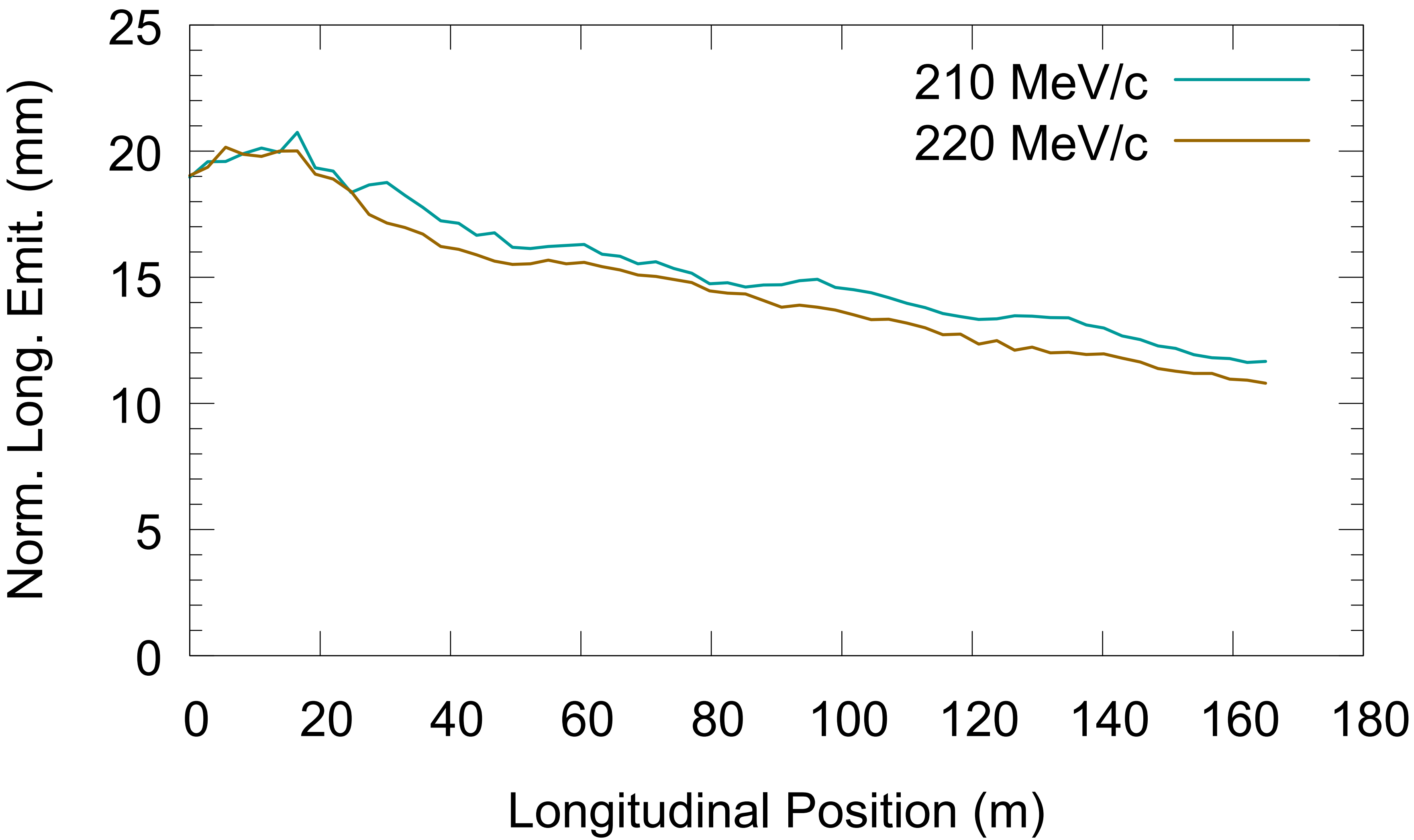
Transmission



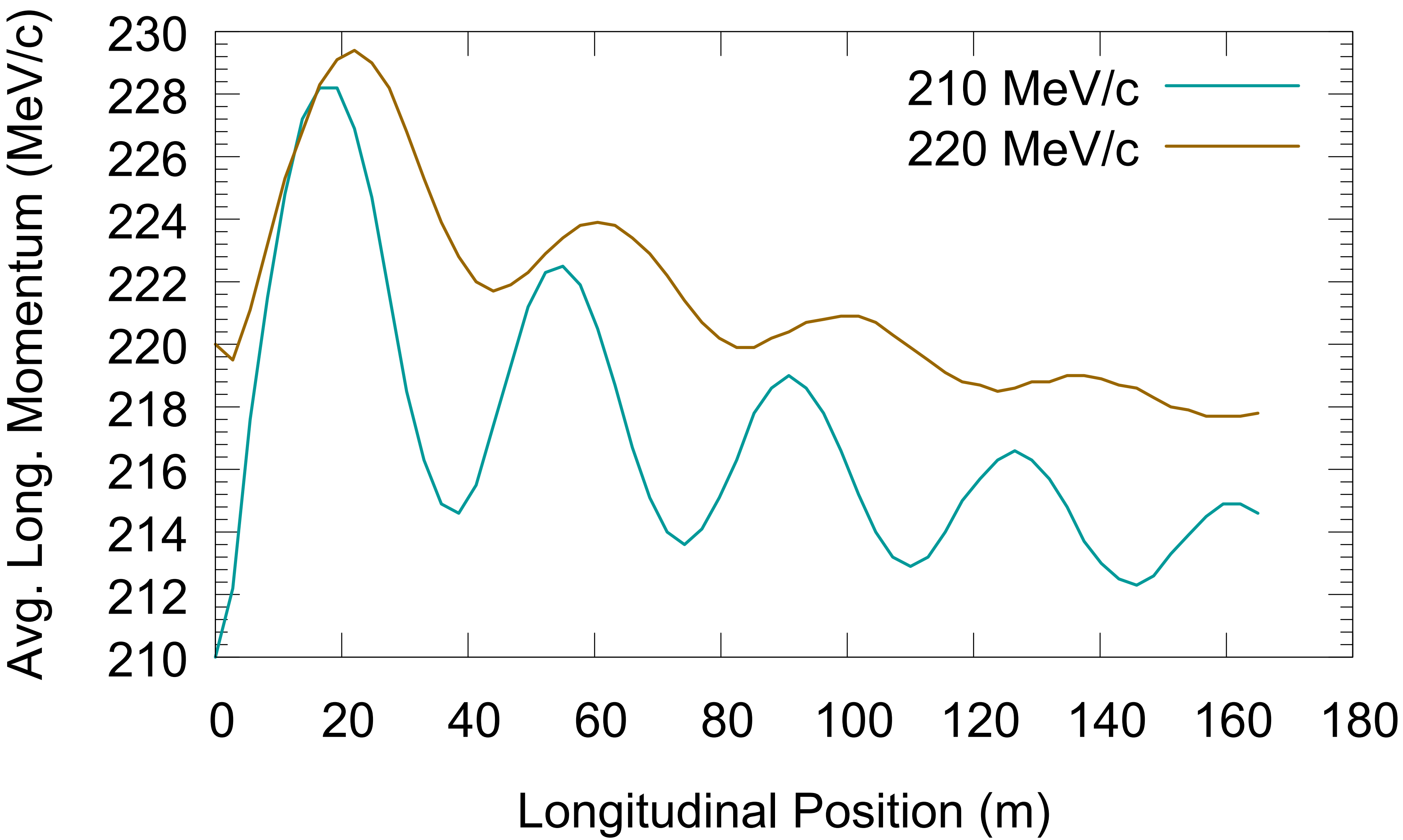
Transverse Emittance



Longitudinal Emittance



Longitudinal Momentum



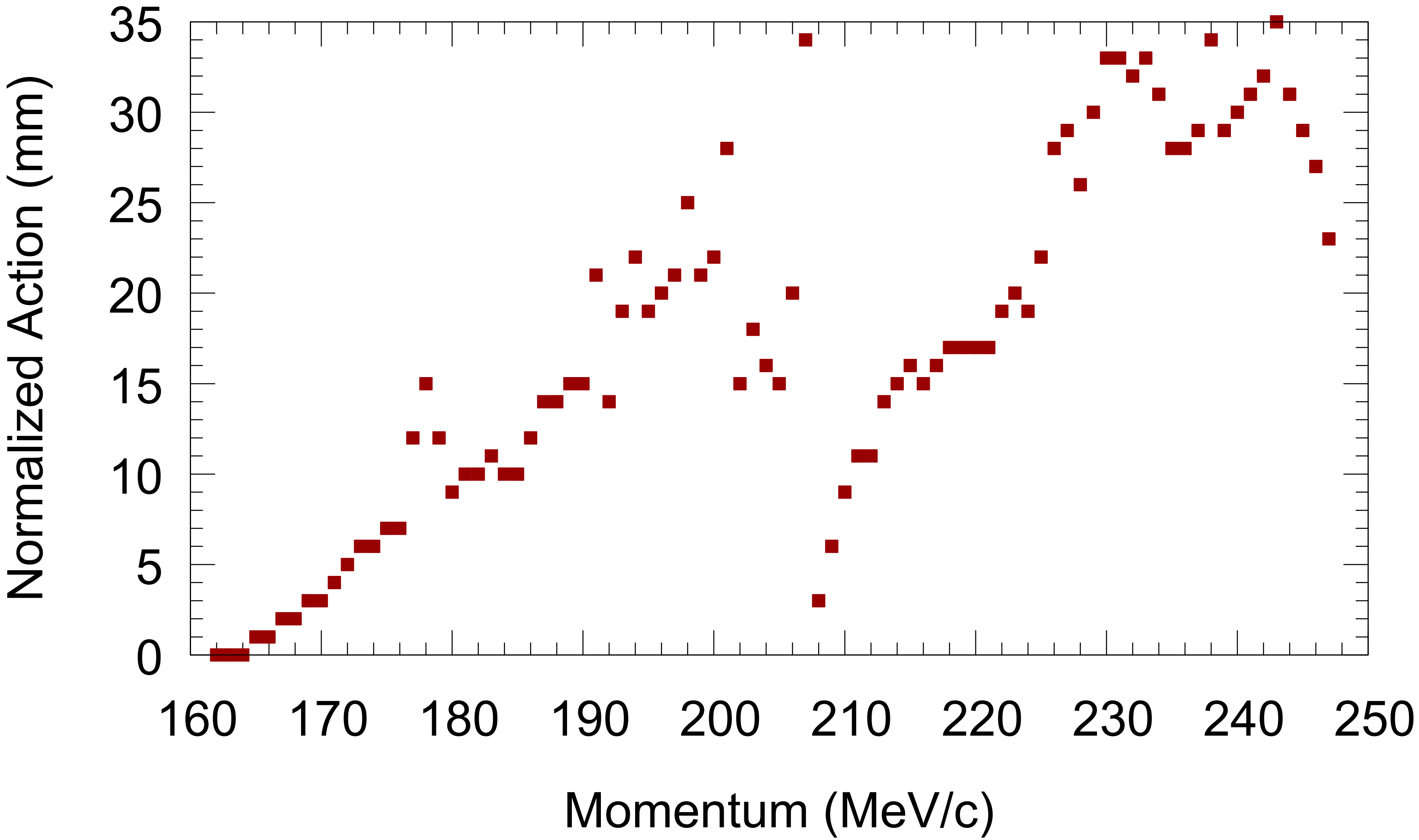
Analysis

- With each distribution, optimized
  - Longitudinal aspect ratio (fixed longitudinal emittance)
  - Cavity timing (ICOOL reference momentum)
  - RF phase
- Optimized for transmission
- Transmission worse at 210 MeV/*c* than 220 MeV/*c*
- Similar emittances for both
- Longitudinal oscillation
  - Initial mismatch arising from missing longitudinal-transverse correlation
  - Can eliminate oscillation (but still mismatch), but not optimal

Dynamic Aperture

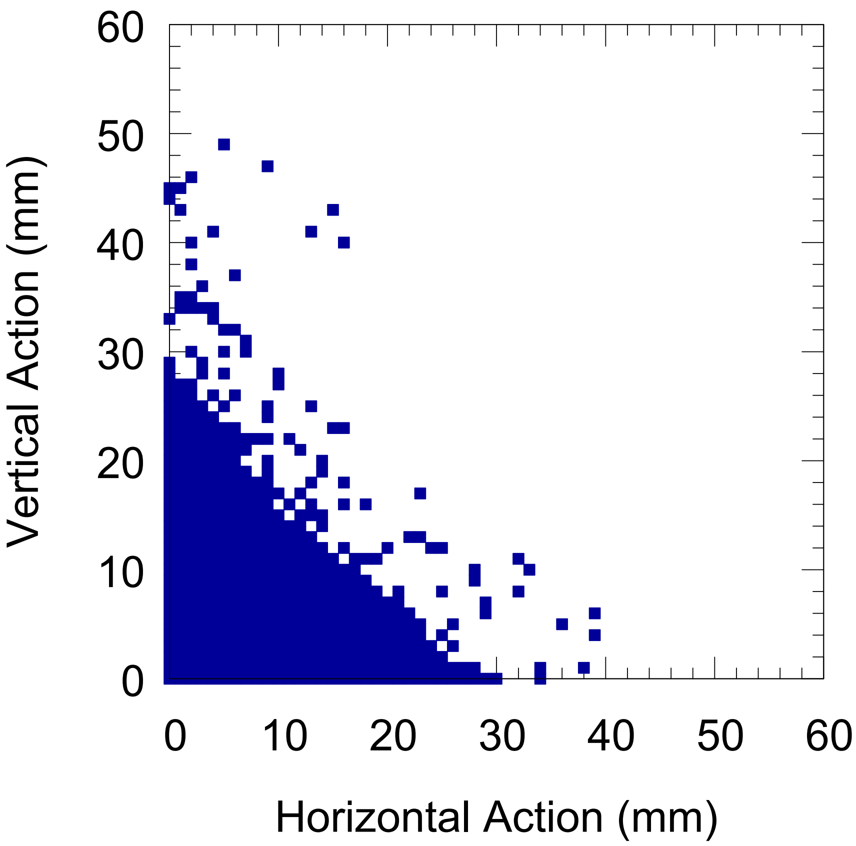
- Examine dynamic aperture over momentum range
- Reduction near integer resonance expected
- Hole near 210 MeV/*c*
- Around 2/3, but not exactly
- Dynamic apertures not very large in general
- Need to analyze more
  - Determine cause of the loss near 210 MeV/*c*
  - Compare to RFOFO

Dynamic Aperture

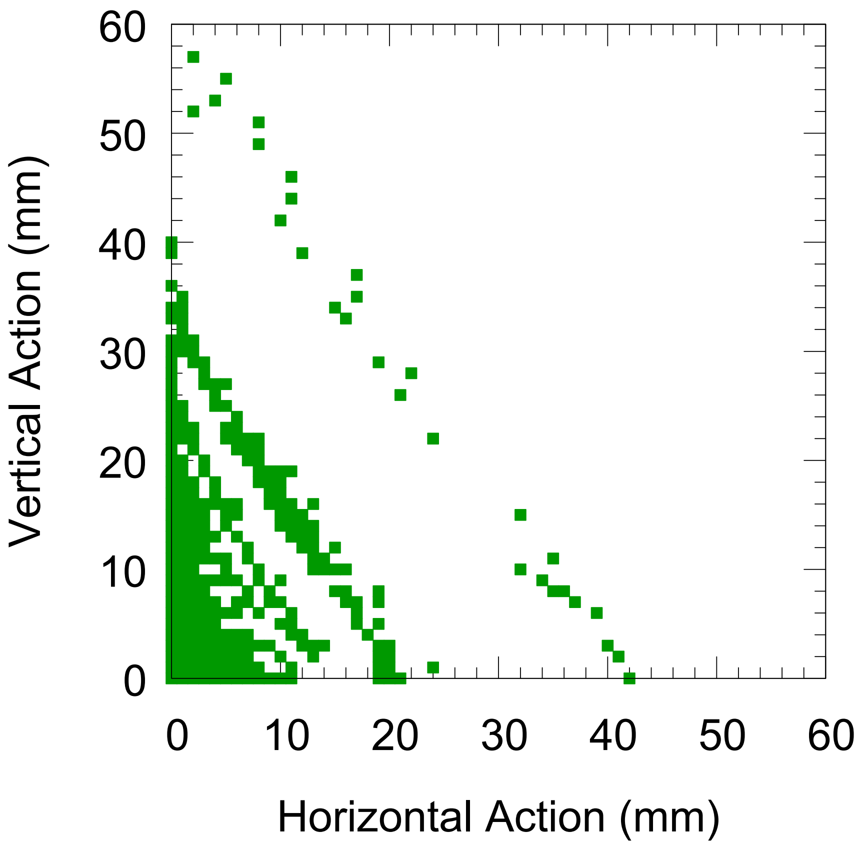


Dynamic Aperture

198 MeV/*c*

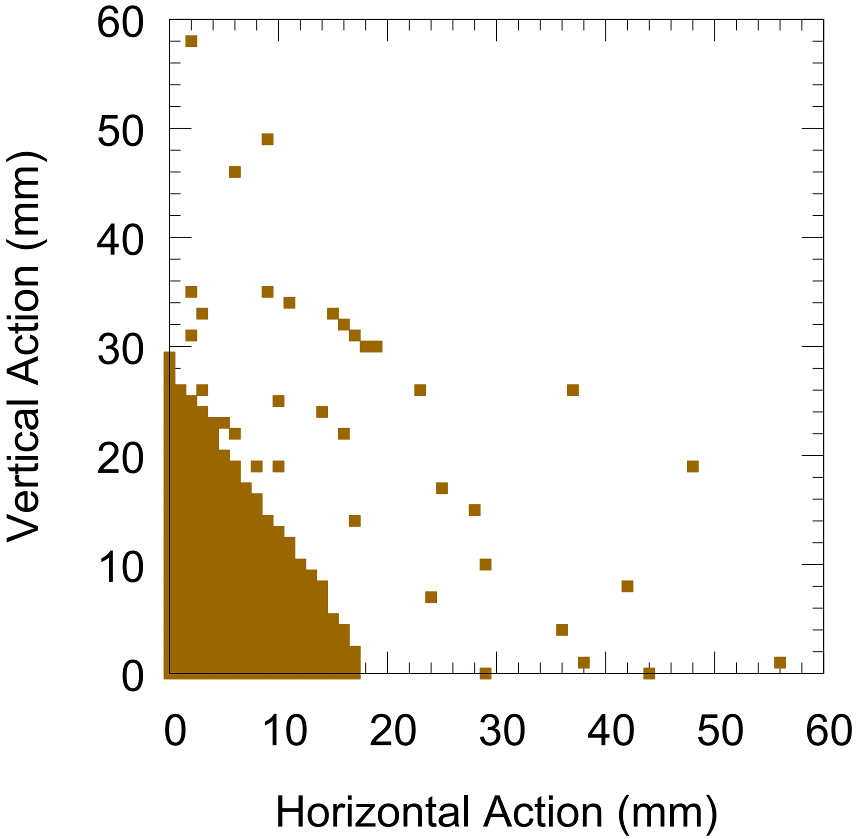


210 MeV/*c*

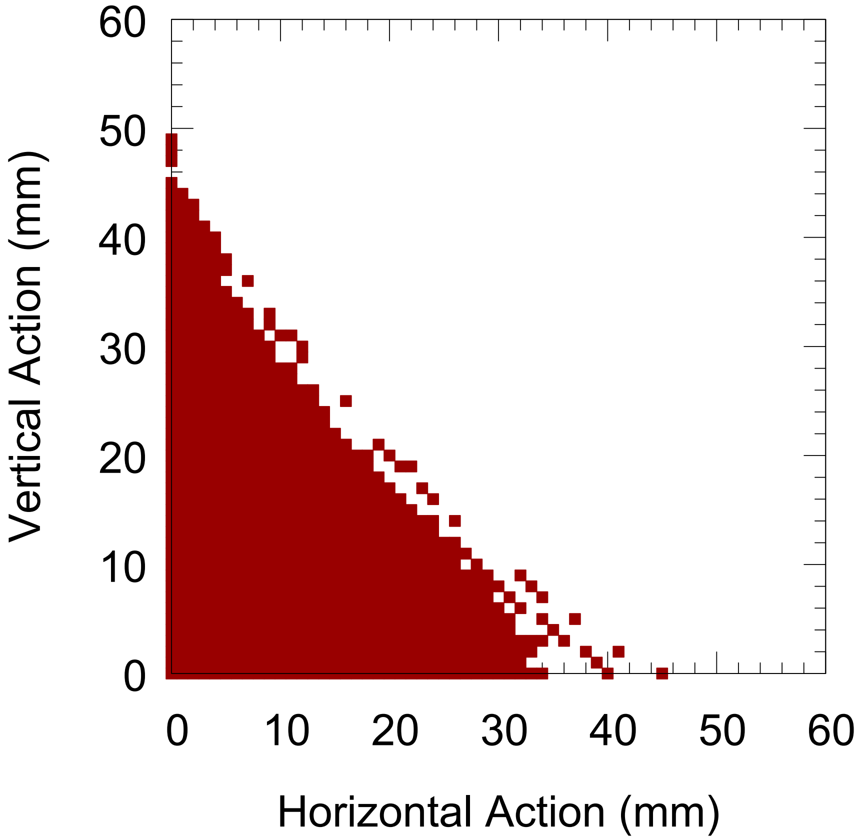


Dynamic Aperture

220 MeV/*c*



231 MeV/*c*



Math

- Find closed orbit at each momentum
- Find linear map M about closed orbit
- Find matrix A (for stable momenta) such that

$$MA = AR \qquad R = \begin{bmatrix} \lambda_1 R_1 & 0 \\ 0 & \lambda_2 R_2 \end{bmatrix}$$

$$R_k = \begin{bmatrix} \cos 2\pi\nu_k & \sin 2\pi\nu_k \\ -\sin 2\pi\nu_k & \cos 2\pi\nu_k \end{bmatrix}$$

- If M symplectic:
  - A can be chosen to be symplectic
  - $\lambda_k = 1$

Math

- Columns of A

$$\begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ a_{1R} & a_{1I} & a_{2R} & a_{2I} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

- For each k: scale a_{kR} and a_{kI} by same factor, and possibly change sign of a_{kI} so that $a_{1R}^T J a_{1I} = 1$
  - Sign change in a_{kI} requires sign change in ν_k
  - Eliminates sign ambiguity in tunes

Math

- Avoid swapping 1 and 2 as you vary parameter (momentum):
  - Swap 1 and 2 so that $A_i^T J A_j \approx J$ where A_i and A_{i+1} are nearby parameter values
  - Choose "horizontal" and "vertical" with $A_0 = I$
- Dynamic aperture
  - Launch particles with coordinates $\sqrt{2J_k}a_{kR}$
  - Dynamic aperture is largest J such that nothing lost for $J_1 + J_2 \leqslant J$
  - Emittance would be average of J_k over distribution